Heavy quark energy loss and electromagnetic response in strongly coupled super-Yang-Mills theory

Lessons from AdS/CFT

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work done with

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Problem: Understand real-time response in thermal QCD at $T \gtrsim T_c$

– Momentum fluctuations

– Electromagnetic fluctuations

– Heavy particle moving through the medium

Address these problems not in QCD, but in another gauge theory:

strongly coupled $SU(N_{\rm c})$, $\mathcal{N}{=}4$ supersymmetric Yang-Mills theory (SYM)

Why study SYM instead of QCD?

- SYM is *simpler* than QCD
- SYM is an example of a system which can be *very strongly coupled*, but still have an *ideal-gas* equation of state $\epsilon = 3P$
- At scales $\gg 1/T$, strongly coupled SYM behaves as a near-ideal fluid
- Strongly coupled SYM is a 4d gauge theory where analytic results can be derived at strong coupling (using AdS/CFT) in *real time* and at *non-zero chemical potential*

AdS/CFT correspondence FAQ

What is AdS/CFT correspondence?

A1: Exact equivalence between a field theory and a string theory

A2: A tool to perform calculations in quantum field theory

When are AdS/CFT methods useful?

When the field theory is strongly coupled, and perturbation theory is invalid. In this sense, AdS/CFT is similar to lattice

Does this mean AdS/CFT is Euclidean?

No. Both real-time and $\mu \neq 0$ are naturally built into AdS/CFT

To what field theories can AdS/CFT be applied?

Typically conformal in the UV and large N_c; various degree of susy

How seriously should one take what AdS/CFT says about field theory?

Pretty seriously, even though the equivalence is not rigorously proven. For problems such as strongly coupled real-time dynamics at $T \neq 0$, AdS/CFT is *the only* analytical tool.

AdS/CFT correspondence FAQ (cont.)

Can QCD be analyzed by AdS/CFT methods?

No (at least at the moment) — duality is not developed for AF theories

If QCD can not be addressed, why should one care about AdS/CFT?

- Field theories which can be analyzed within AdS/CFT are close cousins of QCD. AdS/CFT can treat field theories which exhibit confinement, mass gap, global symmetries, Goldstone bosons, and χSB.

- For theories which can be treated by AdS/CFT methods, analytic results can be derived at *both* strong *and* weak coupling. Thus one can quantitatively compare perturbative and non-perturbative regimes.

- One can try to use AdS/CFT-based models to analyze parts of QCD dynamics which are not sensitive to short-distance physics

- Many important questions in QCD can not be treated by either perturbation theory or lattice methods. For such questions, it is valuable to gain insight from QCD-related models, which can be analyzed by AdS/CFT.

Simplest field theory whose strong coupling regime can be treated by AdS/CFT:

$\mathcal{N}$=4 supersymmetric Yang-Mills theory, $SU(N_c)$, $\lambda = g^2 N_c$

(in the limit of large N_c and large λ)

What is $\mathcal{N}$=4 SYM?

- Gauge fields + 4 fermions + 6 scalars in adjoint of $SU(N_c)$
- Conformal theory, λ is a tunable parameter (does not run)
- Supersymmetric, but SUSY not essential at finite temperature
- $\epsilon = 3P$, $v_s = \frac{1}{\sqrt{3}}$, $\zeta = 0$, at any non-zero temperature
- ϵ, P, η are finite in the limit $\lambda \to \infty$
- $\dfrac{\eta}{s} = \dfrac{1}{4\pi}$ in the limit $\lambda \to \infty$ (P.K., D.Son, A.Starinets, hep-th/0309213)

Use strongly coupled $\mathcal{N}$=4 SYM medium at $T \neq 0$ as a model of strongly interacting QCD medium at $T \neq 0$

In this talk will discuss:

- Heavy quark moving through strongly coupled $\mathcal{N}=4$ SYM (in thermal equilibrium)

  C.Herzog, A.Karch, P.K., C.Kozcaz, L.Yaffe, hep-th/0605158;

  Related:

  J. Casalderrey-Solana, D. Teaney, hep-ph/0605199,

  H. Liu, K. Rajagopal, U. Wiedemann, hep-ph/0605178 $\Leftarrow$ Dust not settled

  J.Friess, S.Gubser, G.Michalogiorgakis, hep-th/0605292 $\Leftarrow$ Dust not settled

- Photon emission from strongly coupled $\mathcal{N}=4$ SYM (in thermal equilibrium)

  P.K., G.Moore, A.Starinets, to appear

Heavy probe energy loss

(C.Herzog, A.Karch, P.K., C.Kozcaz, L.Yaffe, `hep-th/0605158`)

Setup: Particle of mass M ("probe"), moving through a *strongly* interacting thermal medium ($\lambda \to \infty$) with temperature $T \ll M$

Questions:

What is meant by "energy" of a probe?

How localized is the probe?

How to separate energy of the probe from the energy of the medium?

How to define energy loss?

What is the physical mechanism of the energy loss?

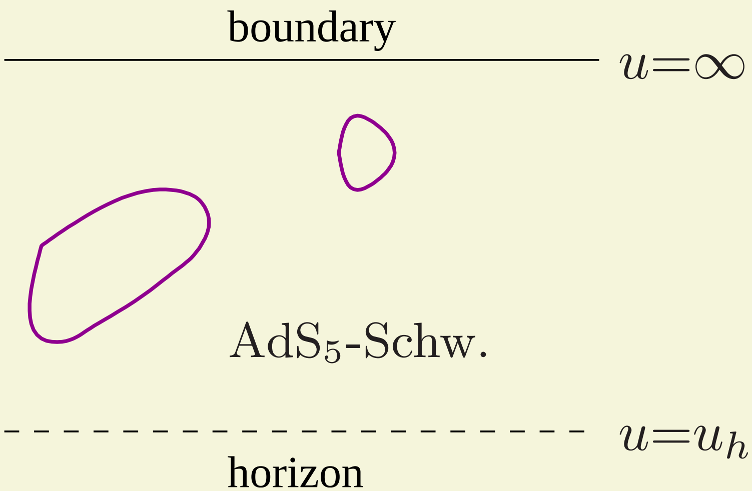
How does energy loss depend on mass, coupling, temperature?

These are non-trivial questions (keep in mind that perturbative language such as "bremsstrahlung", "quark-gluon scattering" etc is useless when coupling is strong)

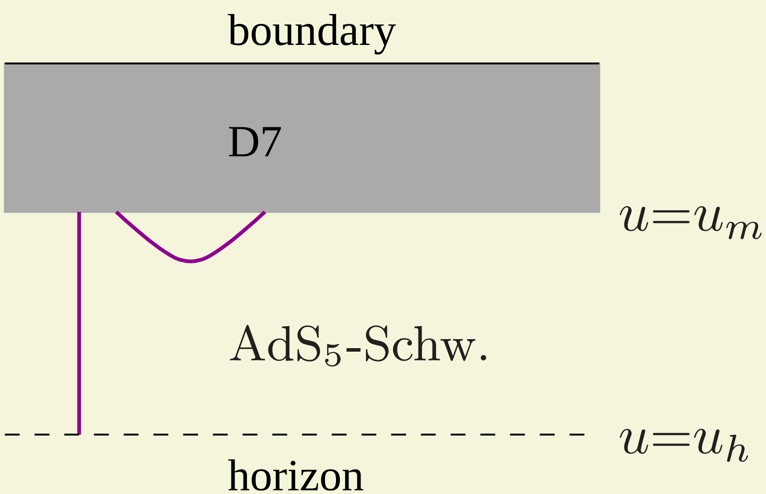
$\therefore$

| it is helpful to look at tractable models where these questions can be addressed |

Model: $\mathcal{N}=4$ SYM + fundamental matter $N_\mathrm{f} \ll N_\mathrm{c}$



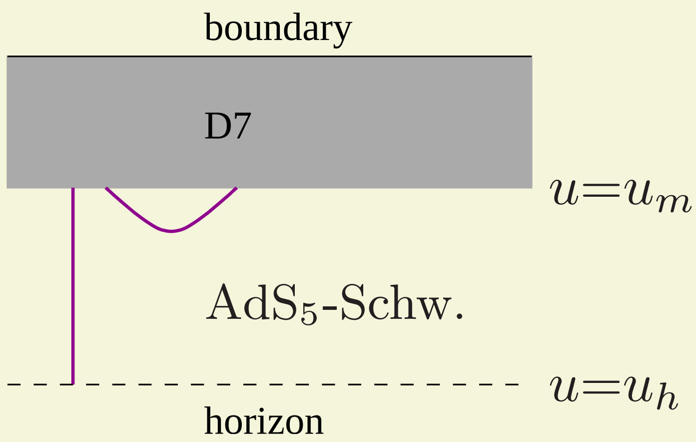
- Adjoint d.o.f. ⇔ closed strings (gravity) in the bulk
- Quantum fluctuations of SYM fields $(T_{\mu\nu}, J_\mu)$ ⇔ classical fluctuations of fields $(g_{\mu\nu}, A_\mu)$ in the bulk
- Finite temperature ⇔ black hole (brane) in the bulk



- When add fundamental d.o.f. ⇔ open strings ending on D7 brane in the bulk (A.Karch, A.Katz, hep-th/0205236)
- "Quark" mass set by u_m
- Quark configuration ⇔ classical string

Energy/momentum of a quark = energy/momentum of a classical string

How heavy a quark can one treat this way?



Static string: $E = \frac{\sqrt{\lambda}}{2\pi} u_m, \quad T = 0$

$$E = \frac{\sqrt{\lambda}}{2\pi}(u_m - u_h), \quad T \neq 0$$

thermal mass

$$E_{\text{rest}} = m - \frac{1}{2}\sqrt{\lambda} T + \ldots$$

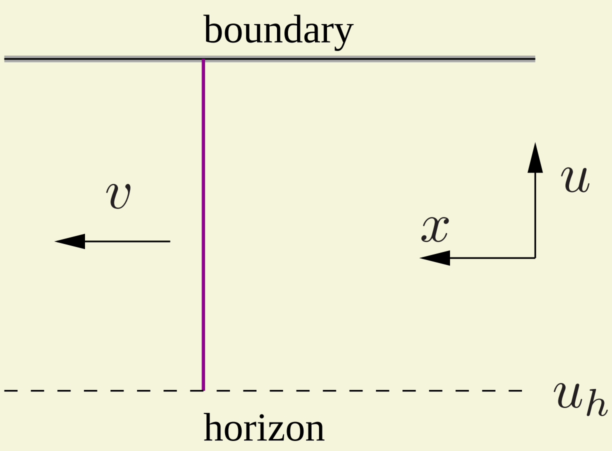
$\Delta m(T)$

When $m > 0.92\,\Delta m(T)$ — D7 above the horizon, string is classical

To analyze energy loss of a heavy quark ($m > \frac{1}{2}\sqrt{\lambda}T$), solve classical equations of motion for a moving string

Stationary analytic solution

Take infinitely heavy "quark" ($u_m \to \infty$), move it at constant speed

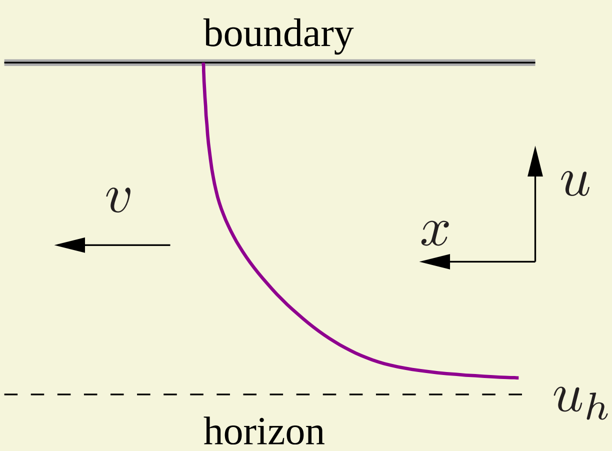


String profile: $x(u,t) = vt$ solves e.o.m.

however:

$-g$ flips sign at $u^4 = \frac{u_h^4}{1-v^2} > u_h^4$

E, P become complex — solution unphysical



$x(u,t) = X(u) + vt$

Can find $X(u)$ s.t. $-g$ is positive everywhere

Source moves at constant speed

Momentum pumped in at the boundary

Momentum leaks off at the horizon

$$\frac{dP}{dt} = -\pi_x^1\Big|_{u=u_h} = -\frac{\sqrt{\lambda}}{2\pi}\frac{v}{\sqrt{1-v^2}}(\pi T)^2 = -\left(\frac{\sqrt{\lambda}T^2\pi}{2M}\right)\left(\frac{Mv}{\sqrt{1-v^2}}\right) = -\mu P$$

Note that friction coefficient μ has a finite $\lambda \to \infty$ limit

Comments

- The same solution $\frac{dP}{dt} = -\frac{\sqrt{\lambda}}{2\pi}\frac{v}{\sqrt{1-v^2}}(\pi T)^2$ is valid for *finite* mass quarks as well; corresponds to external force acting on the probe
- Heavy quark is dual to the *whole string*, not just the endpoint
- The dragging string tail describes the disturbance wake produced by a moving projectile (energy distribution in the wake not calculated yet)
- For the stationary moving string, dE/dt, dP/dt are *finite*, but E and P are *infinite* (from near-horizon behavior). This is due to infinite energy input from the source which has been dragging the quark for infinite time
- To define finite energy/momentum of the probe, this infinity must be cut off and subtracted
- The heavy quark is a quasiparticle whose width is set by μ. Drag constant determines the precision with which the quark energy can be defined
- The coupling of the probe to the surrounding medium is *strong*: energy loss is *not* due to gluon radiation or collisions. In fact, there is no simple energy loss mechanism based on interactions with perturbative quanta — there are no any

Production of real and virtual photons

(e.g. L.D.McLerran, T.Toimela, PRD **31**, 545 (1985), H.A.Weldon, PRD **42**, 2384 (1990))

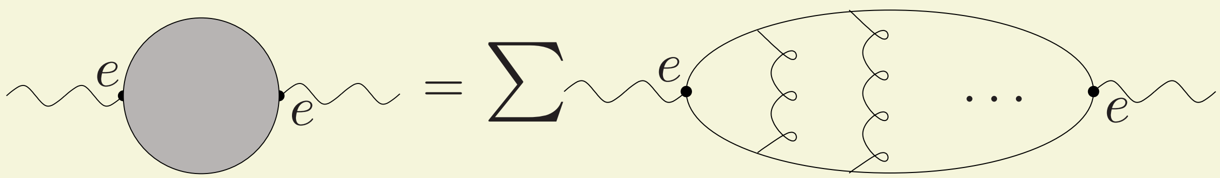
Γ — number of photons per unit time per unit volume

Photon interaction: $e\, J^{\rm EM}_\mu A^\mu$; el. charge e small $\Rightarrow$ photons do not thermalize

$$d\Gamma = \frac{d^3k}{(2\pi)^3}\frac{e^2}{2|\boldsymbol{k}|}\,\eta^{\mu\nu}C^<_{\mu\nu}(k)\Big|_{\text{lightlike }k}$$ where $C^<_{\mu\nu}(x) = \langle J^{\rm EM}_\mu(0)J^{\rm EM}_\nu(x)\rangle$

Virtual photon can decay into a lepton pair: $d\Gamma = \dfrac{d^4k}{(2\pi)^4}\dfrac{e^2 e_\ell^2}{6\pi k^2}\eta^{\mu\nu}C^<_{\mu\nu}(k)\Big|_{\text{timelike }k}$

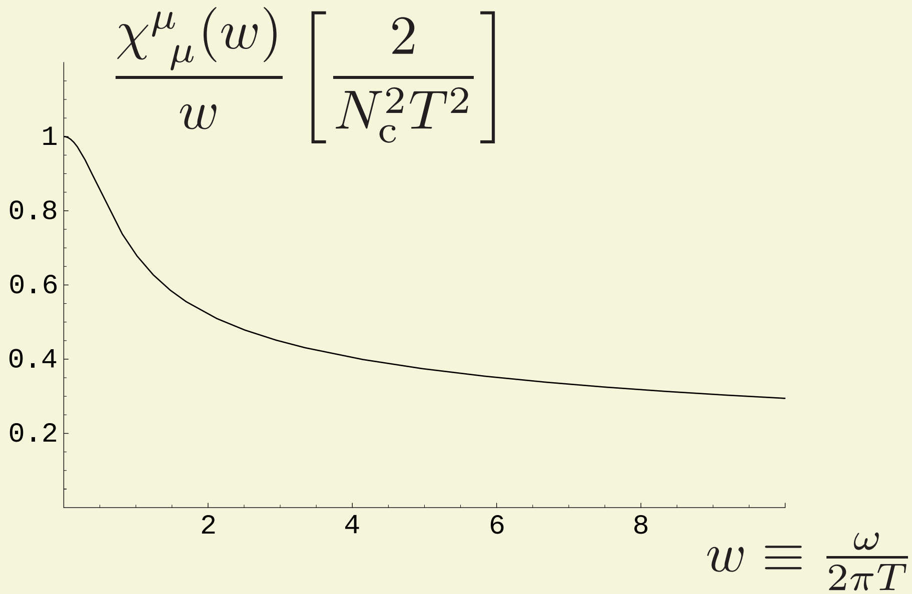
Emission spectra are determined by EM current-current spectral function



true to leading order in e, but to all orders in g

AdS/CFT allows one to evaluate the whole "blob" at large $\lambda \equiv g^2 N_{\rm c}$ in SYM

On-shell photons at strong coupling



$$\chi^{\mu}{}_{\mu}(\omega) = -2\,\mathrm{Im}\,\eta^{\mu\nu}C^{\mathrm{RET}}_{\mu\nu}(\omega, q)\Big|_{\omega=q}$$

Small frequency:

$\chi^{\mu}{}_{\mu}(w) \sim \frac{1}{2}N_c^2T^2w$, in accord with hydro

High frequency:

$$\chi^{\mu}{}_{\mu}(w) \sim \frac{N_c^2T^2}{4}w^{2/3}\left[3^{5/6}\frac{\Gamma(2/3)}{\Gamma(1/3)}\right]$$

Spectral function for on-shell photons can be computed in closed form!

$$\chi^{\mu}{}_{\mu}(w) = \frac{N_c^2T^2w}{8}\left|{}_2F_1\left(1-\frac{1}{2}(1+i)w, 1+\frac{1}{2}(1-i)w; 1-iw; -1\right)\right|^{-2}$$

Emission rate is finite and λ-independent in the limit of large λ

Electric conductivity

Kubo formula: $\sigma = e^2 \lim_{\omega \to 0} \dfrac{\chi_{ii}(\omega, q{=}0)}{6\omega}$

In strongly coupled SYM: $\sigma = e^2 \dfrac{N_c^2 T}{16\pi}$, does not depend on λ

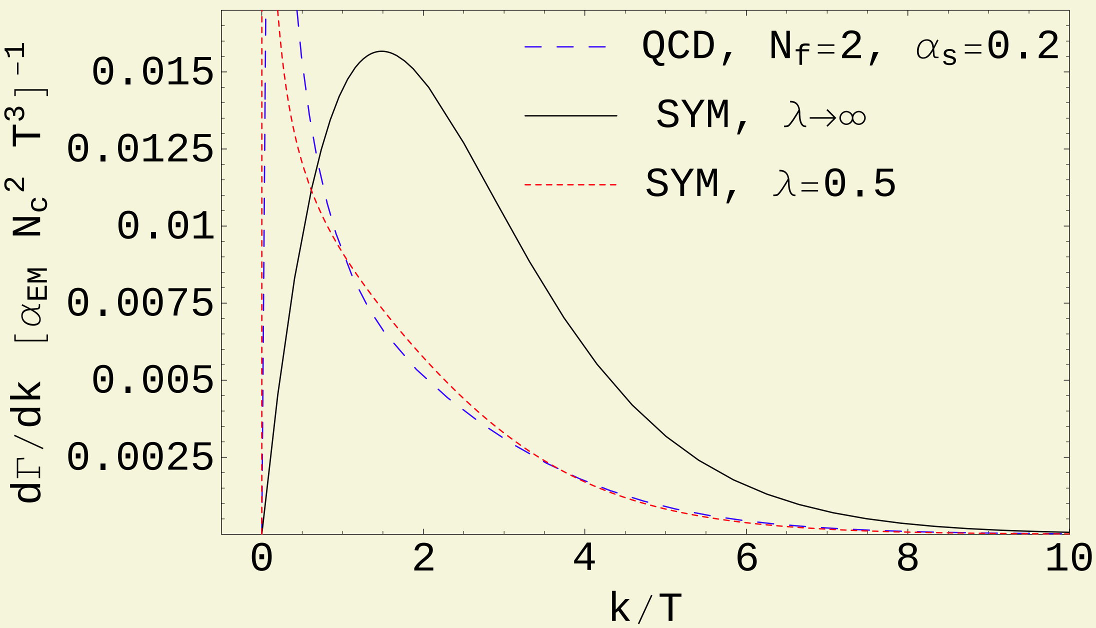
In weakly coupled SYM: $\sigma = e^2 \dfrac{N_c^2 T}{\lambda^2 \ln(1/\lambda)} C$, where $C{=}O(1)$

Normalize to the number of degrees of freedom: $\dfrac{\sigma}{e^2 \Xi}$, where Ξ is charge susceptibility. In strongly coupled SYM,

$$\boxed{\frac{\sigma}{e^2 \Xi} = \frac{1}{2\pi T}}$$

Einstein relation $\dfrac{\sigma}{e^2 \Xi} = D$ is satisfied, as it should.

Photon emission rates



Perturbative QCD rate: when $k/T \gg g_s^4 \ln(1/g_s)$: AMY, hep-ph/0111107

When $k/T \ll g_s^4 \ln(1/g_s)$: spectral function is fixed by hydro, σ computed by AMY, hep-ph/0302165

Strongly coupled SYM: (P.K., G.Moore, A.Starinets,... to appear)

– emission rate for *all* k/T is given by a simple *analytic* expression

– hydro limit is naturally reproduced when $k/T \ll 1$

– at small k/T, the rate is always *greater* in a weakly-coupled theory

– at large k/T, the rate is always *smaller* in a weakly-coupled theory

– as coupling grows, the rate becomes finite and coupling-independent

Strong coupling and weak coupling results are consistent

Conclusions

AdS/CFT allows one to study heavy quark [$m > \Delta m(T)$] moving through a strongly coupled $\mathcal{N}$=4 SYM fluid in real time

At strong coupling, energy loss is neither collisional nor radiative

Real-time spectral functions can be (relatively) easily computed in strongly coupled SYM at finite temperature

Photon emission rate in SYM is given by an explicit analytic formula; dilepton rate is easily computed by solving a simple ODE

In the limit of infinite coupling, drag coefficient, kinetic coefficients and emission rates are *finite*

These same questions about hard probes can be addressed in theories which are more similar to QCD than $\mathcal{N}$=4 SYM

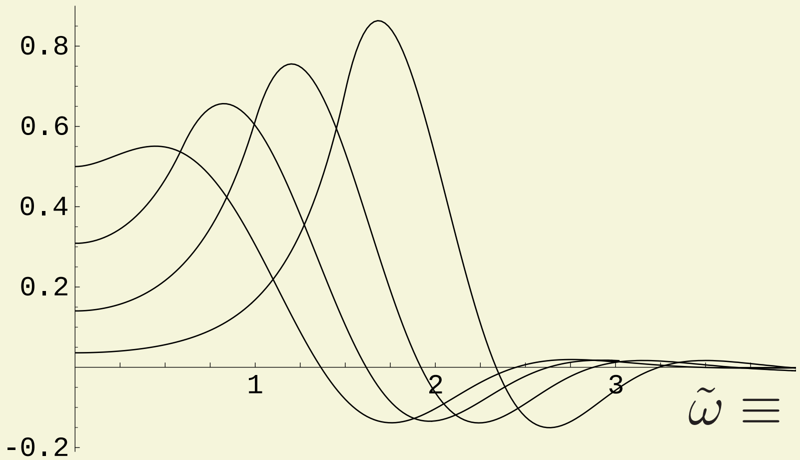
How can SYM results be useful in understanding QCD?

BACKUP SLIDES

Why SYM is a fluid

(P.K., A.Starinets, hep-th/0602059)

$$\frac{1}{\tilde{\omega}}\left(\chi(\tilde{\omega}) - \chi^{T=0}(\tilde{\omega})\right) \ \left[\frac{1}{\pi^2 N_c^2 T^4}\right]$$



$$\tilde{\omega} \equiv \frac{\omega}{2\pi T}$$

$$\chi(\omega, k) = -2\,\mathrm{Im}\,G^{\rm ret}_{xy,xy}(\omega, k)$$

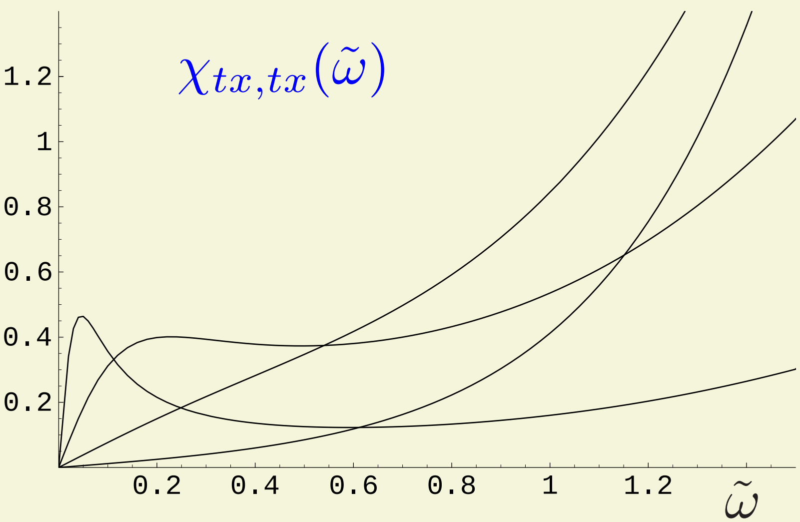
$$\chi(\omega) \sim \omega, \quad \omega \ll 2\pi T$$

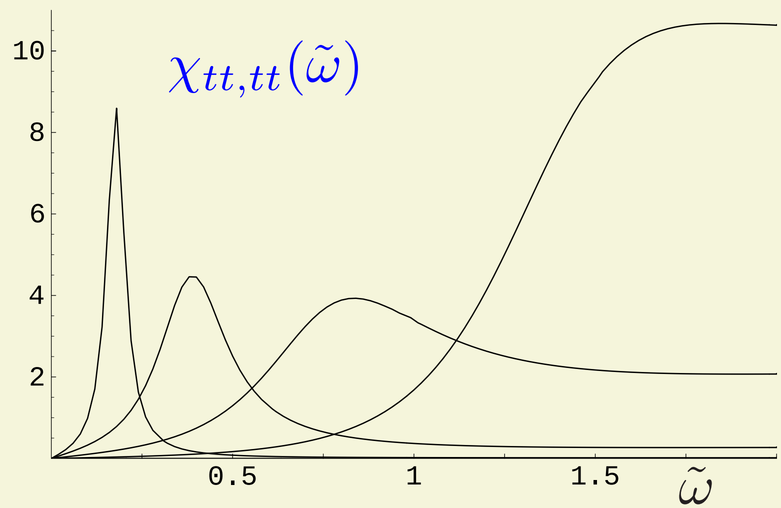
$$\chi(\omega) - \chi^{T=0}(\omega) \sim e^{-\gamma\omega}, \quad \omega \gg 2\pi T$$

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

T^3 by conformal invariance, N_c^2 counts d.o.f.

Spectral function for conserved energy-momentum



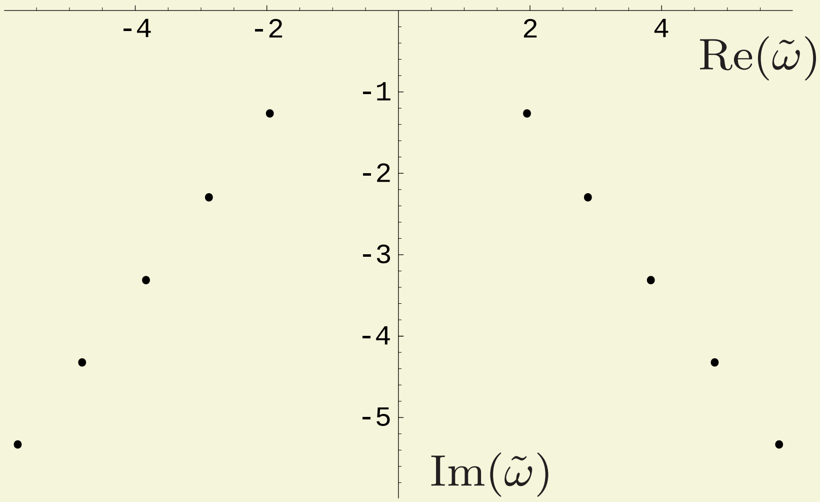


Hydrodynamic peaks are clearly visible in dual classical gravity

Singularities of $G^{\text{ret}}(\omega, k)$

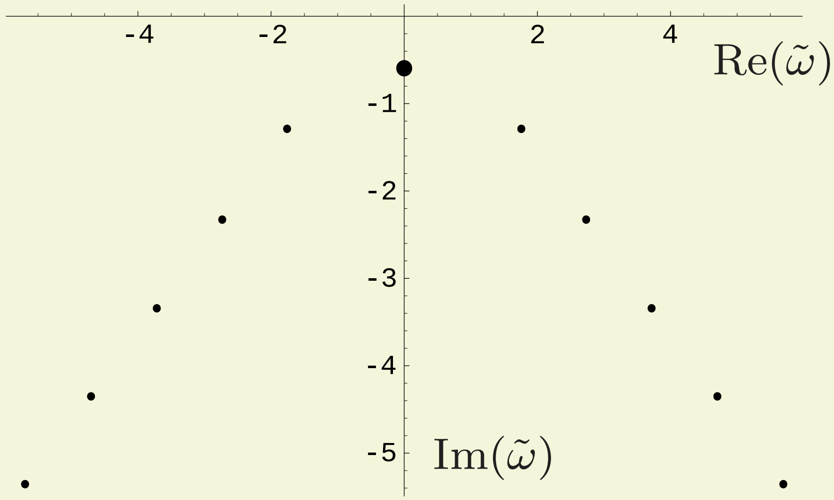
(A.Nunez, A.Starinets, hep-th/0302026, PK, A.Starinets, hep-th/0506184)

$G^{\text{ret}}_{xy,xy}(\omega)$

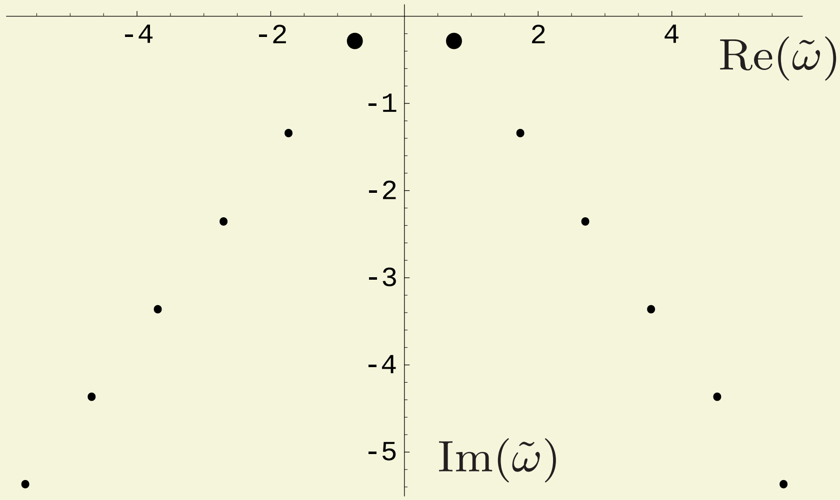


- Infinite series of poles
- $\omega_n = 2\pi n T(\pm 1 - i)$ as $n \to \infty$
- For conserved densities, $\omega_0 \to 0$ as $k \to 0$
- Hydro poles agree with Kubo formula

$G^{\text{ret}}_{tx,tx}(\omega)$

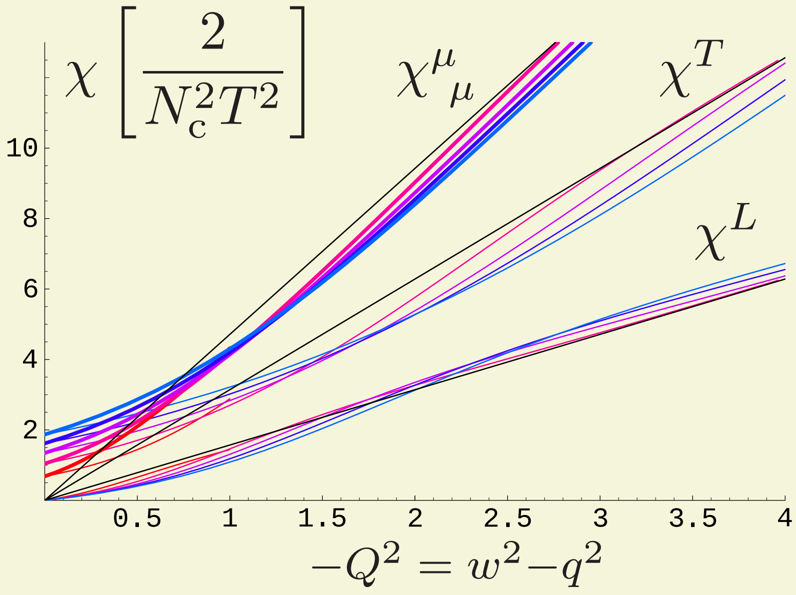


$G^{\text{ret}}_{tt,tt}(\omega)$



Singularities of $G^{\text{ret}}(\omega, k)$ are (quasi)normal modes of the dual gravity background

Dilepton spectrum at strong coupling



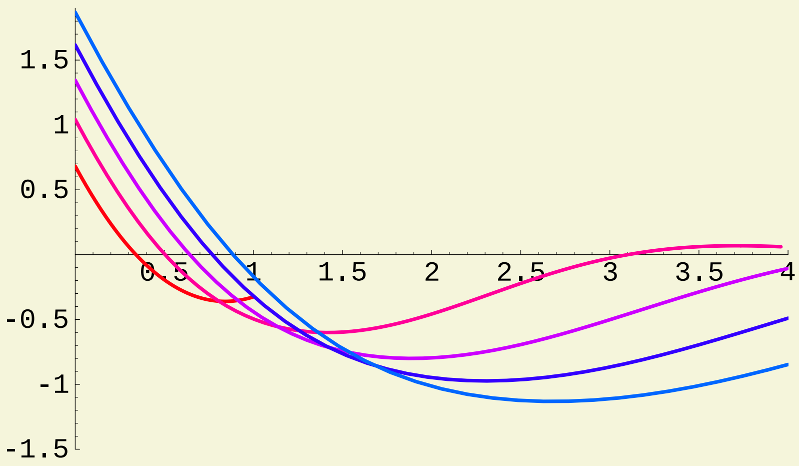
Take 3-momentum along **z**

$\chi^\mu{}_\mu(\omega, q) = \chi^T + \chi^L$, where

$\chi^T = \chi_{xx} + \chi_{yy}$, $\chi^L = -\chi_{tt} + \chi_{zz}$

Colored lines: $w \equiv \frac{\omega}{2\pi T} = 1, 2, 3, 4, 5$

Black lines: zero-temperature correlators

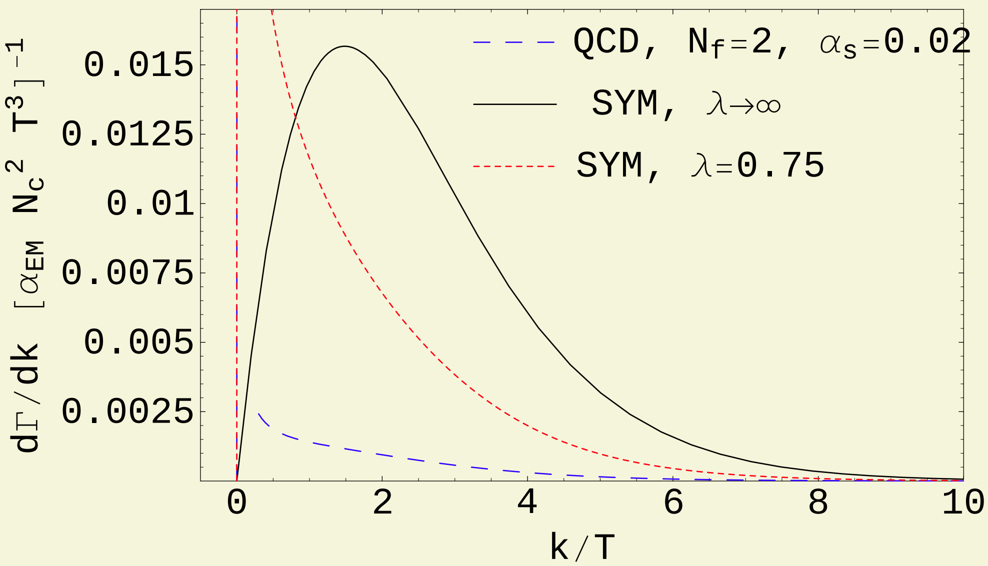


Finite-temperature contribution:

$\chi^\mu{}_\mu - \chi^\mu{}_\mu(T{=}0)$, as a function of $-Q^2$

Oscillations decay exponentially

Compare photon spectrum in QCD and SYM



At the *same value* of (small) coupling, the photon rate in SYM is greater than in QCD