Hard Proceses in Pertrubative QCD

(An introduction/historic review for pp scattering)

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Hard and Electromagnetic Probes of
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Asilomar, Pacific Grove

OUTLINE

- Introduction: From the quark model to QCD
- Self-consistency: antiquarks in hadron-hadron scattering
- How we get away with pQCD: IR safety, factorization, evolution, resummation
- Inclusive annihilation in pQCD
- Using pQCD Corrections
- Getting PDFs from the data
- Using resummation: the Q_T distribution
- Putting it all together: pions and jets in hadronic collisions
- Conclusion as prologue

INTRODUCTION: FROM QUARKS TO QCD

- Spectroscopy and the quark model
  - The discovery of quarks: qqq and $\bar{q}q$ with $q = u, d, s$ generate observed spectrum of baryons and mesons
  - Decay of $\bar{s}s$ states to K, $\bar{K}$ states (OZI rule) indicates continuity of quark lines
  - Non-relativistic wave functions $\longrightarrow$ ratios of magnetic moments μ_n/μ_p etc.

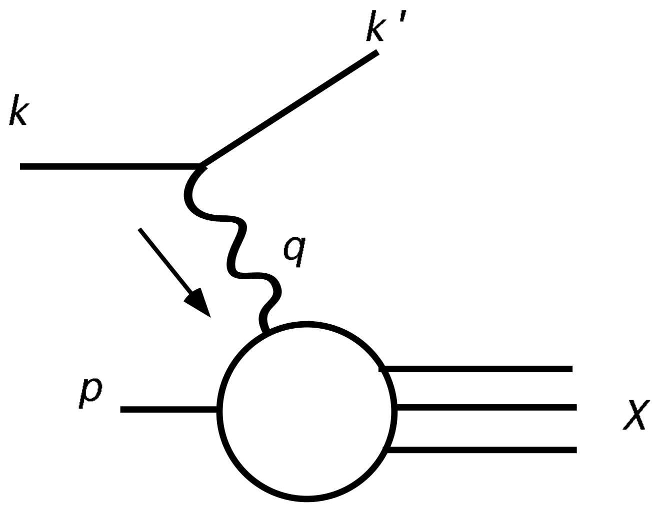
- Dynamical evidence: form factors & structure functions
  - Form factors: ep $\longrightarrow$ ep elastic

$$\frac{d\sigma}{d\Omega_e} = \left[\frac{\alpha_{\mathrm{EM}}^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}\right] \frac{E'}{E} \left(\frac{|G_E(Q)|^2 \ + \ \tau|G_M(Q)|^2}{1+\tau} + \ 2\tau|G_M(Q)|^2 \tan^2\theta/2\right)$$

  - schematically:

$$\frac{d\sigma_{\mathrm{ep}\to\mathrm{ep}}(Q)}{dQ^2} \sim \frac{d\sigma_{\mathrm{ee}\to\mathrm{ee}}(Q)}{dQ^2} \times G(Q) \quad \text{with} \quad G(Q) \sim \frac{1}{\left(1+\frac{Q^2}{\mu_0^2}\right)^2}$$

– Structure functions: ep inclusive



$$\frac{d\sigma}{dE'\,d\Omega} = \left[\frac{\alpha_{\mathrm{EM}}^2}{2SE\,\sin^4(\theta/2)}\right]\left(2\sin^2(\theta/2)F_1(x,Q^2)+\frac{m\cos^2(\theta/2)}{E-E'}\,F_2(x,Q^2)\right)$$

$$\text{with} \quad x = \frac{Q^2}{2p_N\cdot q}$$

– Scaling: $F_2(x, Q^2) \sim F_2(x) \Rightarrow$ Point-like, quasi-free scattering

– $F_2 \sim 2xF_1$: Spin-1/2

– Parton model structure functions

$$F_{2,N}(x) = \sum_q e_q^2 \, x f_{a/N}(x)$$

– Notation: $f_{u/N} = u_N$ etc.

- At the same time, a quark model paradox $\Rightarrow$ color
  - First of all, nobody had $seen$ a quark (confinement), but also
  - A problem with the quark model: quarks have spin-$1/2$ but nucleon quark model wave function was symmetric
- But spin-$1/2$ particles are all fermions – right?
- Fast-forward resolution:
  - Han, Nambu 1965: quarks come in 3 triplets of different colors
  - Quarks in baryons are antisymmetric in quantum number of the group SU(3)

- The birth of QCD: SU(3)
  - A nonabelian gauge theory built on color ($q = q_1 q_2 q_3$):

$$\mathcal{L}_{QCD} = \sum_q \bar{q} \, (i\partial\!\!\!/ - g_s A\!\!\!/ + m_q \,) \, q - \frac{1}{4} F^2_{\mu\nu}[A]$$

  - Think of: $\mathcal{L}_{EM} = K_{\rm e} + J_{EM} \cdot A + (E^2 - B^2)$
  - The Yang-Mills gauge theory of quarks (q) and gluons (A)
    Gluons: like "charged photons". The field is a source for itself.
  - Just the right currents to couple to EM and Weak AND . . .

- Just the right kind of forces: QCD charge is "antishielded" and *grows* with distance

$b_0 = 11 - 2n_{\text{quarks}}/3$ we get:

$$\alpha_s(\mu') = \frac{g_s^2}{4\pi} = \frac{\alpha_s(\mu)}{1 + b_0 \frac{\alpha_s(\mu)}{4\pi} \ln\left(\frac{\mu'}{\mu}\right)^2} = \frac{4\pi}{b_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)}$$

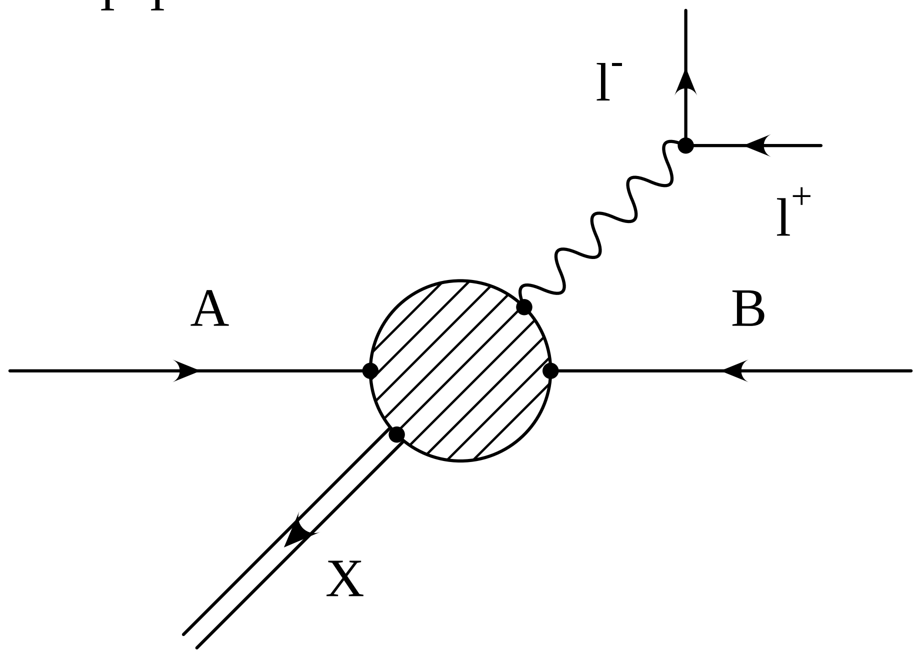
Quantum field theory: every state with the same quantum numbers as uud in the proton . . . is present at least some of the time

So antiquarks are in the nucleon: $uudd\bar{d}$, etc.

What it means: $q\bar{q}$ annihilation processes in NN collisions as d, u from one nucleon collides with $\bar{d}, \bar{u}$ from another

Annihilation into what? Back to quarks, and gluons, yes, but also

$l^+ l^-$



γ, W, Z, H . . .

Which brings us to . . .

SELF-CONSISTENCY:
ANTIQUARKS IN HADRON HADRON SCATTERING

- **The Inclusive Drell-Yan Cross Section**

Parton Model: "Impulse approximation". The template (1970):

$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim \int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a}\bar{\mathrm{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$

$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$

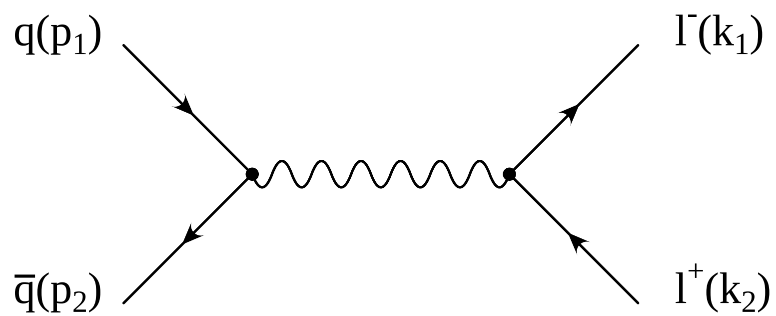
$$\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)$$

The probabilities are $f_{q/N}(x)$'s from DIS!

Recall how it works (with colored quarks) ...

- The Born cross section

$\sigma^{\mathrm{EW,Born}}$ is all from this diagram (ξ's set to unity):



With this matrix element

$$M = e_q \frac{e^2}{\hat{s}} \overline{u}(k_1)\gamma_\mu v(k_2)\overline{v}(p_2)\gamma^\mu u(p_1)$$

- First square and sum/average M. Then evaluate phase space.

- Total cross section:

$$\sigma^{\mathrm{EW,\,Born}}_{q\bar{q}\to\mu\bar{\mu}}(x_1p_1, x_2p_2) = \frac{1}{2\hat{s}}\int \frac{d\Omega}{32\pi^2}\frac{e_q^2e^4}{3}(1+\cos^2\theta)$$
$$= \frac{4\pi\alpha^2}{9M^2}\sum_q e_q^2 \equiv \sigma_0(M)$$

With M the pair mass **and 3 for color average**

Now we're ready for the parton model differential cross section for NN scattering:

Pair mass (M) and rapidity

$$\eta \equiv (1/2)\ln(Q^+/Q^-) = (1/2)\ln[(Q^0+Q^3)/(Q^0-Q^3)]$$

overdetermined $\rightarrow$ delta functions in the Born cross section

$$\frac{d\sigma^{(PM)}_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dM^2 d\eta} = \int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \sigma^{\text{EW, Born}}_{a\bar{a}\to\mu\bar{\mu}}(\xi_1 p_1, \xi_2 p_2)$$
$$\times\delta\left(M^2 - \xi_1\xi_2 S\right)\,\delta\left(\eta - \frac{1}{2}\ln\left(\frac{\xi_1}{\xi_2}\right)\right)$$
$$\times f_{a/N}(\xi_1)\, f_{\bar{a}/N}(\xi_2)$$

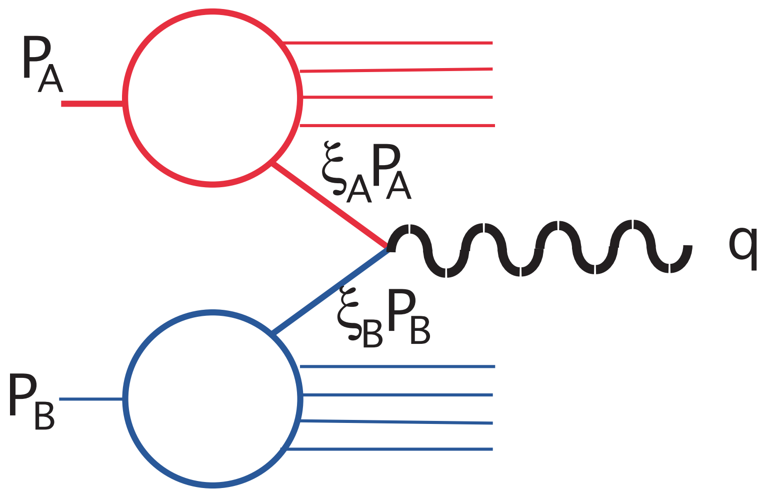
and integrating over rapidity,

$$\frac{d\sigma}{dM^2} = \left(\frac{4\pi\alpha^2_{\text{EM}}}{9M^4}\right)\int_0^1 d\xi_1\, d\xi_2\, \delta\left(\xi_1\xi_2 - \tau\right)\, \sum_a \lambda_a^2\, f_{a/N}(\xi_1)\, f_{\bar{a}/N}(\xi_s)$$

Drell and Yan, 1970 (aside from 1/3 for color)

Analog of DIS: scaling in $\tau = Q^2/S$

- The parton model picture



- **All QCD radiation in the f's . . . but why?**

- **Asymptotic freedom has something to do with this . . . but how? What to do in QFT?**

HOW WE GET AWAY WITH PQCD

- Specific problems for perturbation theory in QCD

1. Confinement

$$\int \mathrm{e}^{-iq\cdot x} \langle 0|\, T[\phi_a(x)\ldots]\,|0\rangle$$

   has no $q^2 = m^2$ pole for ϕ_a that
   transforms nontrivially under color (confinement)

2. The pole at $p^2 = m_\pi^2$

$$\int \mathrm{e}^{-iq\cdot x} \langle 0|\, T[\pi(x)\ldots]\,|0\rangle$$

   is not accessible to perturbation theory (χSB etc., etc.)

- And yet we use infrared safety & **asymptotic freedom**:

$$Q^2\ \hat{\sigma}_{\rm SD}(Q^2,\mu^2,\alpha_s(\mu)) = \sum_n c_n(Q^2/\mu^2)\ {\alpha_s}^n(\mu) + \mathcal{O}\left(1/Q^p\right)$$

$$= \sum_n c_n(1)\ {\alpha_s}^n(Q) + \mathcal{O}\left(1/Q^p\right)$$

- What can we really calculate? PT for color singlet operators
  - $\int {\rm e}^{-iq\cdot x}\langle 0|\, T[J(x)J(0)\ldots]\,|0\rangle$ for color singlet currents

$\rm e^+e^-$ total . . . no QCD in initial state

– Another class of color singlet matrix elements:

$$\lim_{R\to\infty} \int dx_0 \int d\hat{n}\, f(\hat{n})\, \mathrm{e}^{-iq\cdot y} \langle 0|\, J(0) T[\hat{n}_i \Theta_{0i}(x_0, R\hat{n}) J(y)]\, |0\rangle$$

With Θ_{0i} the energy momentum tensor

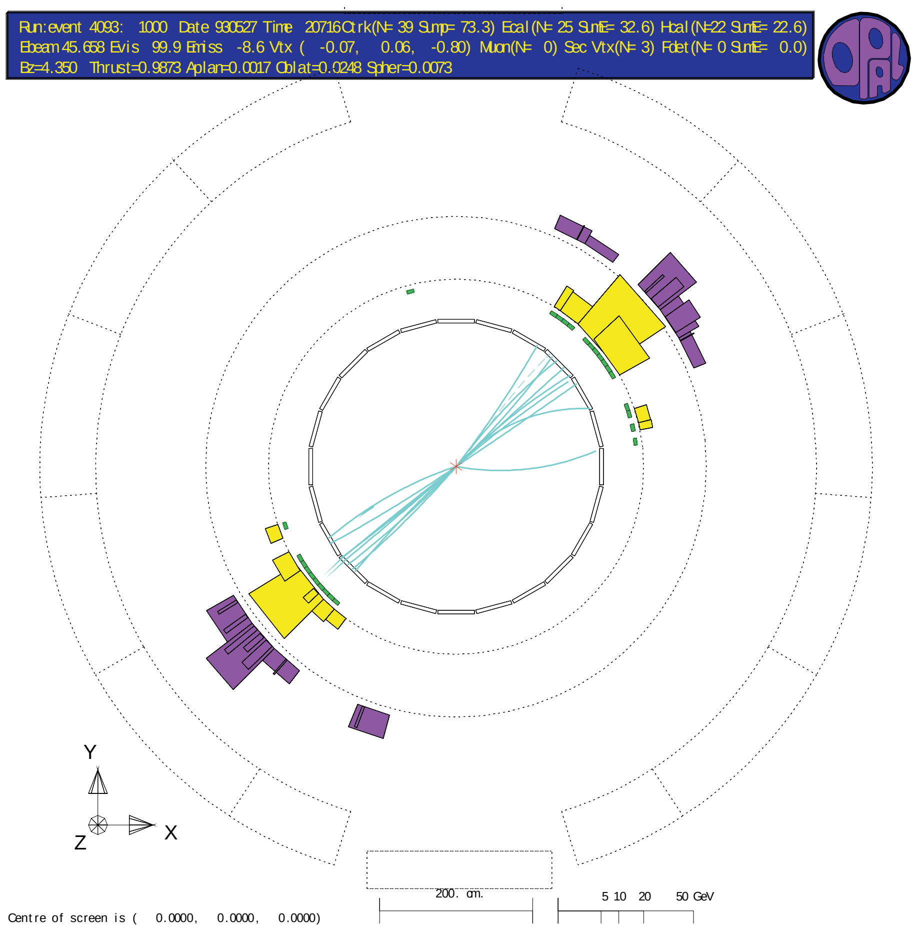
"Weight" $f(\hat{n})$ introduces no new dimensional scale

Short-distance dominated if all $d^k f/d\hat{n}^k$ bounded

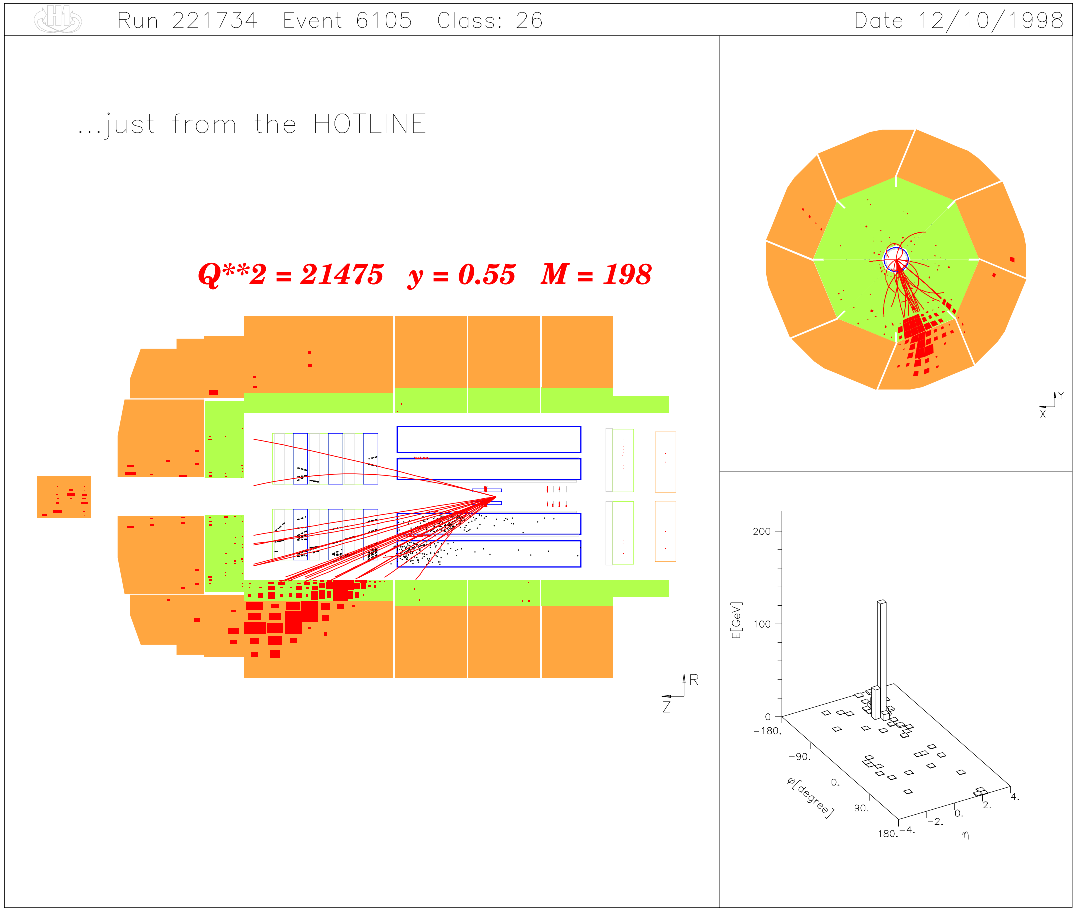
Individual final states have IR divergences, but these cancel in sum over collinear splitting/merging and soft parton emission because they respect energy flow

- The essence of jet computability

- For e^+e^-:

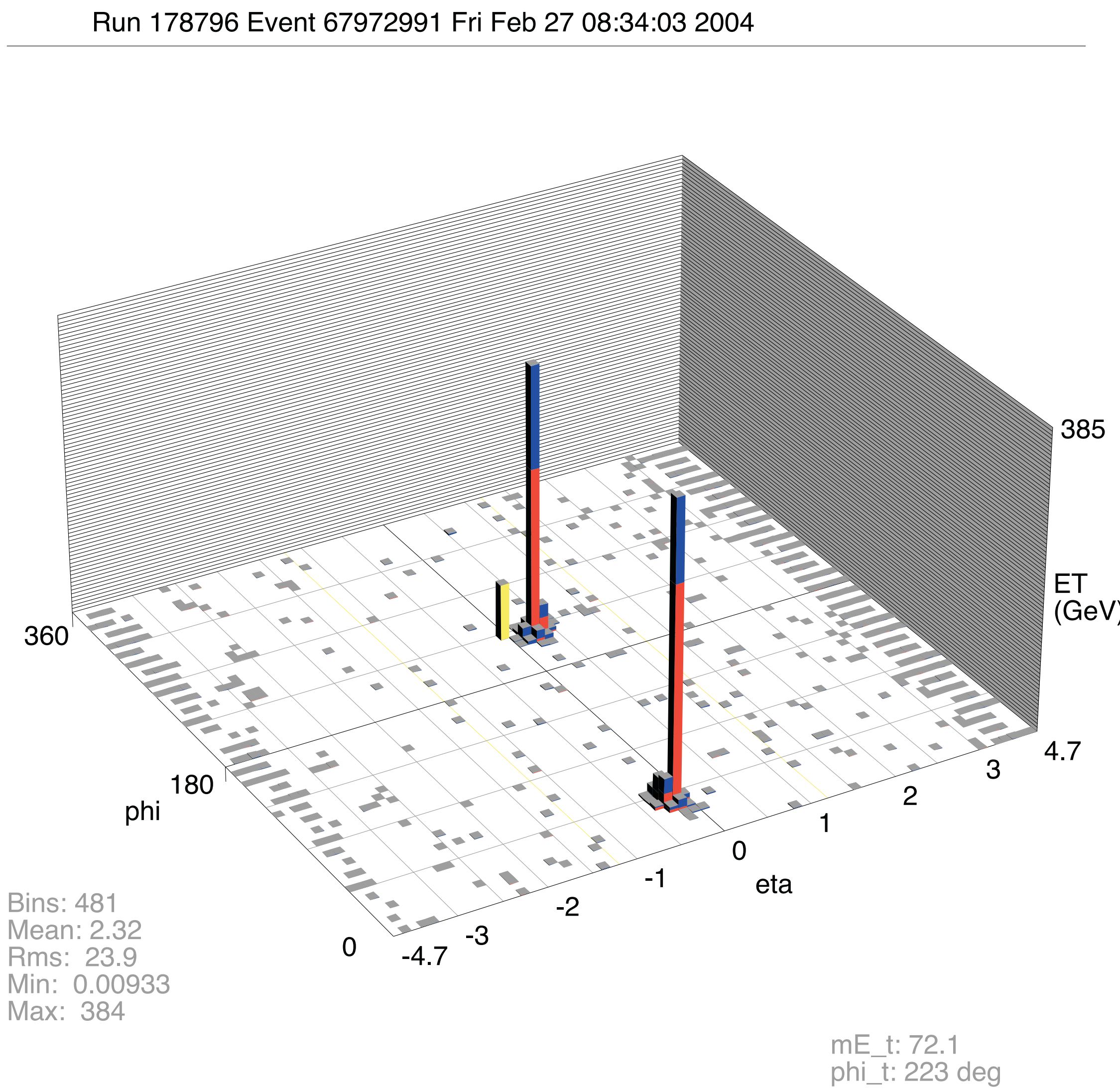

Run:event 4093: 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
X
Z
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

- And for DIS:



Run 221734 Event 6105 Class: 26
Date 12/10/1998
...just from the HOTLINE
Q**2 = 21475 y = 0.55 M = 198

- And in nucleon-nucleon collisions



But what of the initial state? (viz. parton model)

- **Factorization**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(1/Q^p\right)$$

– $\mu =$ **factorization scale**; $m=$ **IR scale** (m *may* be perturbative)

– **New physics** in $\omega_{\rm SD}$; $f_{\rm LD} = f$ and/or D "universal"

– ep DIS inclusive, $\rm pp \rightarrow$ jets, $Q\bar{Q}$, $\pi(p_T)$ . . .

– Exclusive limits: $\rm e^+e^- \rightarrow$ JJ as $m_J \rightarrow 0$

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

$$\mu \frac{d \ln (f \text{ or } D)}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

PDF f <u>or</u> Fragmentation D

- **Wherever there is evolution there is resummation**

$$\ln \sigma_{\rm phys}(Q, m) = \exp \left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- Factorization proofs:
  - (1) $\omega_{\rm SD}$ **incoherent** with long-distance dynamics
  - (2) Mutual incoherence when $v_{\rm rel} = c$:
    Jet-jet factorization Ward identities.
  - (3) Wide-angle soft radiation sees only total color flow:
    jet-soft factorization Ward identities: Wilson lines.
  - (4) Dimensionless coupling and renormalizability
    $\Leftrightarrow$ no worse that logarithmic divergence in the IR:
    fractional power suppression $\Rightarrow$ finiteness

- Classical: Lorentz contracted fields of incident particles don't overlap until the moment of the scattering, creation of heavy particle, etc.!
- Initial-state interactions decouple from the hard process
- Summarized by multiplicative factors: **parton distributions**
- Evolution of partons to jets/hadrons too late to know details of the hard scattering
- Summarized by multiplicative factors: **fragmentation functions**
- "Left-over" cross section for hard scattering is IR safe

INCLUSIVE EW ANNIHILATION IN PQCD

$$\frac{d\sigma_{NN\to\mu^+\mu^-+X}(Q,p_1,p_2)}{dQ^2} = \int_{\xi_1,\xi_2} \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\hat{\sigma}_{a\bar{a}\to\mu^+\mu^-(Q)+X}(Q,\mu,\xi_1 p_1,\xi_2 p_2)}{dQ^2}$$

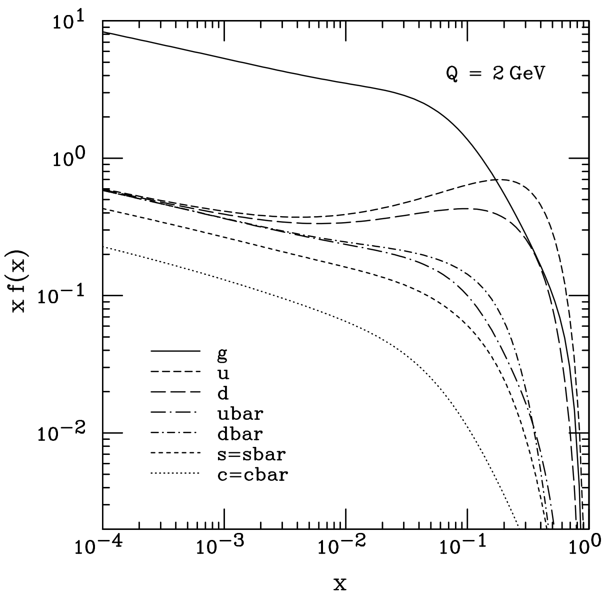
$$\times f_{a/N}(\xi_1,\mu)\, f_{\bar{a}/N}(\xi_2,\mu)$$

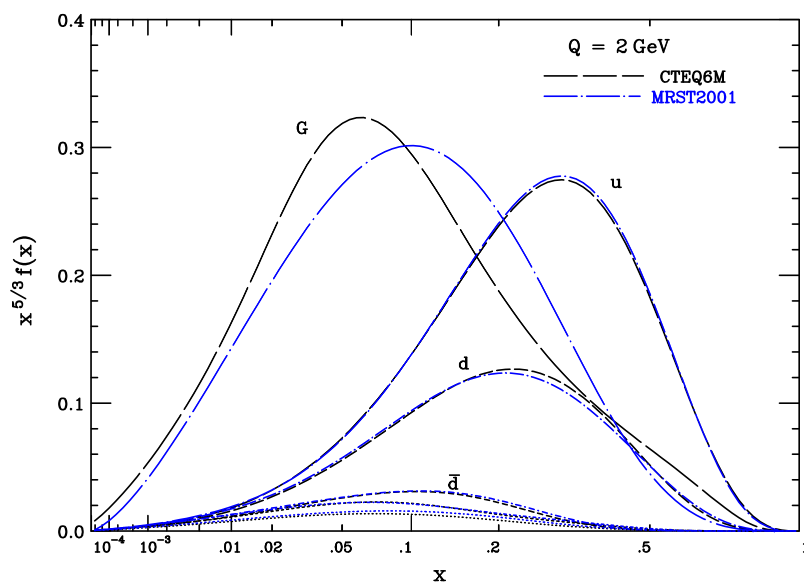
– μ is the factorization scale: separates IR from UV in quantum corrections. μ appears in $\hat{\sigma}$, as $\alpha_s(\mu)$ and as $\ln(\mu/Q)$ so choosing $\mu \sim Q$ can improve perturbative predictions

– Evolution: $\mu df(x,\mu)/d\mu = \int_x^1 P(x/\xi)\, f(\xi,\mu)$ makes energy extrapolations possible.

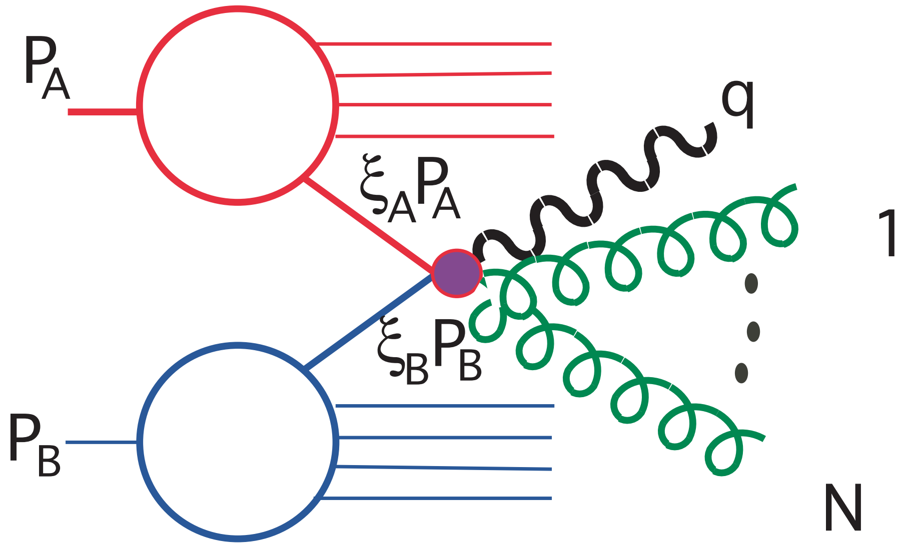
Two portraits of modern parton distributions

* CTEQ6 as seen at moderate momentum transfer:
* Two modern fits compared (note weighting with x)





- The factorized picture



sum N = 0 (PM) to infinity

- High-p_T radiation "has a place to go."
  The rest ($p_T < \mu$) to the PDFs.

USING PQCD CORRECTIONS

The transverse momentum distribution at order α_s

Extend factorization to gluon radiation process:

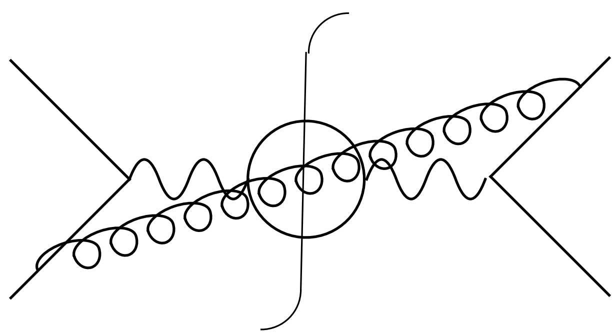
$$q(p_1) + \bar{q}(p_2) \to \gamma^*(Q) + g(k)\,,$$

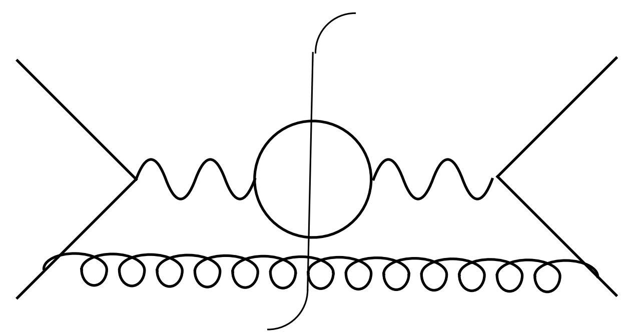
Treat this $2 \to 2$ process at lowest order (α_s) "LO"
in factorized cross section, so that $\mathbf{k} = -\mathbf{Q}$

The result is well-defined for $\mathbf{Q}_T \neq 0$

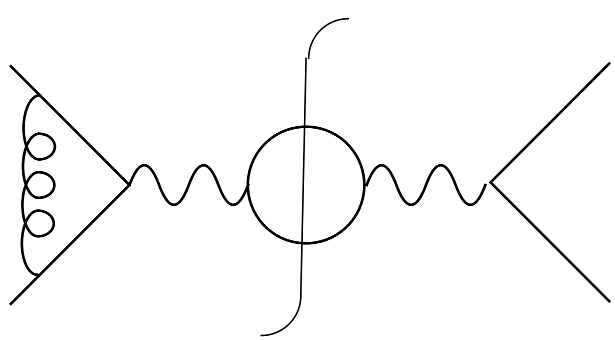
- The diagrams at order α_s

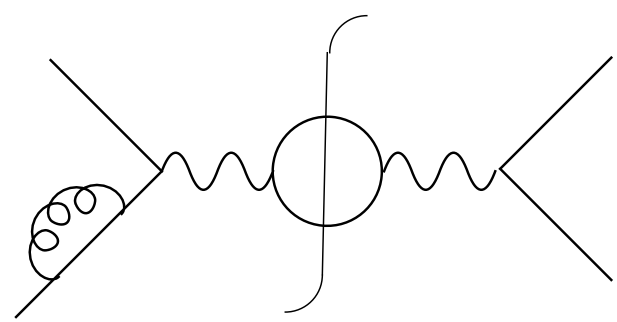
Gluon emission contributes at $Q_T \neq 0$





Virtual corrections contribute only at $Q_T = 0$





$$\frac{d^2\sigma^{(1)}_{q\bar{q}\to\gamma^* g}(z, Q^2, \mathbf{Q}_T)}{dQ^2\, d^2\mathbf{Q}_T} = \sigma_0 \frac{\alpha_s C_F}{\pi^2} \left(1 - \frac{4\mathbf{Q}_T^2}{(1-z)^2 \hat{s}}\right)^{-1/2} \times \left[\frac{1}{\mathbf{Q}_T^2} \frac{1+z^2}{(\mathbf{1}-\mathbf{z})} - \frac{2z}{(\mathbf{1}-\mathbf{z})Q^2}\right]$$

Fine as long as $\mathbf{Q}_T \neq 0$, $z = Q^2/S \neq 1$.

Q_T integral $\to \ln(1-z)/(1-z)$, z integral $\to \ln(Q_T)/Q_T$.

Both off these limits can be dealt with by reorganization, "resummation" of higher order corrections

- Fundamental application: the total cross section

  Integrate over $\mathbf{Q}_T$ at fixed $z = Q^2/S$. $Q_T \to 0$ is singular

  Add diagrams with virtual gluons: *their* Q_T integrals are singular

  Remove (factor) low $\mathbf{k}_T = -\mathbf{Q}_T < \mu$ gluons

  The remainder is now finite at fixed Q_T, $z \neq 1$. Combine with LO $\hat{\sigma}$.

  But the left-over NLO $\hat{\sigma}$ is not a normal function of z!

  Because $d\sigma/dQ^2$ begins at ${\alpha_s}^0$,
  this is next-to-leading order (NLO) here

- $\hat{\sigma}_{\bar{q}q}$ for Drell-Yan at NLO

$$\frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}\to\gamma^* g}(z,Q^2,\mu^2)}{dQ^2} = \sigma_0(Q^2)\left(\frac{\alpha_s(\mu)}{\pi}\right)\left\{2(1+z^2)\left[\frac{\ln(1+z^2)}{1-z}\right]_+ - \frac{\left[(1+z^2)\ln z\right]}{(1-z)} + \left(\frac{\pi^2}{3}-4\right)\delta(1-z)\right\} + \sigma_0(Q^2)\, C_F\, \frac{\alpha_s}{\pi}\left[\frac{1+z^2}{1-z}\right]_+ \ln\left(\frac{Q^2}{\mu^2}\right)$$

  - Plus distributions: “generalized functions” (c.f. delta function)
  - μ-dependence: evolution for hadron-hadron scattering

- What they are, how they work

$$\int_0^1 dx\, \frac{f(x)}{(1-x)_+} \quad \equiv \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)}$$

$$\int_0^1 dx\ f(x) \left(\frac{\ln(1-x)}{1-x}\right)_+ \quad \equiv \int_0^1 dx\ \left(f(x) - f(1) \right) \frac{\ln(1-x)}{(1-x)}$$

and so on . . . where $f(x)$ will be parton distributions

- $f(x)$ term: real gluon, with momentum fraction $1-x$
- $f(1)$ term: virtual, with elastic kinematics

- *A Special Distribution*

- *DGLAP "evolution kernel" = "splitting function"*

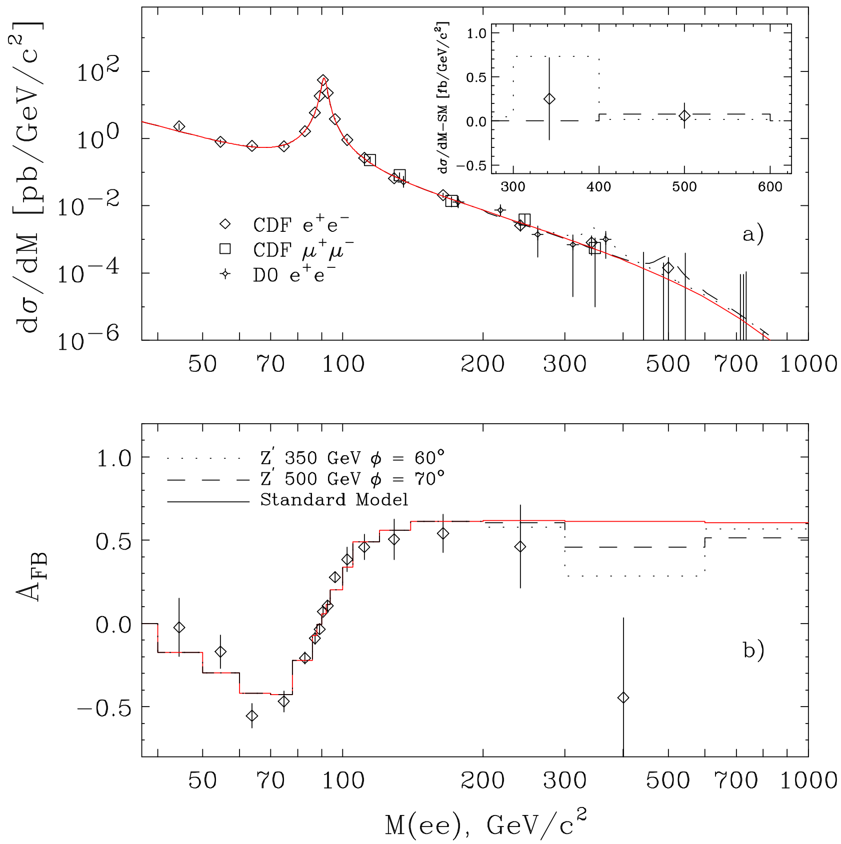
$$P_{qq}(x) = C_F \, \frac{\alpha_s}{\pi} \, \left[\frac{1+x^2}{1-x} \right]_+$$

- Nonsinglet, leading order

Applications

- M-dependence for dileptons at high energy (γ and Z) & forward-backward asymmetry in $\sigma_{\rm Born}$ compared to NLO

A test for "new" physics in the hard scattering



GETTING THE PDFs FROM DATA

W asymmetries at the Tevatron: d/u

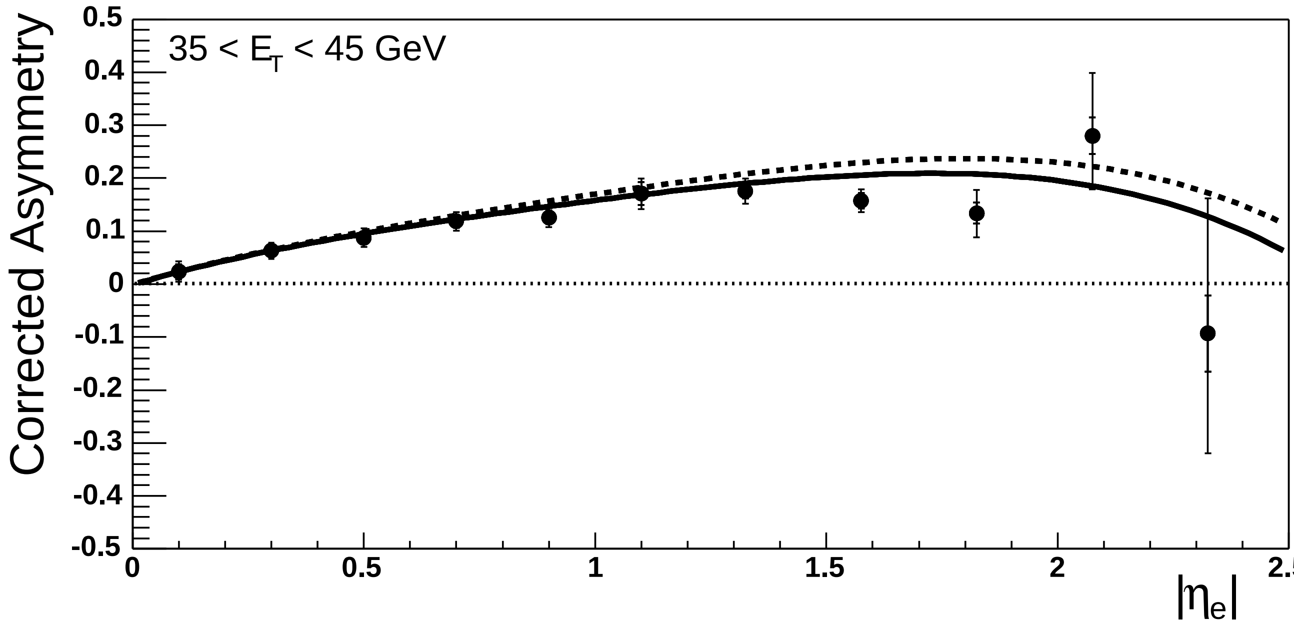
W^+ requires $u\bar{d}$, W^- needs $\bar{u}d$

At LO, since $u_p = \bar{u}_{\bar{p}}$, etc.

$$\frac{d\sigma_{W^+}}{d\eta} = \frac{2\pi G_F}{\sqrt{2}} u_p(x_a = \sqrt{\tau} e^{\eta})\, d_p(x_b = \sqrt{\tau} e^{-\eta})$$

Asymmetry tests d/u as a function of

$$A(y) \equiv \frac{\sigma_{W^+}(\eta) - \sigma_{W^-}(\eta)}{\sigma_{W^+}(\eta) + \sigma_{W^-}(\eta)} = \frac{u_p(x_a)\, d_p(x_b) - d_p(x_a)\, u_p(x_b)}{u_p(x_a)\, d_p(x_b) + d_p(x_a)\, u_p(x_b)}$$



(CDF Collaboration, Phys. Rev. D71, 051104 (2005) hep-ex/0501023)

- Foward fixed target DY ($\tau = M^2/S$) and $\bar{d}/\bar{u}$

  At LO,

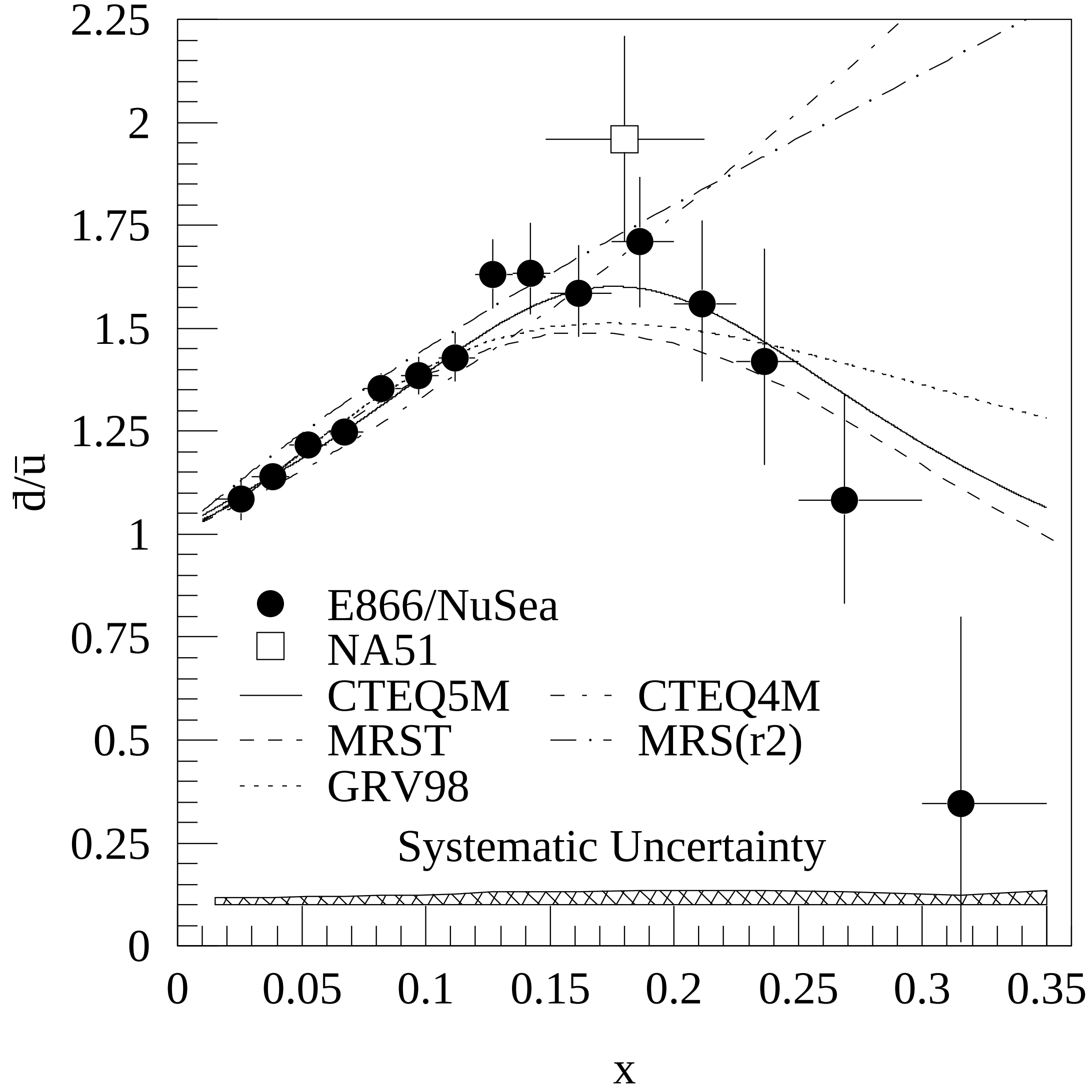
$$\frac{d\sigma_{pN}}{dM^2 d\eta} = \left(\frac{4\pi\alpha_{\rm EM}^2}{9M^4}\right) \sum_a \lambda_a^2 \, f_{a/p}(\sqrt{\tau}{\rm e}^{\eta}, M) \, f_{\bar{a}/N}(\sqrt{\tau}{\rm e}^{-\eta}, M)$$

  Large η; a valence, $\bar{a}$ sea: sensitivity to sea distribution

  E866: compare pp and pd

$$\frac{\sigma_{pD}}{2\sigma_{pp}} \sim \frac{1}{2}\left(1 + \frac{\bar{d}_p(\sqrt{\tau}{\rm e}^{-\eta})}{\bar{u}_p(\sqrt{\tau}{\rm e}^{-\eta})}\right)$$

  Previously unavailable information on the sea ratio



E866/NuSea Collaboration, Phys. Rev. D64, 052002 (2001) hep-ex/0103030

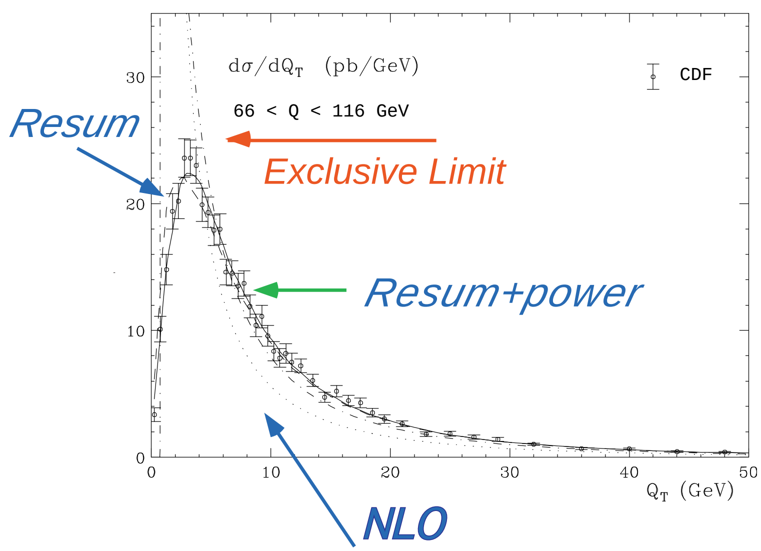
USING RESUMMATION: THE Q_T DISTRIBUTION

- Low Q_T Drell-Yan & Higgs at leading log (LL) $({\alpha_s}^n \ln^{2n-1} Q_T)$

$$\frac{d\sigma(Q)}{dQ_T} \sim \frac{d}{dQ_T} \exp\left[-\frac{\alpha_s}{\pi} C_F \ln^2\left(\frac{Q}{Q_T}\right)\right]$$

$$(C_F = 4/3)$$

- Double jet-soft factorization $\longrightarrow$ double logs (from A. Kulesza, G.S., W. Vogelsang (2002)



- General features:

  Maximum then decrease near “exclusive” limit
  (parton model kinematics) replaces divergence at $Q_T = 0$

  Soft but perturbative radiation broadens distribution

  Typically nonperturbative correction necessary for
  full quantitative description

  Recover fixed order predictions $\sigma^{(1)}$ away from exclusive limit

  Generally requires (Fourier) transform (impact parameter)
  to go beyond leading log

PUTTING IT ALL TOGETHER: OBSERVED HADRONS

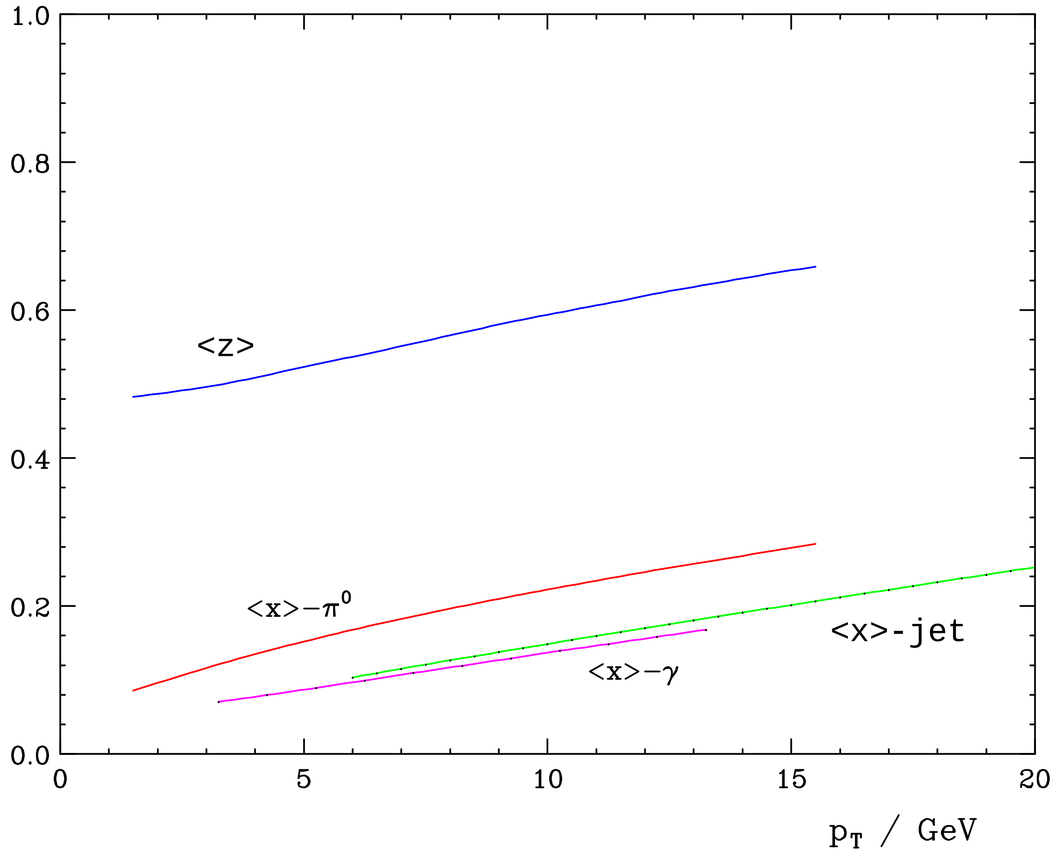
- Pions at fixed target and RHIC (Vogelsang and de Florian, 2004)

$$
\begin{aligned}
\frac{p_T^3\, d\sigma(x_T)}{dp_T} \;=\; & \sum_{a,b,c} \int_0^1 dx_1\, f_{a/H_1}\left(x_1, \mu_F^2\right) \int_0^1 dx_2\, f_{b/H_2}\left(x_2, \mu_F^2\right) \\
& \times \int_0^1 dz\, z^2\, D_{h/c}\left(z, \mu_F^2\right) \\
& \times \int_0^1 d\hat{x}_T\; \delta\left(\hat{x}_T - \frac{x_T}{z\sqrt{x_1 x_2}}\right) \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta}\, \frac{\hat{x}_T^4\; \hat{s}}{2}\, \frac{d\hat{\sigma}_{ab\to cX}(\hat{x}_T^2, \hat{\eta})}{d\hat{x}_T^2 d\hat{\eta}}
\end{aligned}
$$

$\hat{\eta}$: pseudorapidity at parton level

$$
\hat{\eta}_+ = -\hat{\eta}_- = \ln\left[(1 + \sqrt{1 - \hat{x}_T^2})/\hat{x}_T\right]
$$

Averages for distribution x and fragmentation z's



RHIC 200 GeV midrapidity average z for pions, and average x for pions, photons, jets (NLO) Thanks to Werner Vogelsang

– As for the DY Q_T distribution: collinear $f + D$ + soft $\Rightarrow$ double logs

$$\frac{\hat{s}\, d\hat{\sigma}^{(1)}_{ab\to cX}(v,w)}{dv\, dw} \approx \frac{\hat{s}\, d\hat{\tilde{\sigma}}^{(0)}_{ab\to cd}(v)}{dv} \left[A'\, \delta(1-w) + B' \left(\frac{\ln(1-w)}{1-w}\right)_+ \right.$$
$$\left. + C' \left(\frac{1}{1-w}\right)_+ \right]$$

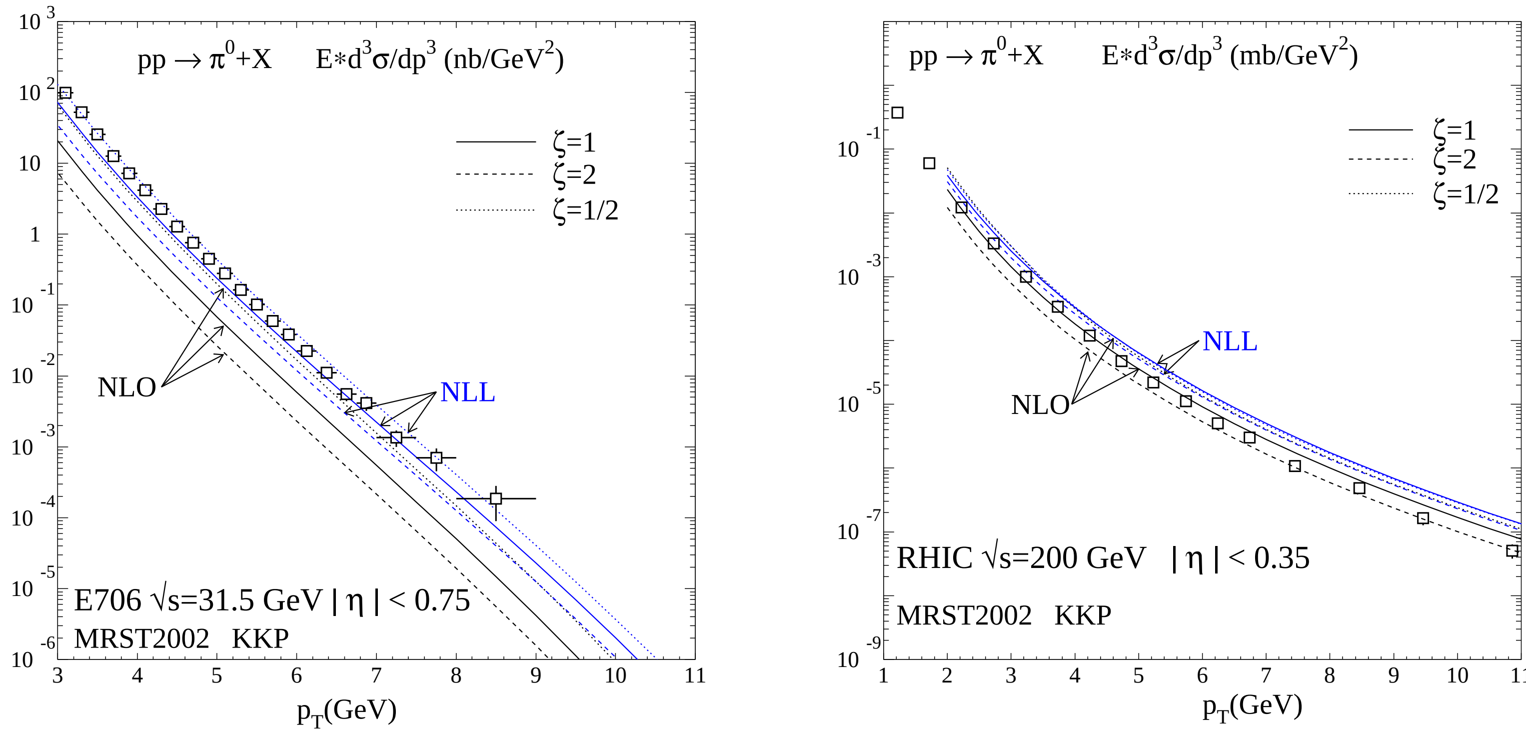
– 1) For resummation, take x_T^{2N} moments:

$$\hat{\sigma}^{(\mathrm{res})}_{ab\to cd}(N) = C_{ab\to cd}\, \Delta^a_N\, \Delta^b_N\, \Delta^c_N\, J^d_N \left[\sum_I G^I_{ab\to cd}\, \Delta^{(\mathrm{int})ab\to cd}_{I\, N} \right] \hat{\sigma}^{(\mathrm{Born})}_{ab\to cd}(N)$$

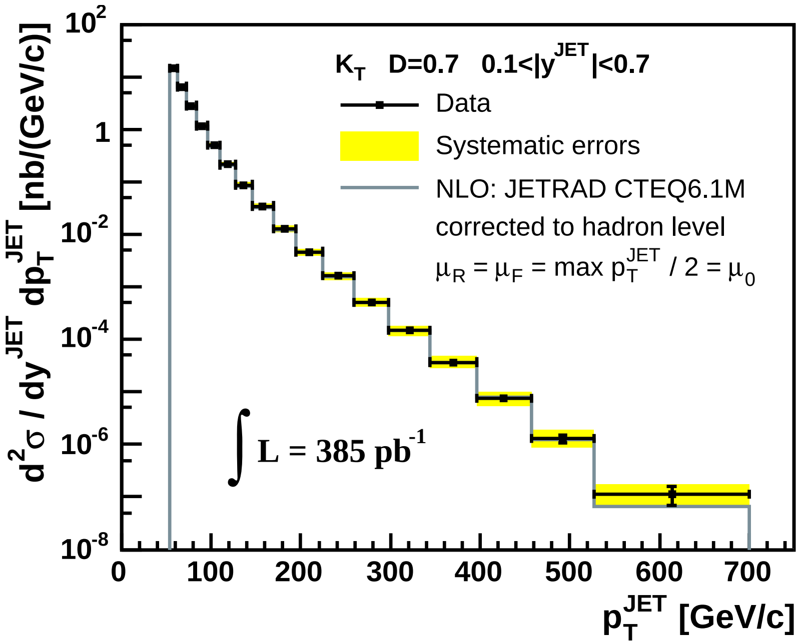
– 2) A typical resummed factor

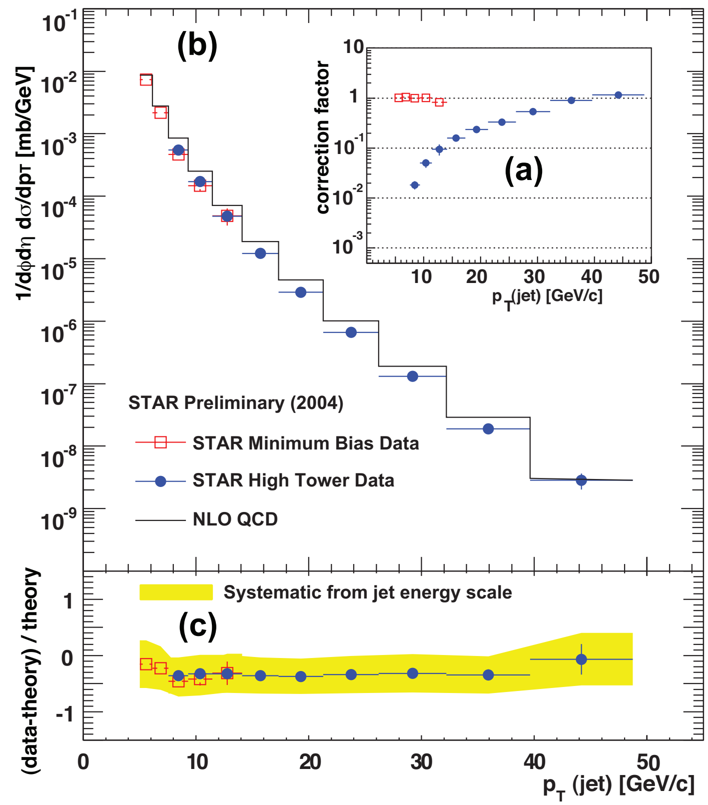
$$\Delta^a_N \quad = \exp\left[\int_0^1 \frac{z^{N-1}-1}{1-z} \int_{\mu_{FI}^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_a(\alpha_s(q^2)) \right]$$
$$A = C_F(\alpha_s/\pi) + \ldots$$

– Invert the moments: resolve a long-standing fixed-target/collider contrast!


pp → π⁰+X E∗d³σ/dp³ (nb/GeV²)
ζ=1
ζ=2
ζ=1/2
NLO
NLL
E706 √s=31.5 GeV | η | < 0.75
MRST2002 KKP
pT(GeV)
pp → π⁰+X E∗d³σ/dp³ (mb/GeV²)
ζ=1
ζ=2
ζ=1/2
NLO
NLL
RHIC √s=200 GeV | η | < 0.35
MRST2002 KKP
pT(GeV)

– And jets at the Tevatron, and now the RHIC





– Nicely settled down.

Conclusions as Prologue

- pQCD formalism works well in pp collisions
  (not least from RHIC data/theory interplay)

- pQCD revolves around factorization and energy flow

- Multiple interactions induce corrections to factorized cross sections
  typically (number of partons) $\times$ (soft scale/hard scale)^2

- Induced radiation redistributes energy flow

- Centrality in AA collisions a control parameter
  for these corrections

- Heavy quark/quarkonia production of special sensitivity
- Nuclear collisions shed light on pQCD and vice-versa
- Enjoy the conference!