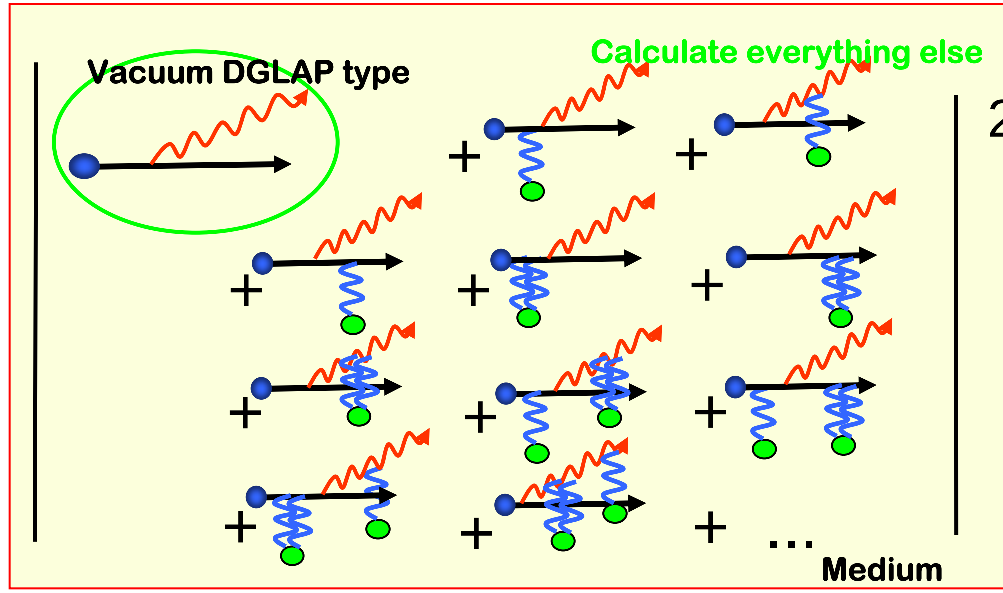
GLV Approach to Multiple Parton Scattering

Ivan Vitev

M.Gyulassy, P.Levai, I.V., Phys.Rev.Lett.85 (2000)
M.Gyulassy, P.Levai, I.V., Nucl.Phys.B594 (2001)



Reaction operator:

- Insertion of one additional unitarized scattering in $d\sigma, dN \sim S^{\dagger}S$
- Initial condition (formal solution to functional recurrence relation)

Not specific to E-loss

Elastic scattering, initial and final state radiative energy loss, …

$$\frac{dN^g_{\rm med}}{d\omega\, d\sin\theta^* d\delta} = \sum_{n=1}^{\infty} \frac{dN^{g\,(n)}_{\rm med}}{d\omega\, d\sin\theta^* d\delta} = \omega \sin\theta^* \sum_{n=1}^{\infty} \frac{2C_R\alpha_s}{\pi^2} \prod_{i=1}^{n} \int_0^{L-\sum_{a=1}^{i-1}\Delta z_a} \frac{d\Delta z_i}{\lambda_g(i)}$$

$$\times \int d^2\mathbf{q}_i \left[\sigma_{el}^{-1}(i)\frac{d\sigma_{el}(i)}{d^2\mathbf{q}_i} - \delta^2(\mathbf{q}_i)\right] \left(-\mathbf{C}_{(1,\cdots,n)} \cdot \sum_{m=1}^{n} \mathbf{B}_{(m+1,\cdots,n)(m,\cdots,n)} \right.$$

$$\left. \times \left[\cos\left(\sum_{k=2}^{m} \omega_{(k,\cdots,n)}\Delta z_k\right) - \cos\left(\sum_{k=1}^{m} \omega_{(k,\cdots,n)}\Delta z_k\right)\right] \right) ,$$

where

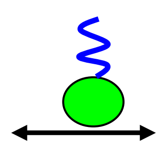
$$\omega_{(j,\cdots,n)} = \frac{(\mathbf{k}-\mathbf{q_j}-\cdots-\mathbf{q_n})^2}{2xE}$$

$$\mathbf{C}_{(j,\cdots,n)} = \frac{\mathbf{k}-\mathbf{q_j}-\cdots-\mathbf{q_n}}{(\mathbf{k}-\mathbf{q_j}-\cdots-\mathbf{q_n})^2}$$

$$\mathbf{B}_{(j+1,\cdots,n)(j,\cdots,n)} = \mathbf{C}_{(j+1,\cdots,n)} - \mathbf{C}_{(j,\cdots,n)}$$

- Controlled approach to coherence

- Independent of the details of the momentum transfer



$$V \sim \frac{\alpha_s}{q^2+\mu_D^2}\delta(q_0) \qquad \mu_D = g^2T^2\left(1+\frac{n_f}{6}\right) \qquad \lambda_D \sim \frac{1}{\mu_D}$$

Analytic Limits of Delta E

$$\frac{d\Delta E}{dx} = \frac{2C_R\alpha_s}{\pi} E\int_0^L \frac{d\tau}{\lambda(\tau)}\int_0^\infty \frac{dk_\perp^2/\mu^2}{k_\perp^2/\mu^2(1+k_\perp^2/\mu^2)}\left[1-\cos\left(\frac{(k_\perp^2/\mu^2)\mu^2(\tau-\tau_0)}{2xE}\right)\right], \quad x=\frac{\omega}{E}$$

Coherent regime

$$x > x_c = \frac{(\tau-\tau_0)\mu^2}{2E}$$

On the relevance of density, rapidity density and transport coefficients

$$\Delta E \sim \int_{z_0}^{L} \rho(z) z dz \sim \frac{1}{2}\rho L^2$$

$$\Delta E \sim \int_{z_0}^{L} \rho(z)\frac{z_0}{z} z dz \sim \rho z_0 L \sim \frac{dN}{dy} L$$

$$\Delta E^{(1)} \approx \frac{C_R\alpha_s}{4}\frac{\mu^2 L^2}{\lambda_g}\mathrm{Log}\frac{2E}{\mu^2(L)L}+\dots,$$

– **Static medium**

$$\Delta E^{(1)} \approx \frac{9\pi C_R\alpha_s^3}{4} L \frac{1}{A_\perp}\frac{dN^g}{dy}\mathrm{Log}\frac{2E}{\mu^2(L)L}+\dots,$$

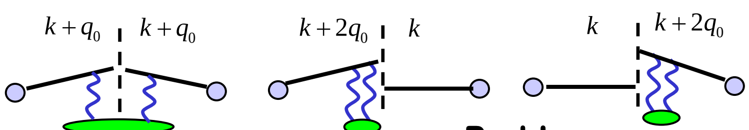
– **1+1D Bjorken**

Can find corrections $\left(Log\frac{2E}{\mu^2 L}+\frac{4}{\pi}+\frac{1}{4}+\dots\right)$

1+3D expansion

$$\Delta E \approx \frac{9\pi C_R\alpha_s^3}{4}\frac{1}{A_\perp}\frac{dN^g}{dy}\frac{L'}{1+\beta_T L'/L} Log\frac{2E}{\mu^2(L')L'}$$

Where $L' = L/(1-\beta_T)$ **Same energy loss**



Problem

$$(\omega)\frac{dN^g(k_\perp)}{d\omega d^2k_\perp} \rightarrow (\omega)\frac{dN^g(k_\perp-q_0)}{d\omega d^2k_\perp}$$

From the analytic formulas we can match the full numerical results: central Au+Au

Static $\alpha_s = 0.3,\ L = 6\ fm,$ $\mu = 0.5\ GeV,\ \lambda_g = 1.5\ fm,\ \hat{q} = 0.2\ GeV^2/fm$

1+1D $\frac{dN}{dy} = 1200,\ \alpha_s = 0.3,\ L = 6\ fm,$ $A_\perp = 120\ fm^2,\ \mu = 0.5\ GeV$

Scales in Thermalized QGP

- **Experimental: Bjorken expansion**

$$\frac{dN^g}{dy} \simeq \frac{3}{2}\left|\frac{d\eta}{dy}\right|\frac{dN^{ch}}{d\eta} \qquad \frac{dN^g}{dy} = 1200$$

$$\rho_{\exp}(\tau) = \frac{1}{A_\perp \tau}\frac{dN^g}{dy}, \qquad A_\perp = 120 \ \ fm^2$$

$$\tau_0 = 0.6 \ \ fm$$

$$\Rightarrow \ \rho_{\exp}(\tau_0) = 17 \ \ fm^{-3}$$

- **Theoretical: Gluon dominated plasma**

$$\rho_{theory}(T) = \#DoF \int_0^\infty \frac{1}{e^{p/T}-1}\frac{4\pi p^2 dp}{(2\pi)^3} = \frac{\#DoF}{\pi^2}\varsigma[3]\times T^3$$

where $\#DoF = 2(polarization)\times 8(color), \ \varsigma[3] = 1.2$

$$T = 400 \ \ MeV$$

- **Energy density**

$$\varepsilon_{theory}(T) = \frac{\pi^4}{30\varsigma[3]}\times \rho_{theory}(T)\times T$$

$$\varepsilon_{\exp}(\tau_0) = 18 \ \ GeV.fm^{-3} \geq 100\times 0.14 \ \ GeV.fm^{-3}$$

- **Transport coefficients (not a good measure for expanding medium)**

$$\mu_D \approx gT, \ \ g = 2-2.5 \ \ (\alpha_s = \frac{g^2}{4\pi} = 0.3-0.5)$$

$$\sigma^{gg} = \frac{9\pi\alpha_s^2}{2\mu_D^2}, \quad \lambda_g = \frac{1}{\sigma^{gg}\rho}$$

$$\mu_D = 0.8-1 \ \ GeV$$
$$\lambda_g = 0.75-0.42 \ \ fm$$

$$\hat{q} = \frac{\mu_D^2}{\lambda_g} = \frac{9\pi\alpha_s^2}{2}\rho$$

$$\hat{q} = 1-2.5 \ \ GeV.fm^{-1}$$

- **Define the average for Bjorken** $\langle\langle \hat{q} \rangle\rangle = \frac{2}{(L-z_0)^2}\int_{z_0}^{L} \hat{q}(z) z dz$

$$\langle\langle \hat{q} \rangle\rangle = 0.35-0.85 \ \ GeV^2.fm^{-1}$$

- **GLV results are consistent with these scales and do not require exotic interpretations**

Comparison to Other Models

- **Comparison to Wang & Wang** (not directly comparable)

  For practical purposes equivalent to GLV 1st order in opacity. Formulated in terms of gluon correlation

  Not probabilistic, In cold nuclear matter.

$$\xi^2 = \left(\frac{3\pi\alpha_s(Q^2)}{8r_{0\perp}^2}\right)\int \frac{dy^-}{2\pi}\, e^{i0p^+y^-}\left\langle p\middle|F^{+\perp}F_\perp^{\ +}\middle|p\right\rangle \theta(y^-)$$

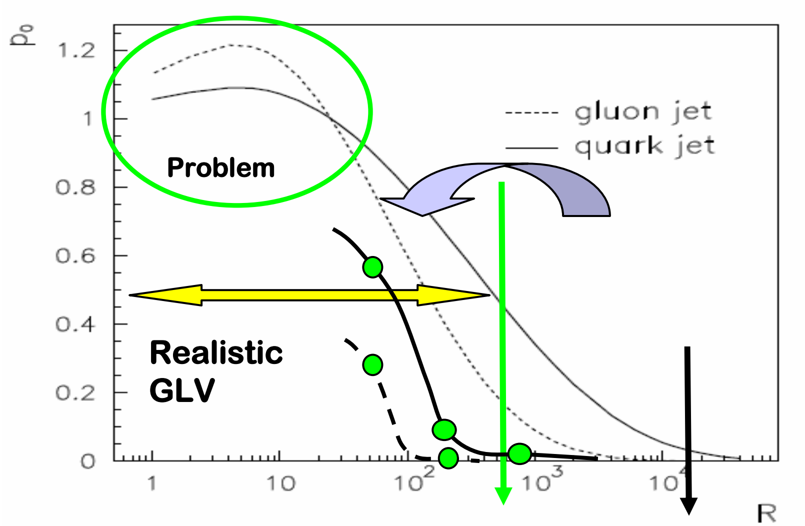
- **Comparison to AMY** (not directly comparable)

  Average implementation, α_s is used as a parameter, Formulated as a local rate possible $\sim L^2$?

  Not probabilistic

Quote: $T = 370\ MeV \quad \alpha_s = 0.3$

$P_0 = e^{-\langle N_g \rangle}$ • **Comparison to SW** (directly comparable)



$\omega_c = \hat{q}L^2/2$

$R = \omega_c L$

$\hat{q}_{200\ \text{GeV}} \approx 0.4\ \text{GeV}^2/\text{fm}$ $\hat{q}_{200\ \text{GeV}} \approx 14\ \text{GeV}^2/\text{fm}$

Negative gluon number and jet enhancement from energy loss? $\omega_c(L = 5\ fm)$

- **Elastic energy loss** (analytically comparable)

$$R(rad./el.) = \frac{\Delta E^{rad}}{\Delta E^{el}} = \frac{\frac{C_R\alpha_s}{4}\frac{\mu^2L^2}{\lambda_g}Log\frac{2E}{\mu^2L}}{\mu\frac{L}{\lambda_R}Log\frac{(4)E}{2\mu}} \qquad \lambda_g = \lambda_R\frac{C_A}{C_R}$$

$$= \frac{\frac{C_R\alpha_s}{4}\frac{\mu^2L^2}{\lambda_g}Log\frac{2E}{\mu^2L}}{(1+\log(2))\frac{\mu}{2}\frac{L}{\lambda_R}Log\frac{(4)E}{2\mu}} = \frac{C_A\alpha_s}{2}(\mu L)\frac{Log\frac{2E}{\mu^2L}}{Log\frac{(4)E}{2\mu}}(1+Log(2))$$

$\alpha_s = 0.3,\ L = 6\ \ fm,\ E = 50\ GeV$

$R(rad./el.) = 4-2$