CALCULATING THE JET QUENCHING PARAMETER FROM AdS/CFT

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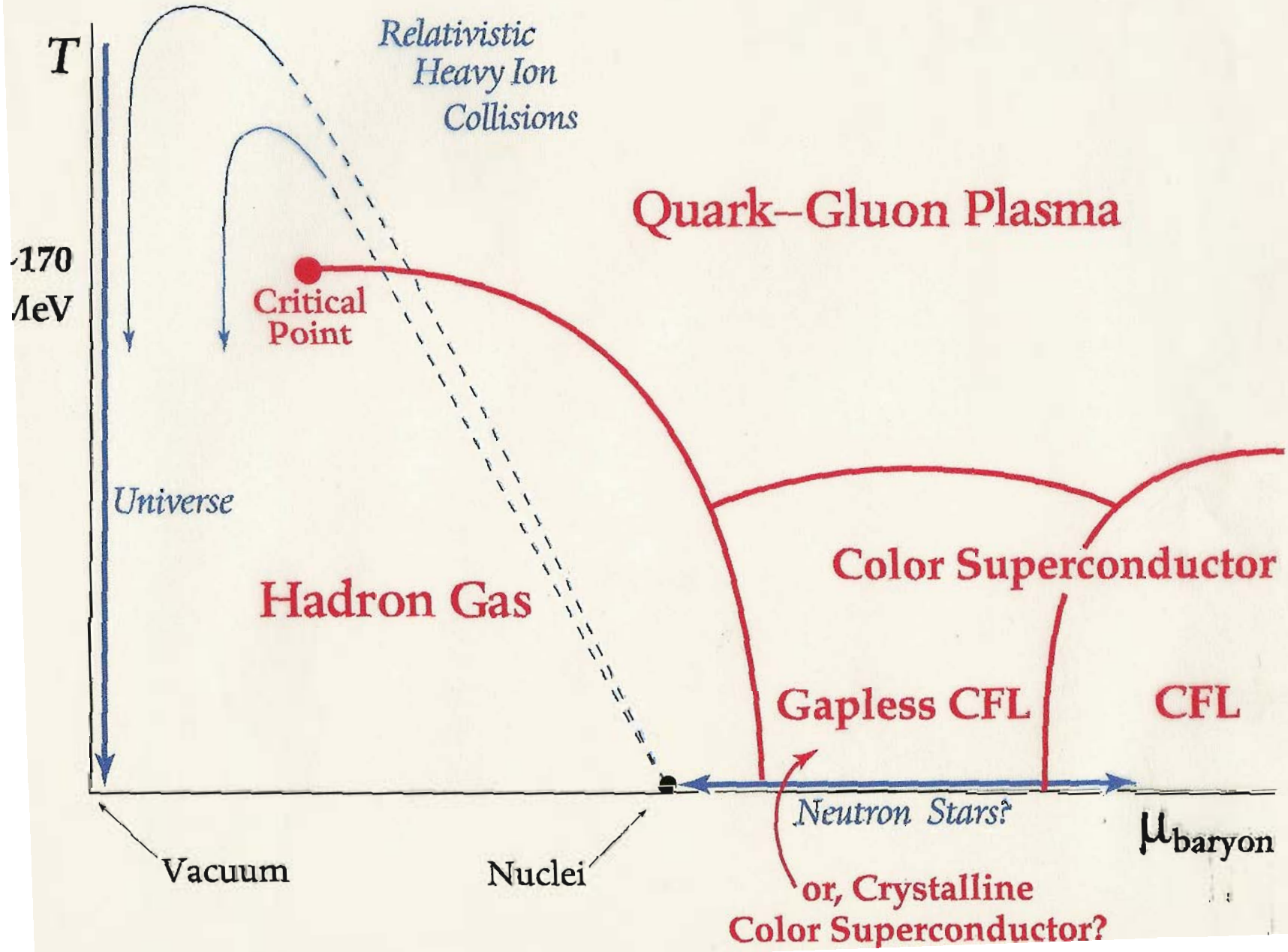
EXPLORING *the* PHASES *of* QCD



$T \neq 0$; $\mu = 0$

- vertical axis
- we know a lot from lattice QCD. eg →
- QCD describes a transition

FROM	TO
gas of hadrons	plasma of quarks and gluons
with chiral symmetry badly broken	with chiral sym. almost restored.

- $T_c \simeq 175 \pm 15$ MeV
- The transition is a smooth crossover, like ionization of a gas, occurring in a narrow range of T.

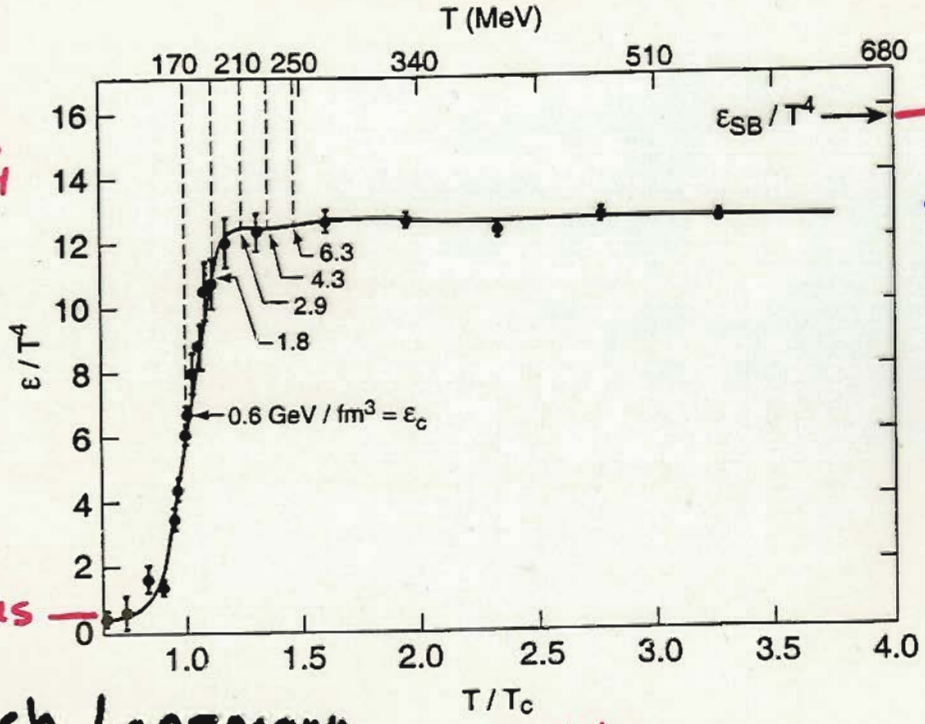
IF $m_s \gtrsim \frac{1}{5} m_s^{physical}$, and so in nature

NB: In world with $m_u = m_d = m_s$, crossover if $m_q \gtrsim \frac{1}{15} m_s^{physical}$

Bielefeld Columbia

T(MeV), assuming $T_c = 170$ MeV.
(estimate is $140 < T_c < 190$)

$\frac{\varepsilon}{T^4}$



ideal QGP

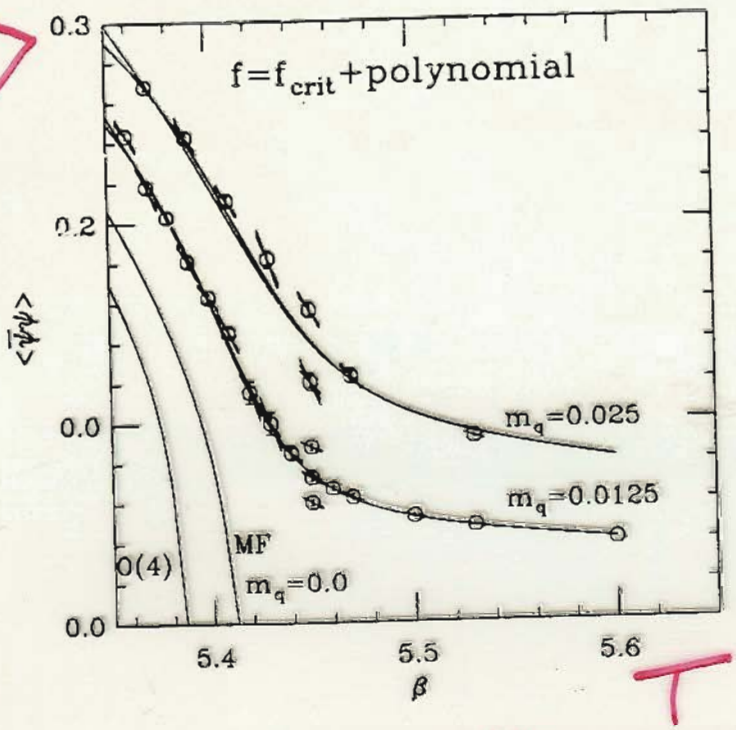
pion gas

DECONFINEMENT
(IONIZING THE HADRONS)

Karsch Laermann Peikert (Heine)

T/T_c

$\langle\bar{\psi}\psi\rangle$



Blum DeTar MILC collab.

CHIRAL SYMMETRY RESTORATION
(MELTING THE VACUUM)
ON THE LATTICE

$N_f = 2$
$m_q \neq 0$
$\therefore$ smooth crossover

T
(funny units)

EXPLORING QGP PROPERTIES

"Making QGP" is not a yes/no question:
No sharp boundary between hadrons, QGP.

Goal of RHIC: create matter ① that is above the crossover ② and study its properties. ③

①: RHIC data (on v_2) tell us interactions sufficient to yield ~equilibrated matter, expanding collectively as a fluid, by a time ~ 0.6 – 1 fm.
After that hydrodynamics (ideal hydro; zero mean free path; ideal liquid not ideal gas) describes "bulk" of particles ($p_T \lesssim 1$-2 GeV) well.

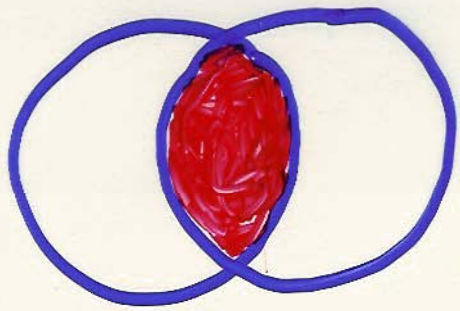
②: RHIC data (dE_T/dy) tell us
$\varepsilon(1\,\text{fm}) \underset{\text{NB}}{>} 5\,\text{GeV/fm}^3 \Rightarrow$ above crossover

So, on to ③.....

TOWARD MEASURING SHEAR VISCOSITY

Elliptic flow indicates extent of early equilibration.

Look at non-head-on collisions:

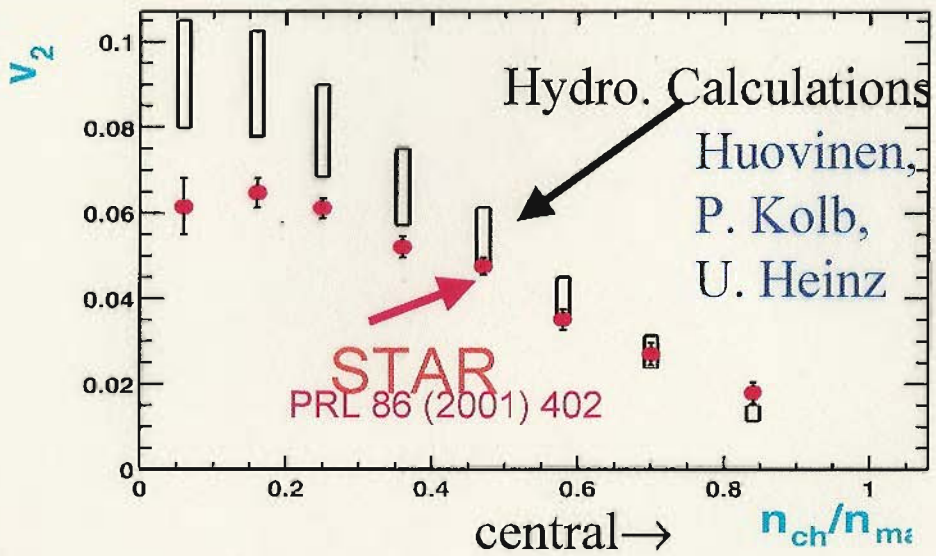


If just lots of p-p collisions followed by free streaming, then final state momenta uniformly distributed in azimuth angle ϕ.

If interaction → equilibration → pressure, pressure gradients → collective flow.

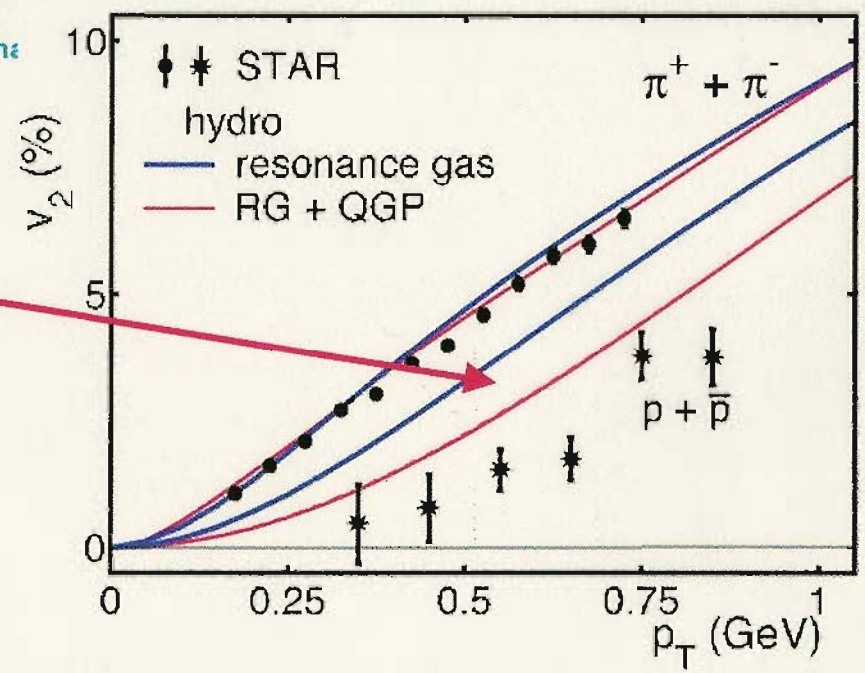
If this happens early, before circularizes by free streaming, then nonzero $V_2 \sim \langle \cos 2\phi \rangle$.

v_2 predicted by hydrodynamics



pressure buildup → explosion happens fast → early equilibration !

Hydro can reproduce magnitude of elliptic flow for π, p. BUT
must add QGP to hadronic EOS!!



Similar conclusion reached by CM Ko, et al., Kapusta, et al., Bleicher, et al., among others…

talk by B. Jacak

- Ideal hydrodynamics based on <u>assumption</u> of local eqbm.
- Hydro never agreed with data before RHIC. (At SPS, $v_2^{data} \sim \frac{v_2^{hydro}}{2}$
- At RHIC, hydro does good job of describing v_2, spectra for $P_T < 1\text{-}2$ GeV
- <u>MEANS</u>: "hydro works" by $t \sim 0.6\text{-}1$ fm

  Heinz Kolb

- Challenge to theory: how can ~equilibration occur so quickly? Strong interactions? Strong color fields → plasma instabilities?

  Mrowczynski
  Rebhan
  Romatschke
  Strickland
  Arnold Moore
  Yaffe, ...

- <u>MEANS</u>: "small" shear viscosity η.

  Teaney: $\eta/s < O(\frac{1}{10})$

  [cf water: $\eta/s > 10$]

CHALLENGES: Real extraction of η requires hydro calculations with $\eta \neq 0$.

Muronga; Heinz Song Chaudhuri

Should we be surprised if/that the QGP turns out to be liquid-like?

1) No. At $T \sim$ few T_c, coupling not small

2) But .... Lattice shows ε/T^4 reaches 80% of its value in an ideal-gas-QGP (ie noninteracting) already just above T_c. Doesn't this imply interactions are "just a 20% correction"???

3) $N=4$ SUSY QCD can teach us a lesson:

- $\varepsilon/T^4 = 75\%$ of its value in a noninteracting SUSY-QGP

ubser
ebanov
Tseytlin

- interactions <u>very</u> strong.

blicastro
on Starinets

$\eta/s = \frac{1}{4\pi} \rightarrow$ m.f.p. ~ spacing

ovtun
on Starinets

- a liquid with lower viscosity per entropy than water

- ideal hydro!

- Teaney uses v_2 data to suggest η/s of real world QGP ~ as small.

η/s IN $N=4$ SYM vs. IN QCD

- $N=4$ SYM is supersymmetric, but susy badly broken at $T \neq 0$.
- $N=4$ SYM has lots of extra fields:
  - 1 gluon
  - 6 scalars
  - 8 fermion d. of f.

  } all in adjoint rep. of color, like gluons in QCD

  But: difference in # of d. of f. in the 2 theories cancels in ratio η/s, since η & s both proportional to # of d of f.
- $N=4$ SYM has no quarks – ie no fields in fund. rep. of color.

  BUT: Buchel + Liu showed that $\frac{\eta}{s} = \frac{1}{4\pi}$ in <u>any</u> gauge theory that

has a gravity dual. η/s is a "universal" quantity. Theories with quarks with gravity duals are known. Not known whether QCD is one, or in what approximation it becomes one.

- $N=4$ SYM is conformal. QCD is <u>not</u> conformal for $T < T_c$, or for $T \to \infty$. But, for $2T_c < T < 10+ T_c$, QGP looks quite conformal. [No scale except T.] $\Rightarrow$ fig.
- η/s calculated for $N_c \to \infty$. Calculating $1/N_c$ corrections is hard, and has not been done.

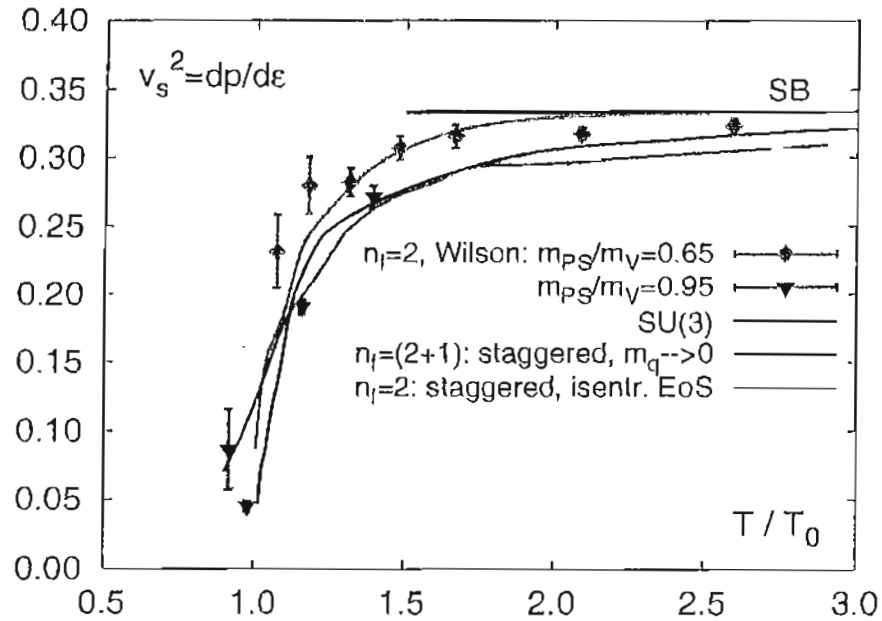


FIGURE 6. The velocity of sound in QCD vs. temperature expressed in units of the transition temperature T_0. Shown are results from calculations with Wilson [22] and staggered fermions [10] as well as for a pure SU(3) gauge theory [21]. Also shown is the resulting v_s^2 deduced from Eq. 3 [19].

- $\eta/s = \frac{1}{4\pi}$ in $\underbrace{g^2 N_c}_{\text{'t Hooft coupling.}} \to \infty$ limit.

$\equiv \lambda$

Note:

$N_c = 3,\ \alpha_s = \frac{1}{2} \leftrightarrow g^2 N_c = 6\pi \simeq 19$

So, λ <u>is</u> large in the QGP at RHIC

and, $1/\lambda$ corrections to η/s have been calculated by Buchel + Liu

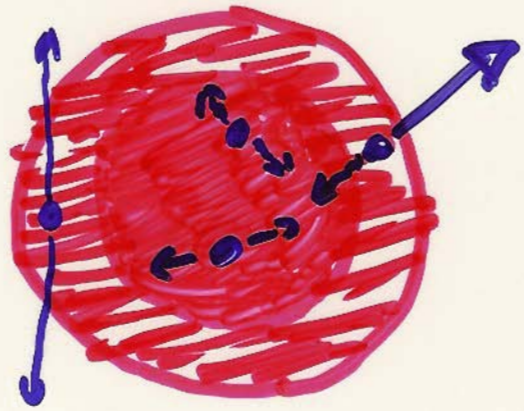
$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \underbrace{\frac{135\ \zeta(3)}{8\,(2\lambda)^{3/2}}}_{\simeq \frac{7.2}{\lambda^{3/2}} \simeq .09 \text{ for } \lambda = 6\pi} + \cdots\right)$$

Correction to $\frac{1}{4\pi}$ is small for $\lambda = 6\pi$

TOWARD MEASURING OPACITY AND PERHAPS v_{sound} AND T^3

"Jet quenching": RHIC data suggests that the rare high-p_T particles produced in initial hard scatterings are efficiently stopped. "Parton energy loss" to the point that matter is opaque.

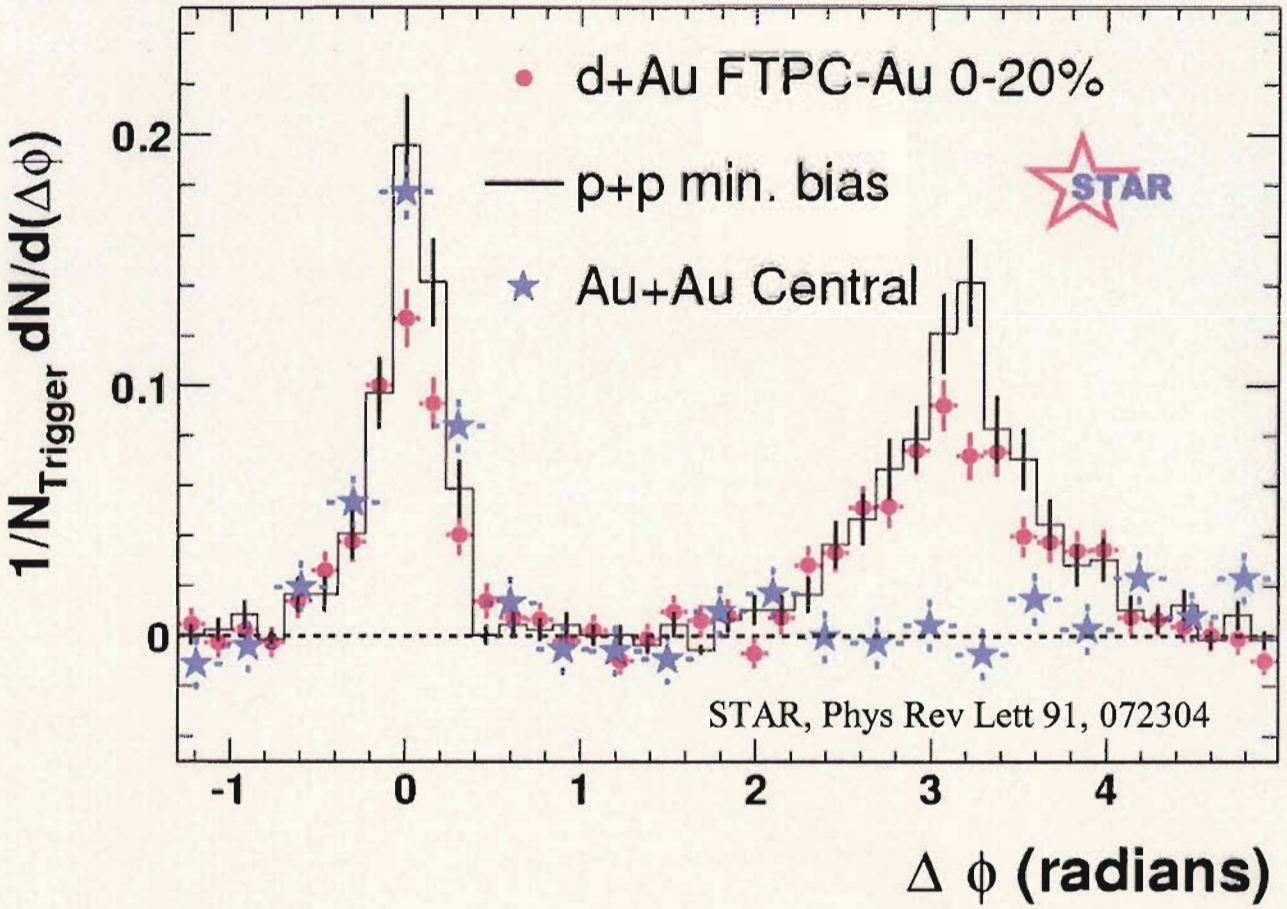
Picture Suggested:



Ingoing, and interior, jets quenched.

Should see <u>some</u> back to back jets at any p_T, and more at higher p_T.

What is known: recoiling hadrons are suppressed

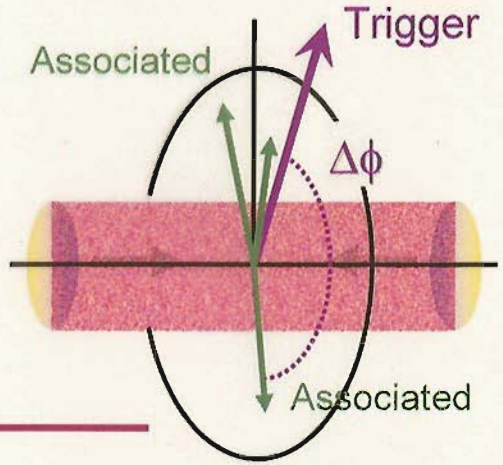


Compare to d+Au: suppression is final-state effect

Evolution of $\Delta\phi$ correlations at RHIC

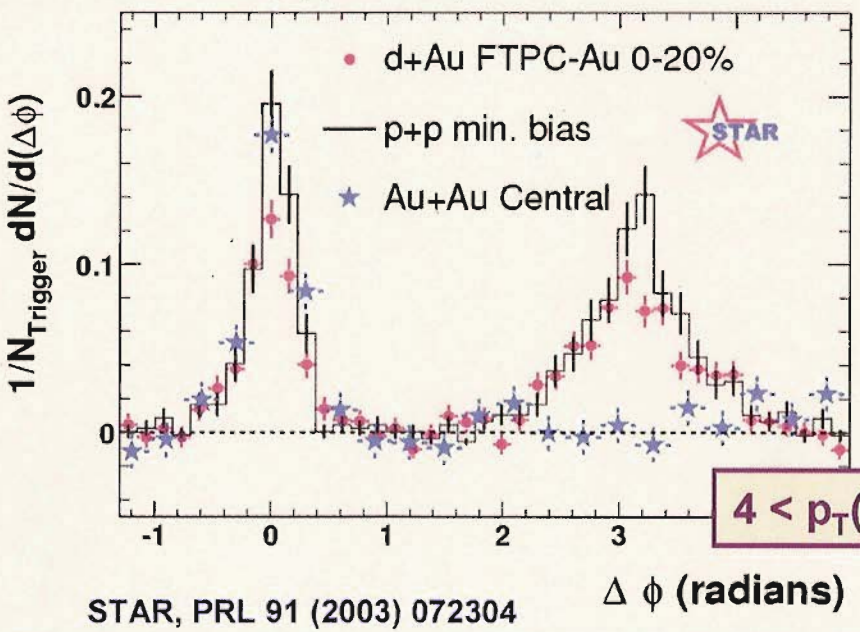
$\Delta\phi$ correlations

- "Trigger-associated" technique valuable for tagging jets in high-multiplicity environment (vs. jet-cone algorithms)
- Probes the jet's interaction with the QCD medium
- Provides stringent test of energy-loss models



Higher p_T → Away-side suppression

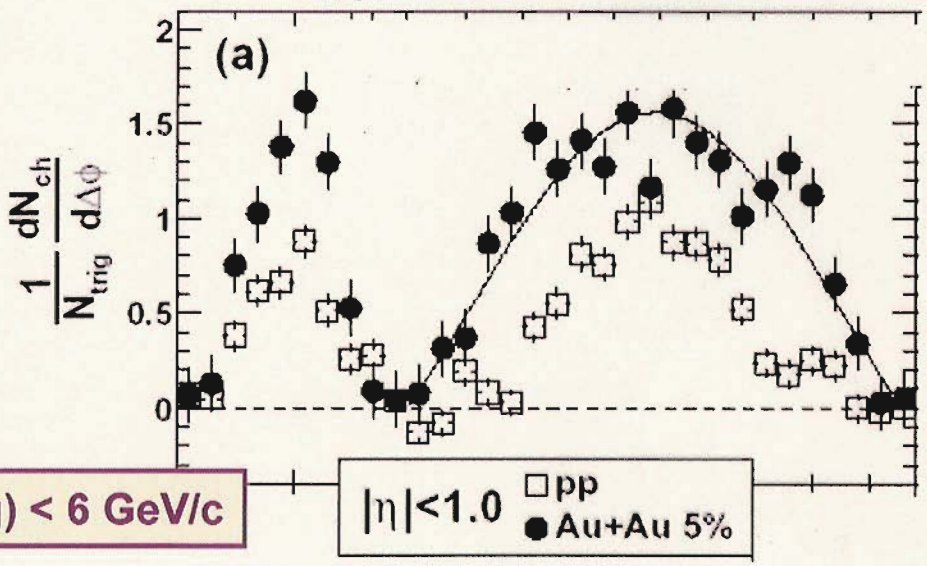
p_T(assoc) > 2 GeV/c



STAR, PRL 91 (2003) 072304

Lower p_T → Away-side enhancement

p_T(assoc) > 0.15 GeV/c

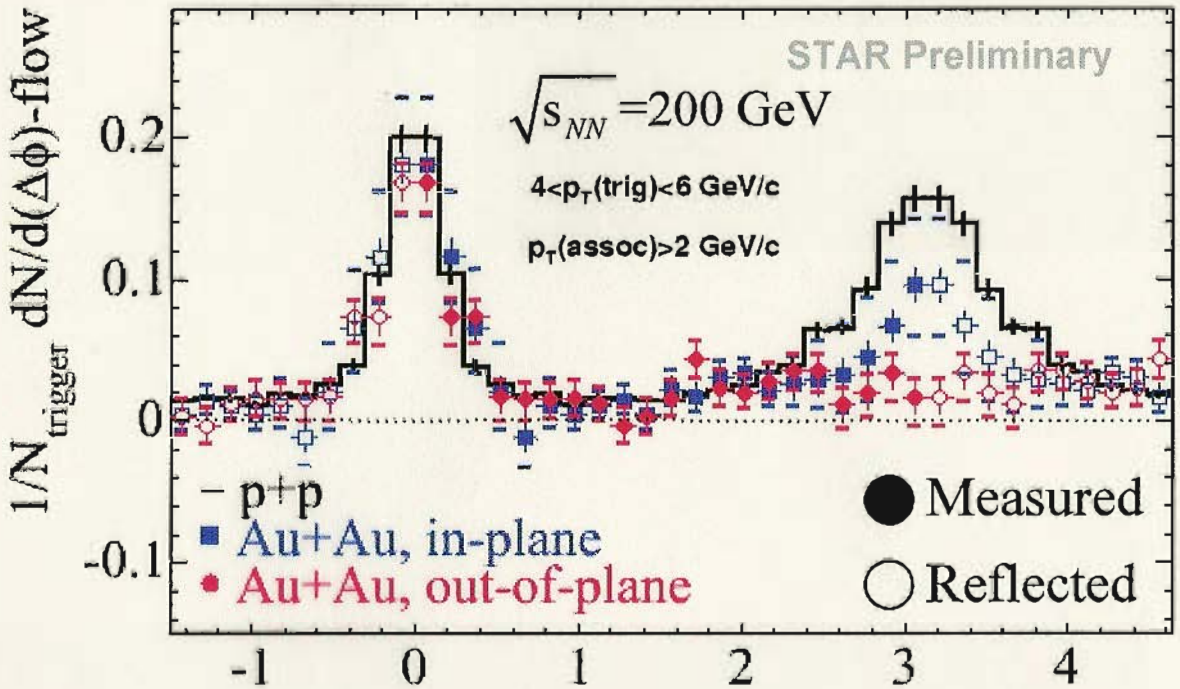


$4 < p_T$(trig) < 6 GeV/c

STAR, nucl-ex/0501016

Path Length Dependence

di-hadron, 20-60% Central



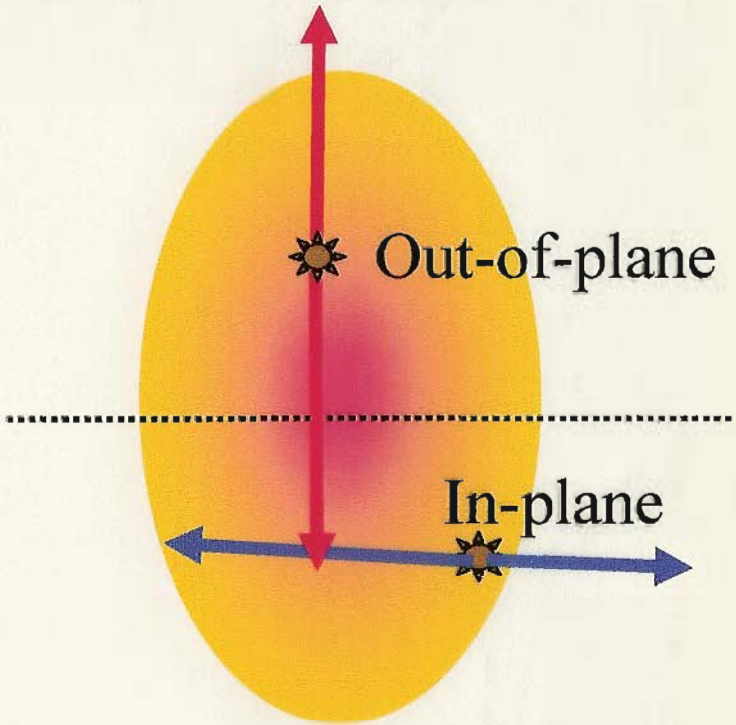
K. Filimonov DNP03

Δ φ (radians)

Suppression larger out-of-plane

Background Subtracted

See J. Bielcikova *et al.*, (nucl-ex/0311007) for background derivation



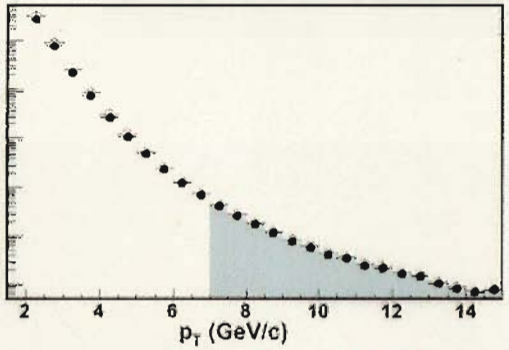
$$\Delta E_{GLV} \sim L^n(\phi)$$

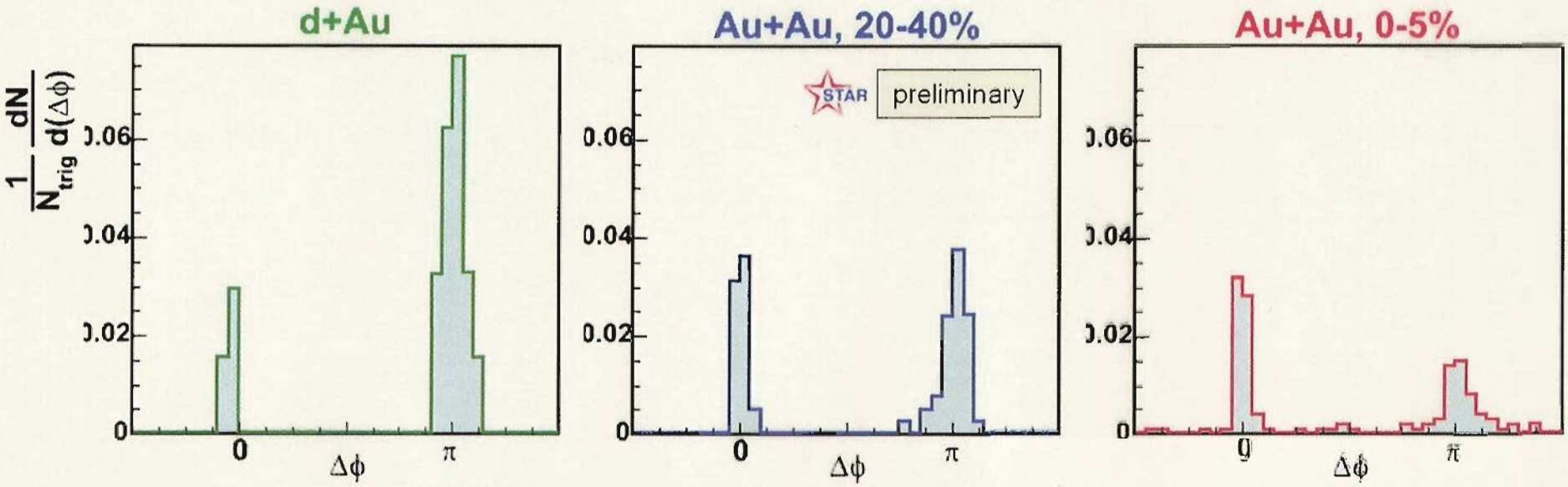
Emergence of dijets w/ increasing p_T(assoc)

- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15$ GeV/c

$p_T(\text{assoc}) > 7$ GeV/c





- Narrow peak emerges cleanly above vanishing background

WHAT DO WE LEARN ABOUT THE MEDIUM FROM HOW A HARD PARTON LOSES ENERGY PLOWING THROUGH IT?

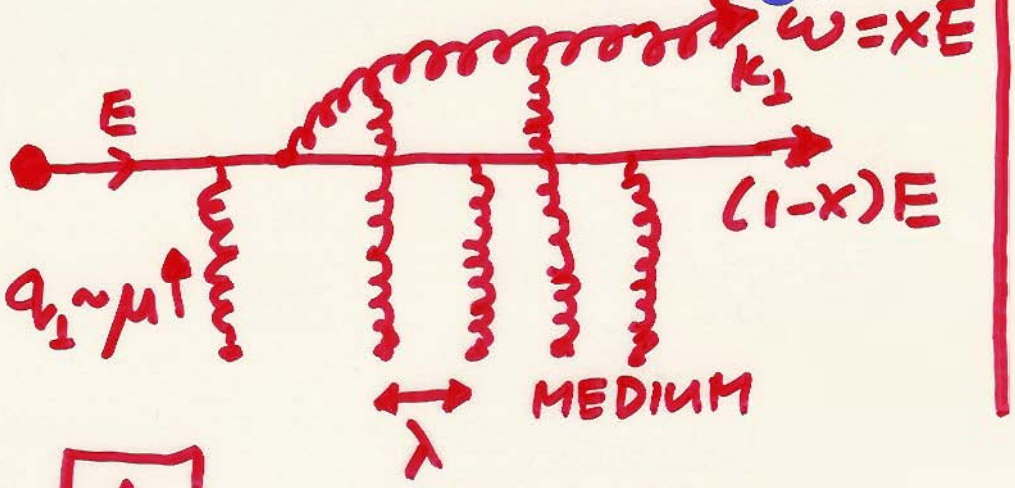
Perturbative formalism for calculating parton energy loss: Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov, Wiedemann, Gyulassy, Wang, Wang, Levai, Vitev, ...

Sensitive to medium through one parameter, called $\hat{q}$.

Energy lost by gluon radiation.

Spectrum of radiated energy:

$$\omega \frac{dI}{d\omega} \sim \left|\right.$$



$$\left.\right|^2$$

$$\sim \alpha_s \sqrt{\frac{\hat{q}}{\omega}}\, L_- \quad \text{for } \omega < \hat{q} L_-^2$$

where $\hat{q} = \frac{\mu^2}{\lambda}$ and L_- = distance travelled in medium

$\hat{q}$ is p_T broadening per L_- travelled

$$\langle k_\perp^2 \rangle \sim \hat{q} L_-$$

Energy lost by leading parton is

$$\Delta E \sim \alpha_s \hat{q} L_-^2$$

Perturbative intuition:

$\hat{q} \sim \frac{\mu^2}{\lambda}$ ← (Debye screening length)$^{-2}$

λ ← "mean free path"

$\sim n_{\text{scatterers}} \cdot$ (Dimensionless measure of σ)

Implications of RHIC data:

Eskola Honkanen Salgado Wiedemann → Fig

Dainese Loizides Paic

$\bar{\hat{q}} \sim (5\text{-}15)\ \text{GeV}^2/\text{fm}$

- the $\gtrsim 5$ more reliable than the $\lesssim 15$.
- current data can really only provide a lower bound, since quenching so effective

Time averaged $\hat{q}$. Relation to $\hat{q}(\tau)$ depends on assumptions, but $\bar{\hat{q}} \sim \hat{q}\left(\frac{L_-}{2}\right) \sim \hat{q}(1\text{-}1.5\,\text{fm})$

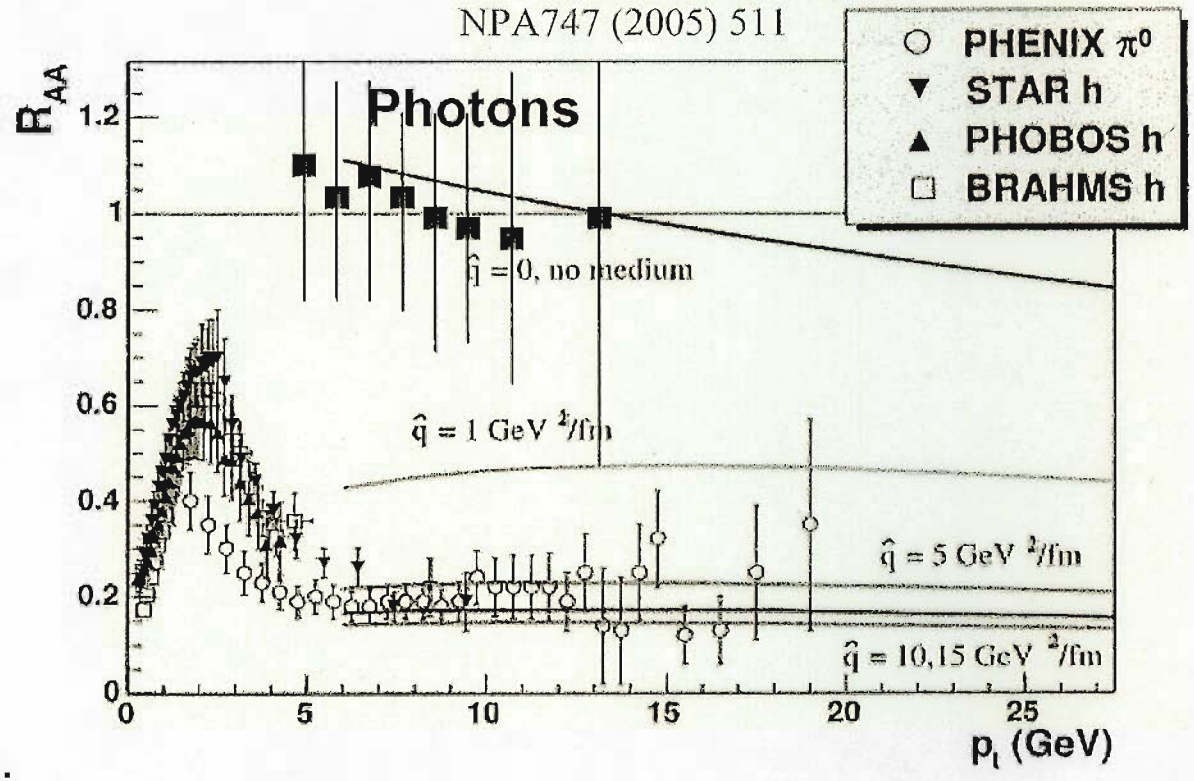
The suppression of leading hadrons

Parton energy loss calculations account for:

- Nuclear modification factor
- Centrality dependence
- Back-to-back correlations
- R_{AA} = 0.2 is a natural limit due to surface emission

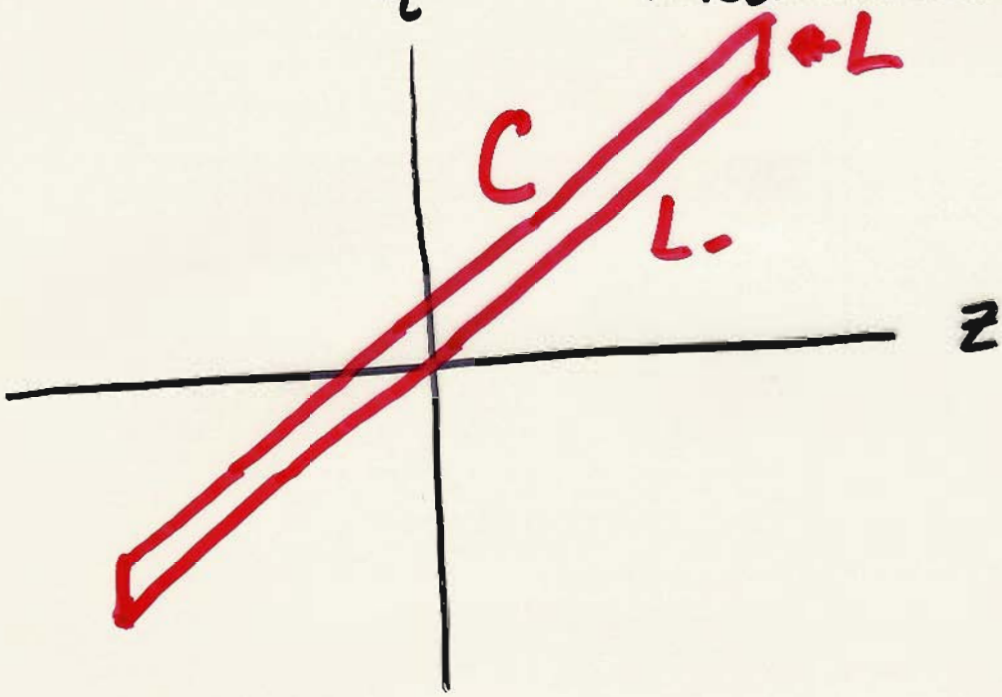
indicates very opaque medium.

Eskola, Honkanen, Salgado, Wiedemann
NPA747 (2005) 511



$\hat{q} \leftrightarrow$ LIGHT-LIKE ADJOINT WILSON LOOP

Wiedemann



Contour C : a rectangle,

L_- distance travelled along light cone

L transverse

$$L_- \gg L \; ; \quad L \sim \frac{1}{k_T}$$

$$\langle W^A(C) \rangle_T \propto \exp\left[-\frac{\hat{q}}{4} L_- L^2 \right]$$

for $LT \ll 1$.

Reproduces perturbative definition.

Provides nonperturbative definition!

$\hat{q}$ IN N=4 SYM FROM AdS/CFT

H. Liu, KR, Wiedemann

Use AdS/CFT to calculate Wilson loop at nonzero T, in large-N_c and large $\lambda \equiv g^2 N_c$ limits. [NB: Minkowski space crucial, ∴ inaccessible to lattice QCD]

$$\langle W^A(C)\rangle = \langle W^F(C)\rangle^2 + \mathcal{O}\left(\frac{1}{N_c}\right)$$

Prescription: (Maldacena; Witten; Gubser Klebanov Polyakov; Maldacena; Rey; Yee; ....)

$$\langle W_F(C)\rangle = \exp[-S]$$

where S is action of an extremized world sheet in a 5Dimensional Anti-deSitter space with a black hole in it, with boundary C at $r=\infty$.

r: 5th dimension.

We live at $r=\infty$

BH horizon at $r=r_0$

TRANSLATION DICTIONARY

N=4 SYM gauge theory in 3+1 Mink.

Type II-B String Th in $AdS_5 \times S_5$

$$\frac{\lambda}{4\pi N_c} = g_{string}$$

If g_{string} is small, string theory calculation reduces to solving classical GR problems in certain metric. No stringy corrections. That's why we need $N_c \to \infty$.

$$T = T_H = \frac{r_0}{\pi R^2}$$

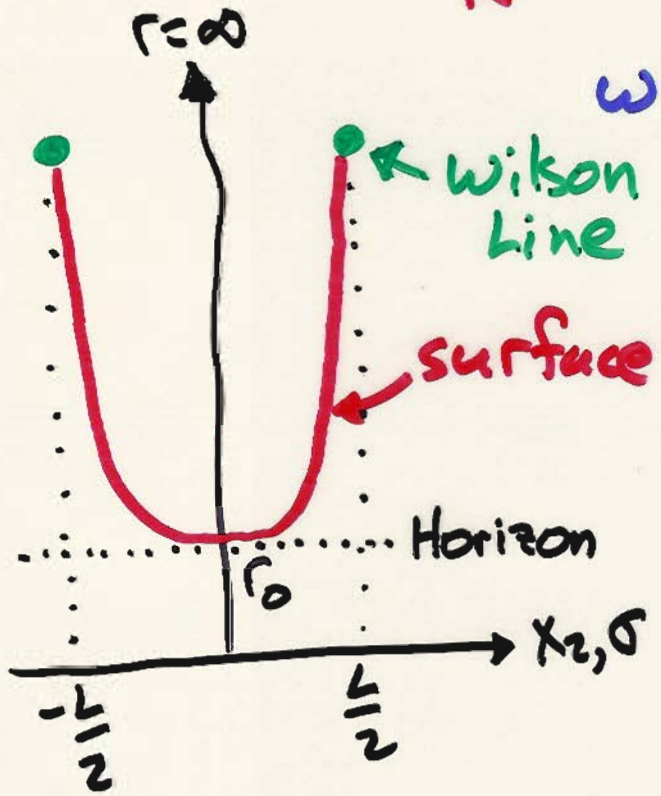
where R is curvature of AdS

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

where $\frac{1}{2\pi\alpha'}$ is string tension

$$ds^2 = -\left(\frac{r^2}{R^2}+f\right)dx^+dx^- + \frac{1}{2}\left(\frac{r^2}{R^2}-f\right)\left(dx^{+2}+dx^{-2}\right) + \frac{r^2}{R^2}\left(dx_2^2+dx_3^2\right) + \frac{1}{f}dr^2$$

where $f = \frac{r^2}{R^2}\left(1-\frac{r_0^4}{r^4}\right)$



Surface: $X^\mu(\sigma,\tau)$, $\sigma \to x_2$, $\tau \to x^-$

$x^+ = \text{const}$, $x_3 = \text{const}$

Need: $r(\sigma)$

b.c.: $r(\pm L/2) = \infty$

NB: $L_- \gg L$

$$S(C) = \frac{i}{2\pi\alpha'}\int d\sigma\, d\tau \sqrt{-\det g_{\alpha\beta}}$$

where $g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$

$$= \frac{\sqrt{2}\, r_0^2 L_-}{2\pi\alpha' R^2}\int_0^{L/2} d\sigma \sqrt{1 + r'^2 R^2/f r^2}$$

Extremize. → E of M for $r(\sigma)$: $r'^2 = \gamma^2 r^2 f/R^2$ (int. const.)

$\Rightarrow r' = 0 \Leftrightarrow f = 0 \Leftrightarrow r = r_0$

∴ SURFACE KISSES HORIZON, FOR ANY L!

Can now do the integrals:

$$\frac{L}{2} = \int_0^{L/2} d\sigma = \int_{r_0}^{\infty} \frac{dr}{r'} = \frac{R^2}{\gamma}\int_{r_0}^{\infty} \frac{dr}{\sqrt{r^4 - r_0^4}}$$

$$= \frac{aR^2}{\gamma r_0} \quad \text{with } a = \frac{\sqrt{\pi}\,\Gamma(5/4)}{\Gamma(3/4)} \simeq 1.311$$

and hence

$$S(c) = \frac{1}{2\sqrt{2}}\pi\sqrt{\lambda}\, L_- L\, T^2 \sqrt{1 + 4a^2/\pi^2 T^2 L^2}$$

where we used $T = r_0/\pi R^2$ + $\sqrt{\lambda} = R^2/\alpha'$.

Finally, must subtract self-energy of q and $\bar{q}$, given by two disconnected surfaces "hanging" from $r=\infty$ to $r=r_0$ at const. X_2

$$S_0 = \frac{1}{\sqrt{2}} a\sqrt{\lambda}\, L_- T$$

And, $\langle W_A \rangle = \langle W_F \rangle^2$ means $S \to 2S$.

$$2(S(c) - S_0) = \underbrace{\frac{\pi^2}{4\sqrt{2}\,a}\sqrt{\lambda}\, T^3 L^2 L_-}_{\frac{1}{4}\hat{q}\,!} + \mathcal{O}(T^5 L^4 L_-)$$

WHAT DO WE LEARN?

$$\hat{q}_{SYM} = \frac{\pi^{3/2}\Gamma(5/4)}{\sqrt{2}\,\Gamma(3/4)}\sqrt{\lambda}\,T^3 = \underline{\underline{18.87\sqrt{\alpha N_c}\,T^3}}$$

- $\hat{q}$ is <u>not</u> proportional to S, or $n_{scatterers}$, as these are proportional to N_c^2. Also, $\varepsilon \sim N_c^2$ so $\hat{q} \not\propto \varepsilon^{3/4}$.

  [Redo calculation in (p+1)-dim SYM, find $\hat{q}$ & S have different T-dep. for $p \neq 3$]

- $\hat{q}$ is better thought of as measuring T^3!

- Try some numbers: $N_c = 3$, $\alpha = \frac{1}{2}$

  $\hat{q}_{SYM} = 3.2, 7.5, 14.7$ GeV^2/fm

  for $T = 300, 400, 500$ MeV

  - right ballpark, but a little low?

$$\bar{\hat{q}} \equiv \frac{2}{L_-^2}\int_{\tau_0}^{\tau_0+L_-} \tau\,\hat{q}(\tau)\,d\tau \;;\; T(\tau) = T_0\left(\frac{\tau_0}{\tau}\right)^{1/3}$$

$\Rightarrow \bar{\hat{q}} = 10$ GeV^2/fm, $L_- = 2$ fm $\Leftrightarrow T(1\text{fm}) = 440$ MeV

- too hot

IMPLICATIONS

Either: In going from $N=4$ SYM to QCD (reduce # of adjoints by $\frac{2}{15}$; add quarks) $\hat{q}$ goes somewhat up.

- Evaluate $\hat{q}$ in $N=2^*$, see if it is increasing.
- Evaluate $1/\lambda$ corrections
- More generally, evaluate $\hat{q}$ in as many theories with gravity duals as possible, and see whether there is a "universal" combination. (Not $\hat{q}/s$. Maybe $\hat{q}/T^3$?) If not universal, see whether $\hat{q}/T^3 \uparrow$ as SUSY $\downarrow$.

- Buchel

$$\hat{q}_{KW} = \sqrt{\frac{27}{32}}\,\hat{q}_{N=4}$$ KW: a different conformal theory

$$\hat{q}_{KS} = \left[1 - 3.123\left(c_s^2 - \tfrac{1}{3}\right)\right]\hat{q}_{KW}$$ KS: a nonconformal theory

$\simeq 0.85\,\hat{q}_{KW}$ for $c_s^2 - \frac{1}{3} = .05$, as in QCD at $T \sim 1.5\,T_c$

Maybe corrections due to nonconformality are small?

- Poritz: possibly examples with only $N=1$ SUSY in which $\hat{q} > \hat{q}_{N=4}$
- Really want examples where "15" reduced. $\to N=2^*$ ($N=4$ with masses for some fields
- Recent calculation of drag on a <u>heavy</u> quark moving through $N=4$ SYM:

$$\frac{dp}{dt} = -\frac{\pi\sqrt{\lambda}}{2}\,T^2\,\frac{p}{m}$$

Herzog et al; Gubser; Teaney, Casalderrey; ....

This drag coefficient is also <u>not</u> $\propto N_c^2$, ie not a measure of # of d. of f. !

This is a calculation of a different quantity in a different regime. But, also related to energy loss so would be nice to relate to our calculation...

<u>Or</u>: Current extraction of $\hat{q}$ from RHIC data is misleading us, because there are actually additional sources of energy loss.

- HIC @ LHC
  - much higher energy jets
  - see actual jets, not just leading hadrons
  - study jet modification
    → more discriminating observables.

CAN WE MEASURE (OR BOUND) ν AND DEMONSTRATE DECONFINEMENT?

$$\nu \sim \frac{\varepsilon}{T^4} \sim \frac{s}{T^3} \sim \frac{s^4}{\varepsilon^3}$$

We know: $\varepsilon(1\,\text{fm}) > 5\,\text{GeV}/\text{fm}^3$

We can estimate $s(1\,\text{fm})$ from final state entropy, assuming equilibration before 1 fm:

$s(1\,\text{fm}) = 33 \pm 3\ \text{fm}^{-3}$ Muller, KR

Can we use jet quenching observables to get upper bound on T^3?

Challenge to theory: $\hat{q} \leftrightarrow T^3$ in QCD

Challenge to exp + theory:

reliable upper bound on $\hat{q}$.

(Need jet modification, not quenching)

Other routes to T, also hard:

photons? J/ψ?

FOR YOUR AMUSEMENT

If $s(1\,\text{fm}) = 33\ \text{fm}^{-3}$

if $L_- = 2\,\text{fm}$

if $\alpha = \frac{1}{2}$

if $\hat{q}_{QCD} = 18.87\sqrt{3\alpha}\,T^3$

if $\bar{\hat{q}}_{\text{from RHIC}} = 2.25\ \text{GeV}^2/\text{fm}$

THEN: $T(1\,\text{fm}) = 268\ \text{MeV}$

AND, using $s = \frac{2\pi^2}{45}\nu T^3$ and $s = \frac{33}{\text{fm}^3}$

$\rightarrow$ $\nu = 30$, as in lattice QCD at $T = 1.5\,T_c$.

So: Watch with interest how $\bar{\hat{q}}_{RHIC}$ evolves, and on the theory side how large corrections to $\hat{q}_{QCD}$ vs $\hat{q}_{SYM}$ appear to be.