Evolution at small x: The Color Glass Condensate

Heribert Weigert
The Ohio State University

Hard Probes 2006,
Asilomar, CA

Outline

1. Motivation: gluons form the CGC
   - Current and planned collider experiments
   - Enhanced gluon production at high energies
   - CGC: why the name
2. JIMWLK evolution: properties of the CGC
   - Gluons in observables
   - The evolution equation
   - The saturation scale
3. Experiment
   - Geometric scaling @ HERA
   - Erasing the Cronin effect @ RHIC
4. Overview and outlook

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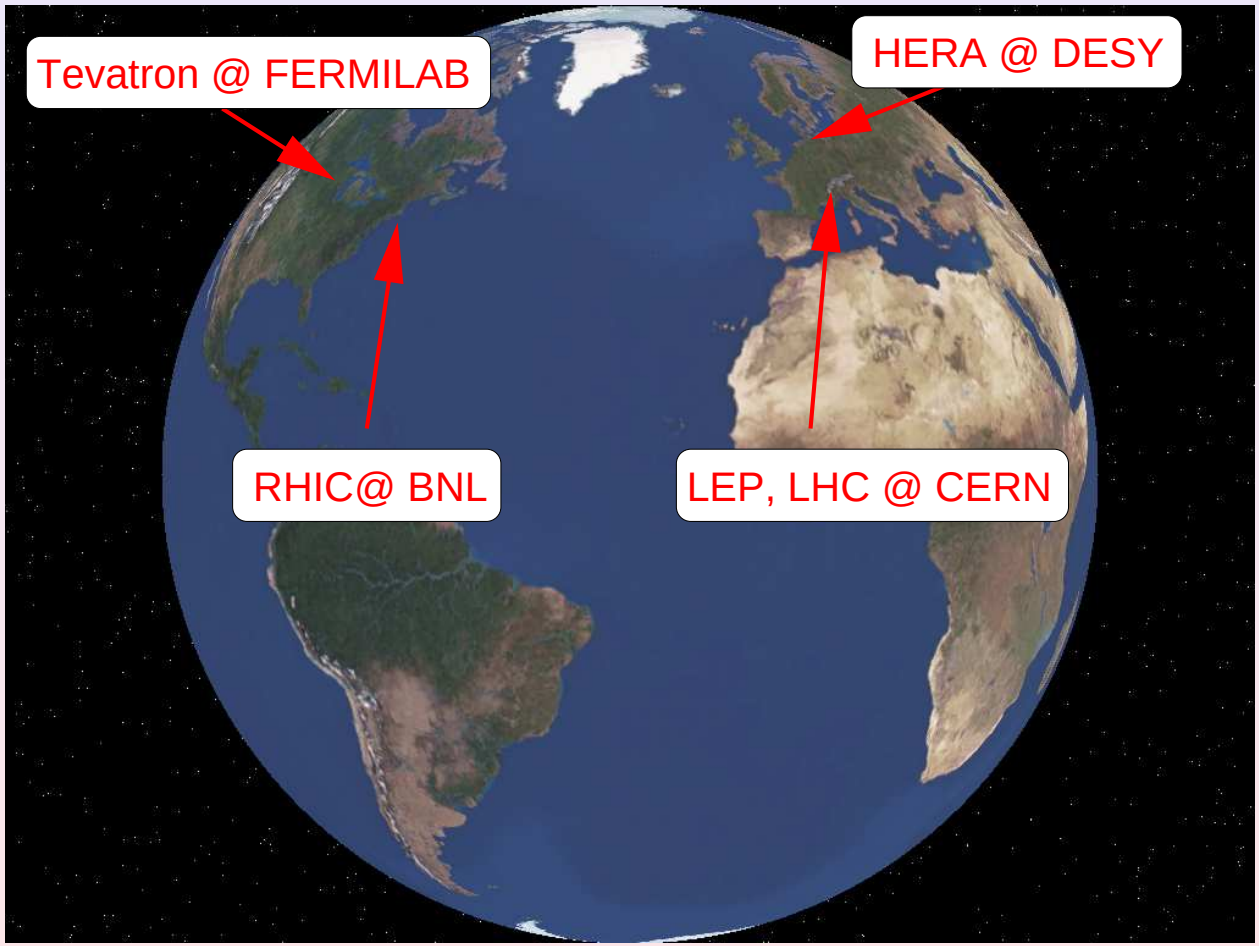
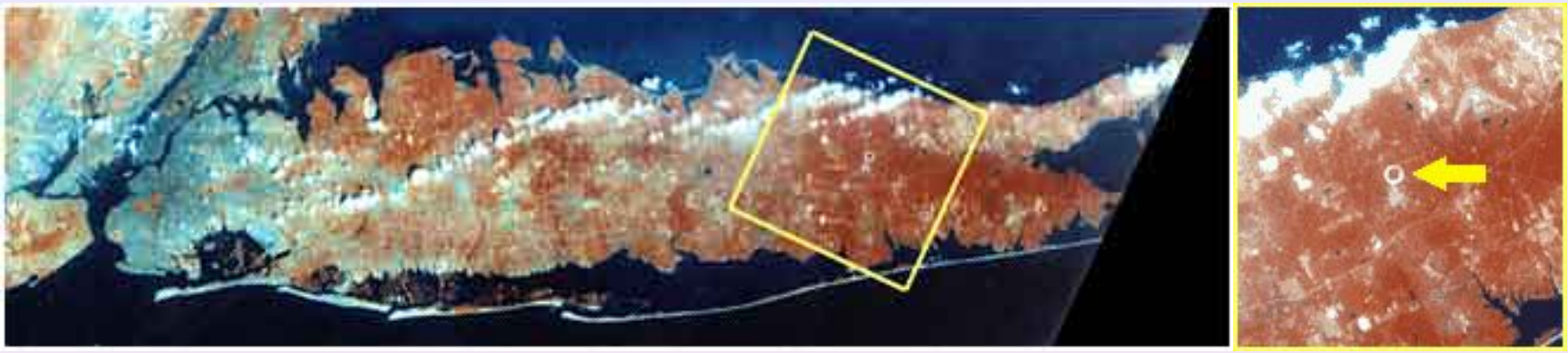
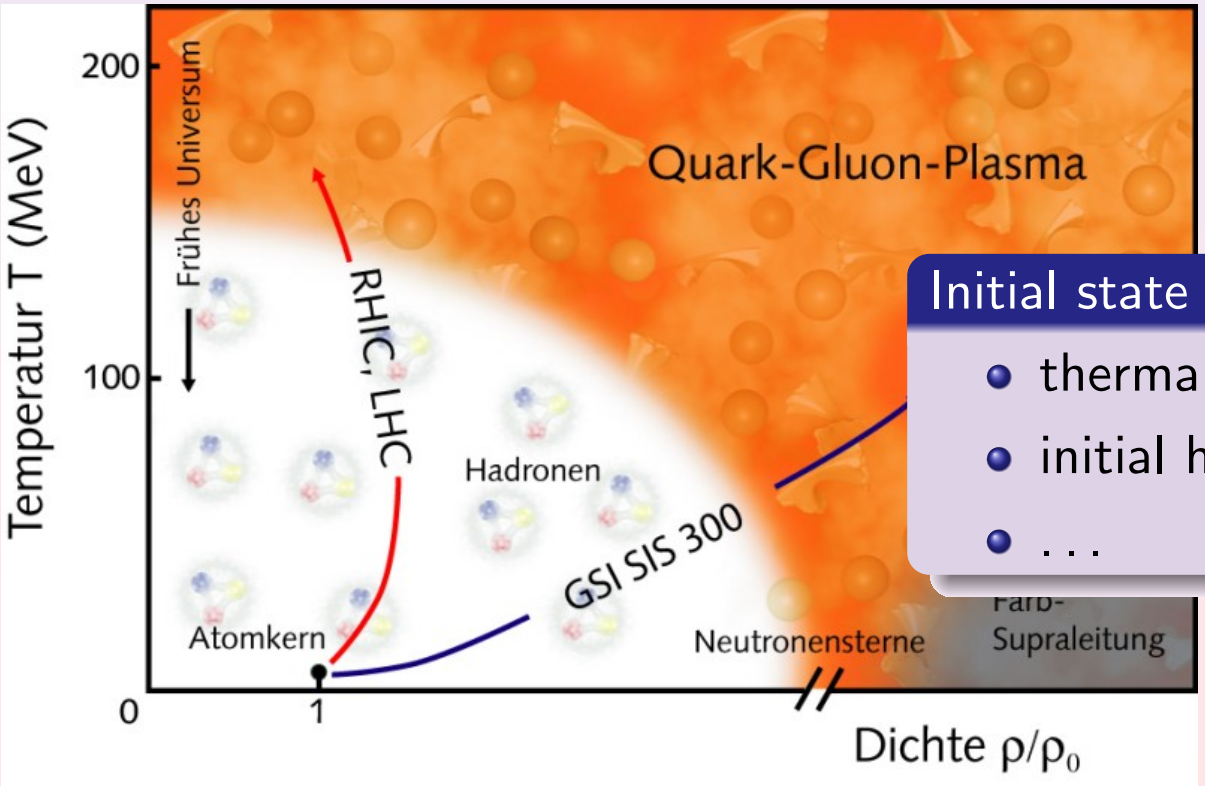
Current collider experiments worldwide



RHIC: searching for the Quark Gluon Plasma





Initial state determines final state via

- thermalization times
- initial hard particle production
- ...

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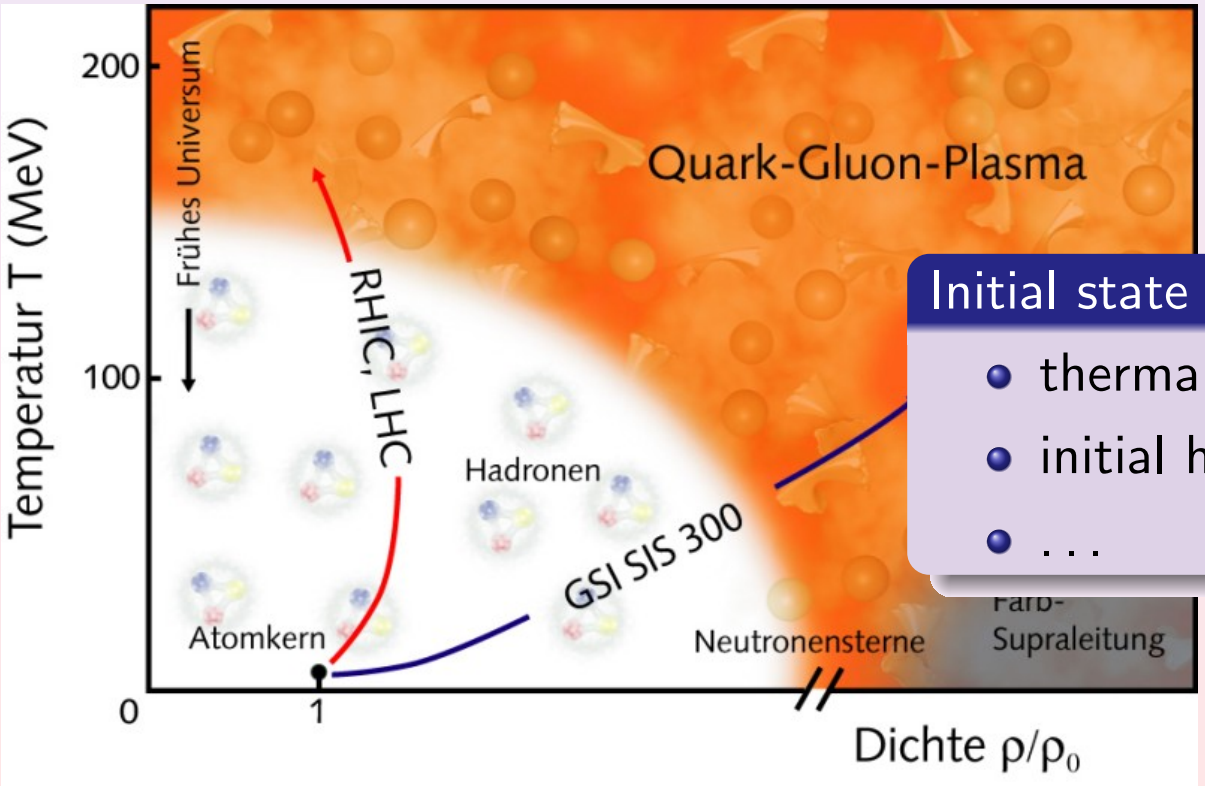

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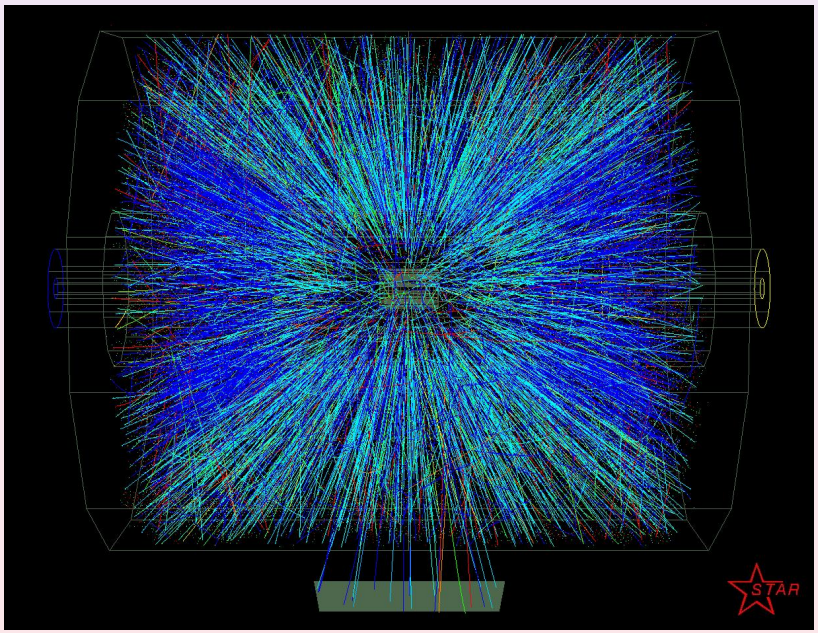
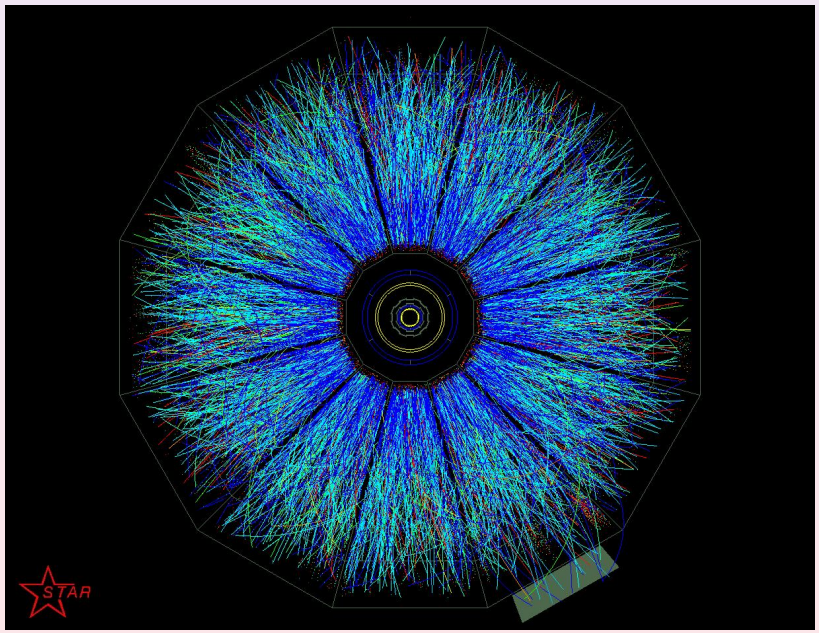


side view



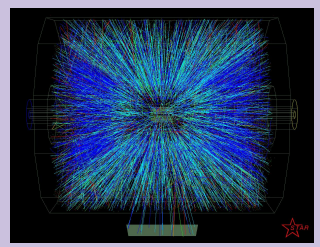
front view

Particle production at modern colliders

Large amounts of energy available: 200-14000 m_{proton}

heavy ions @ RHIC

- 200GeV/nucleon pair — QGP



particle & heavy ions @ LHC

- 4TeV/nucleon pair — QGP
- 14TeV to cross production thresholds — Higgs

amount of data/event $\equiv$ capacity of phone lines in Europe

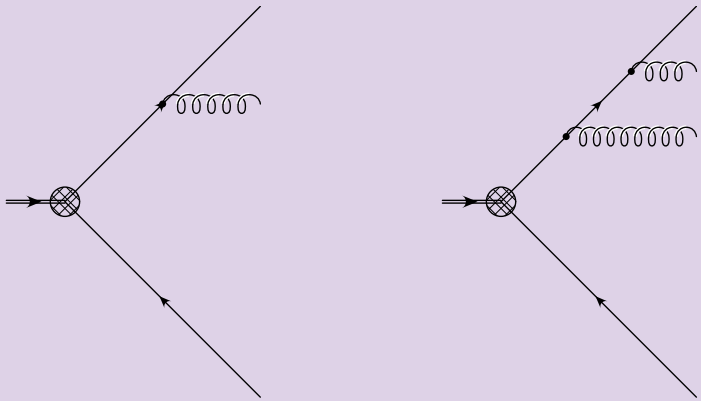
QCD drives particle production

- serious backgrounds for particle searches
- new physics phenomena

copious gluon production → Color Glass Condensate CGC

From photons to gluons

photon-like contributions



- enhanced by phase space integrals $\frac{dE}{E}\frac{d\theta}{\theta}$ ➡ $\alpha_s \ln E \ln \theta$
- all orders calculation needed $\sum_{n=0}^{\infty} (\alpha_s \ln E)^n \ldots$

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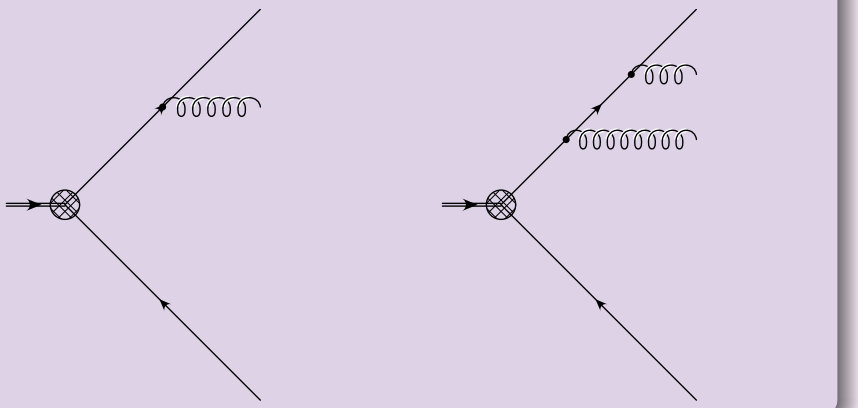
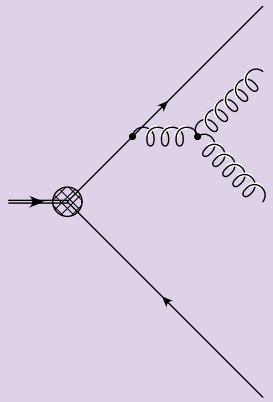


QCD: charged gluons



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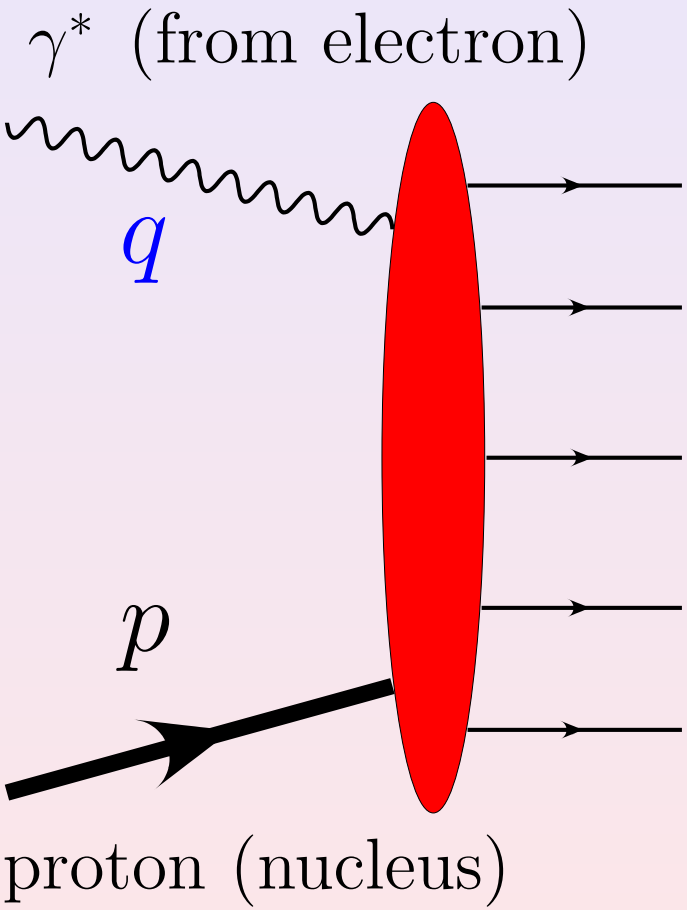
- gluons charged ➡ radiation nonlinear in QCD

Kinematic variables: transverse resolution vs energy



- $Q^2 := -q^2 \gg 0$ spacelike! transverse resolution $\Delta \boldsymbol{r} \sim \frac{1}{Q}$

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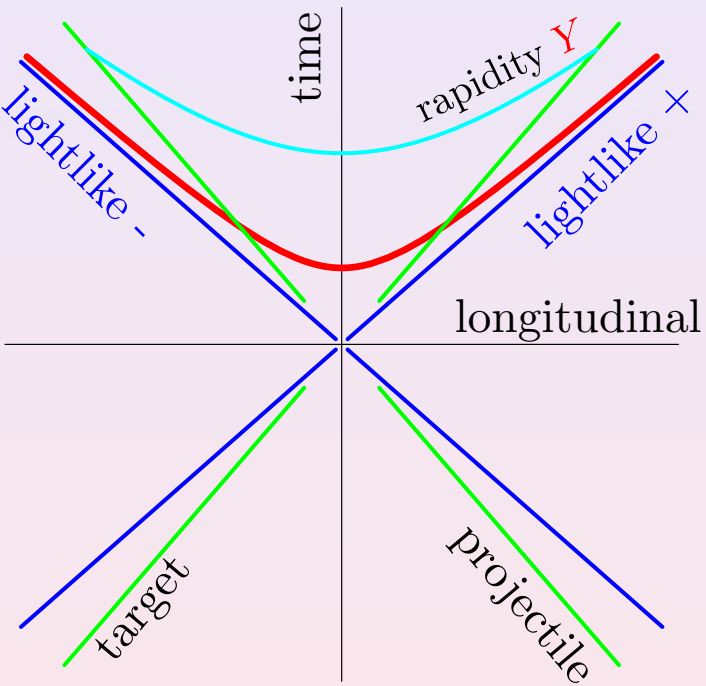

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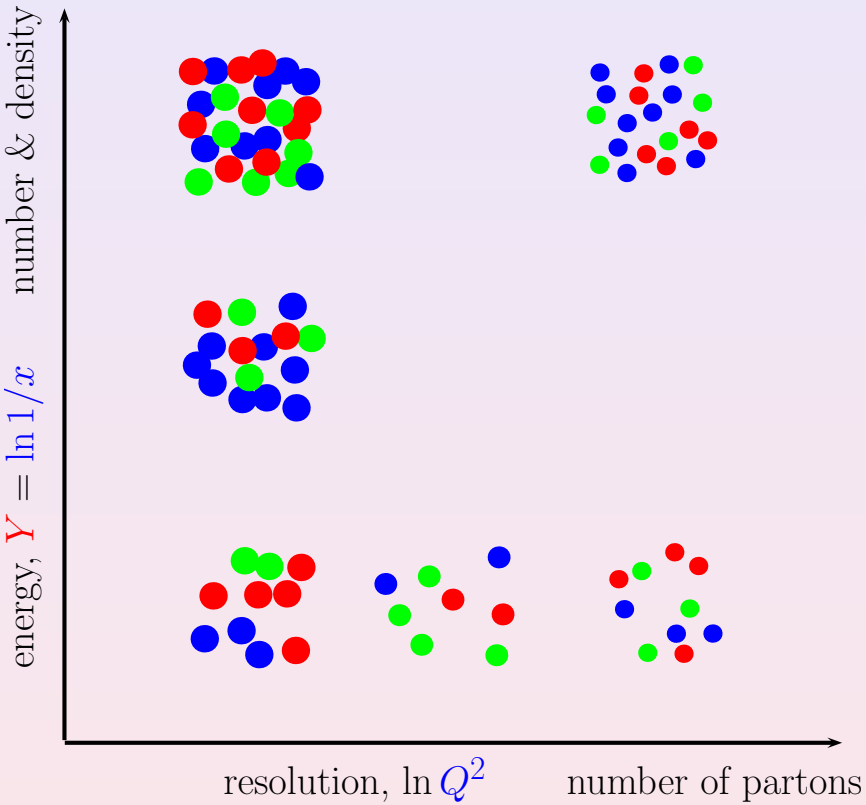
- $Y = \ln \frac{1}{x} \propto \ln E_{\mathrm{rest}}$ all used synonymously

Large energies mean large densities



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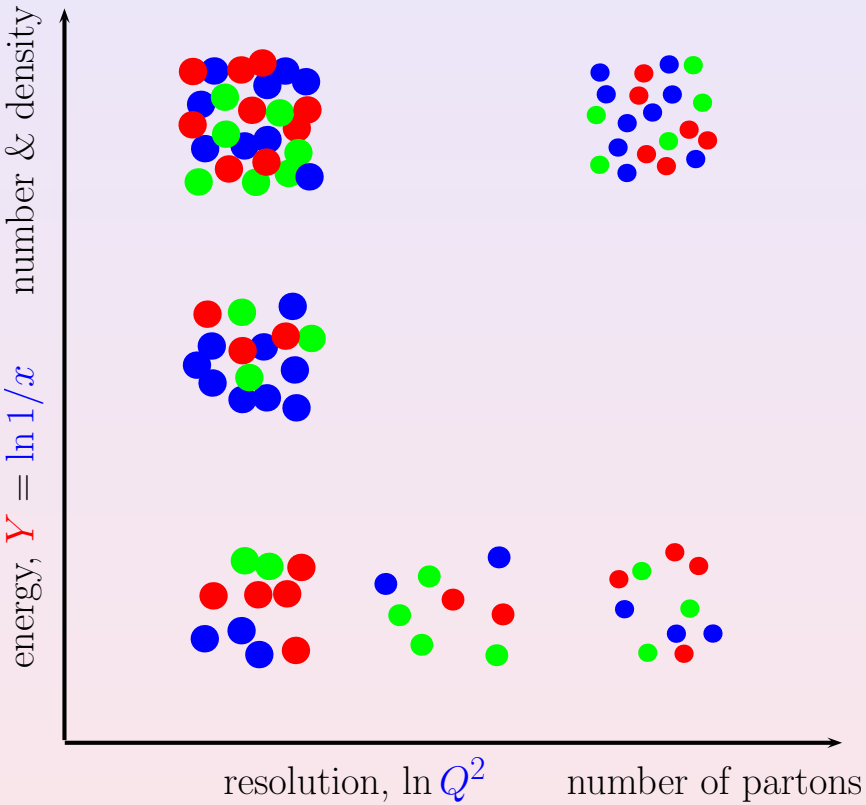

- density ⟶ finite correlation length R_s

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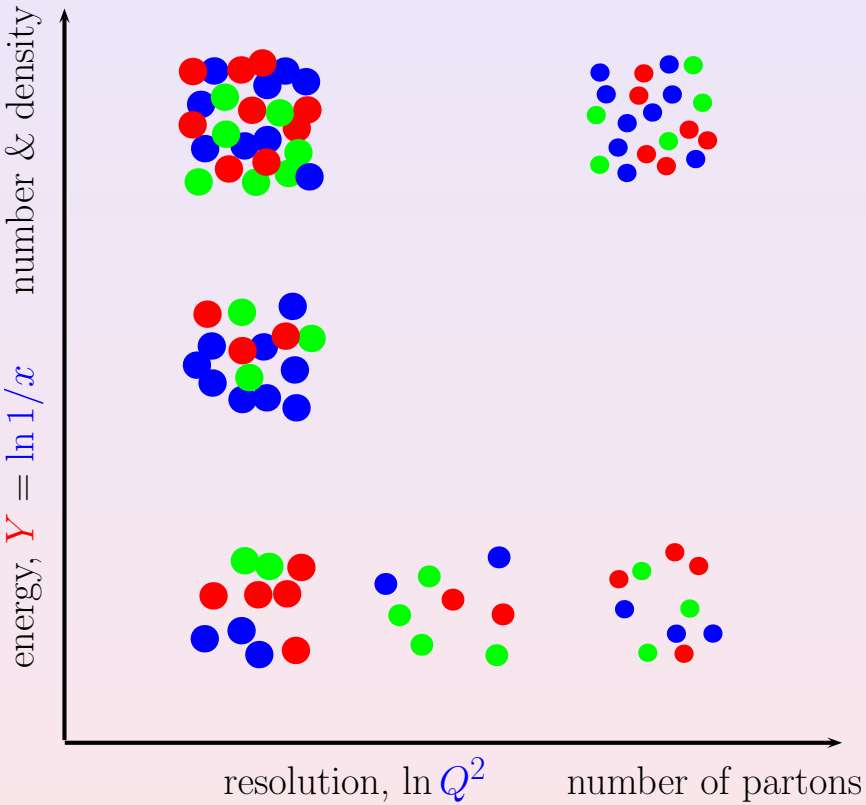

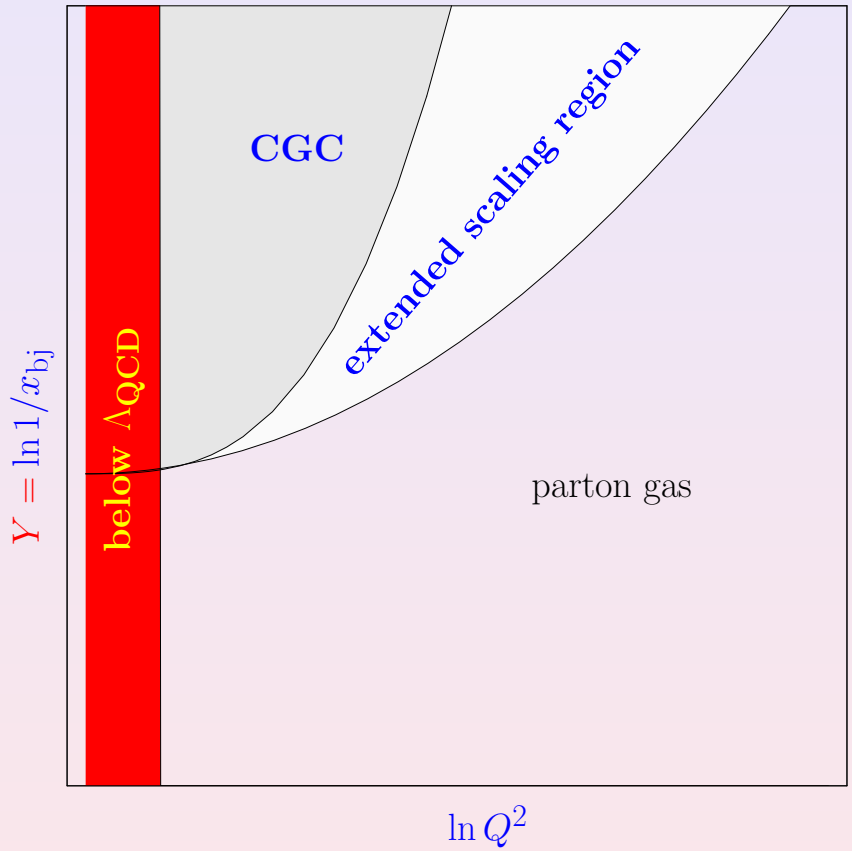

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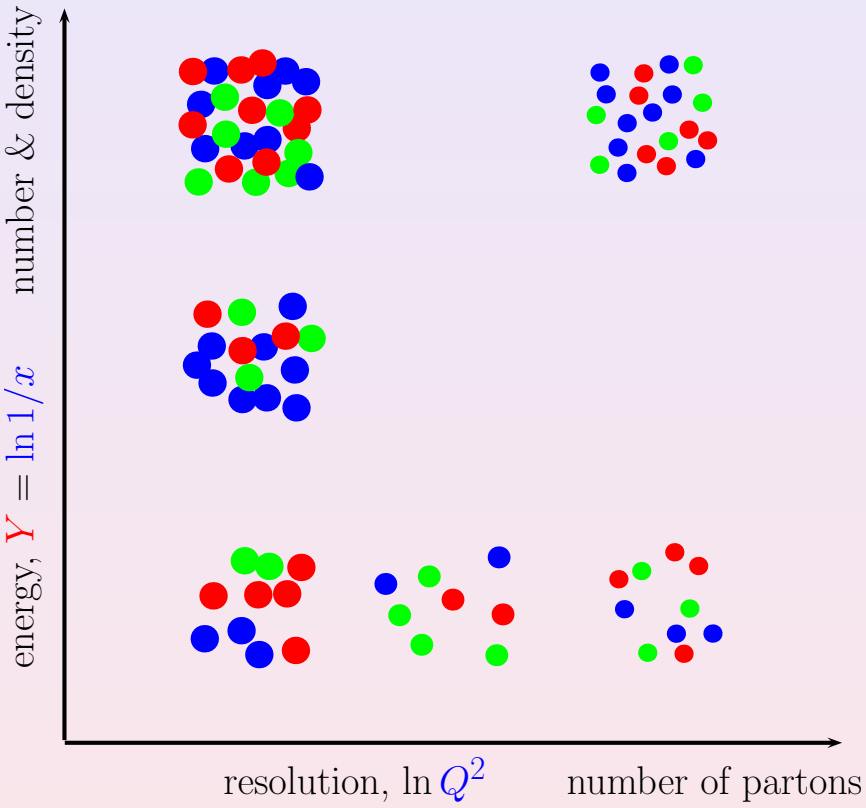
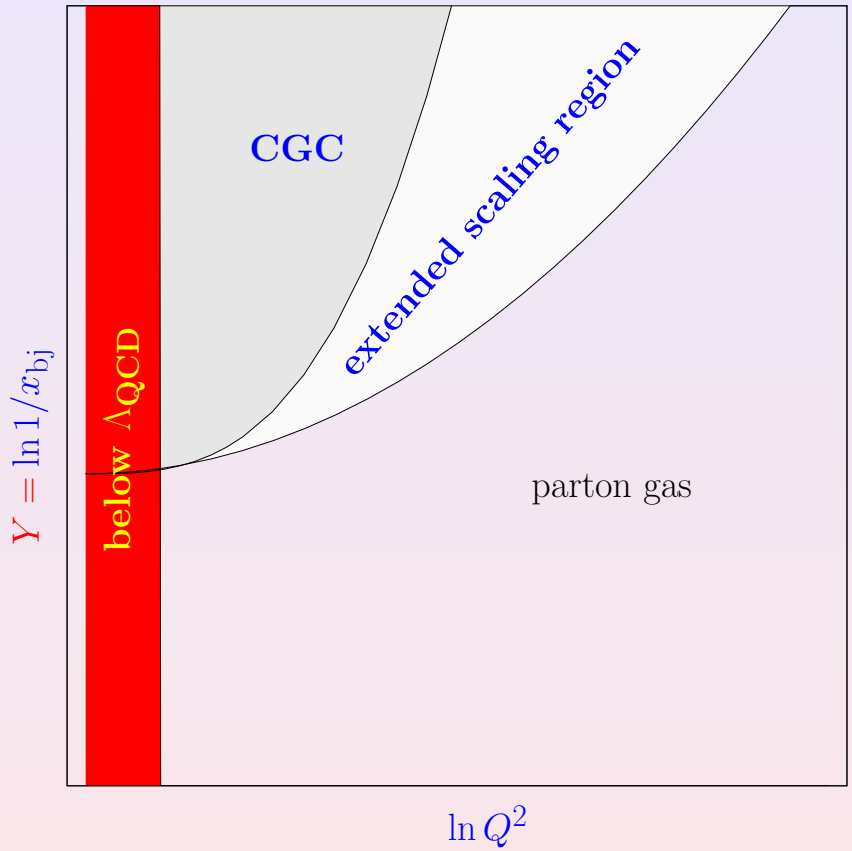




- density ⟶ finite correlation length R_s
- Why CGC?

Why the name?

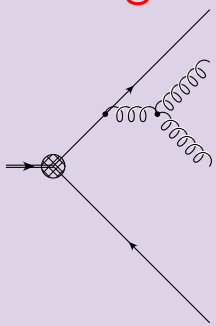
Color Glass Condensate

QCD	fields evolve slowly relative to natural scales	phase space density $\sim 1/\alpha_s$ & saturates

[nothing more specific implied!]

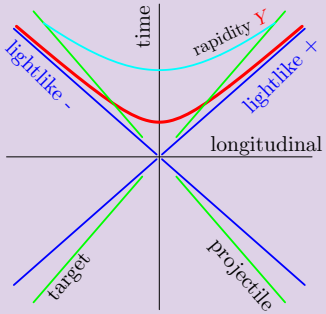
QCD

quarks and gluons



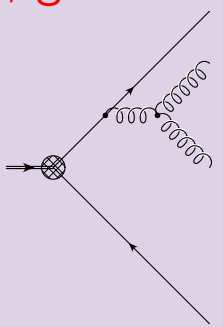
time scales

energy, time dilation



density

energy, gluons charged



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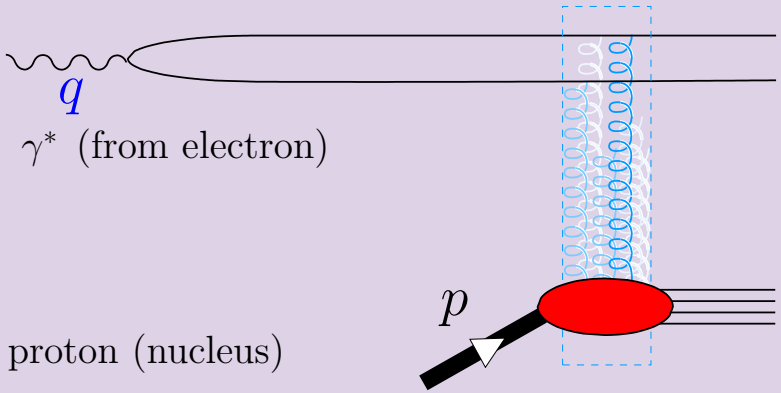
Gluon production at increasing energy

the photon splits



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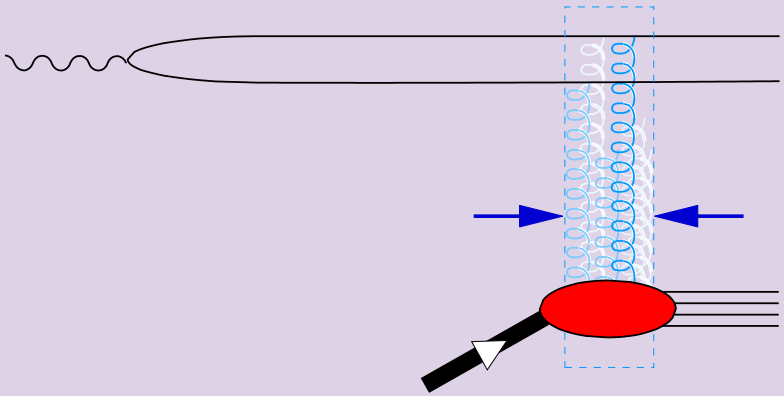

- target gluons Lorentz contracted to $\delta(x^-)$

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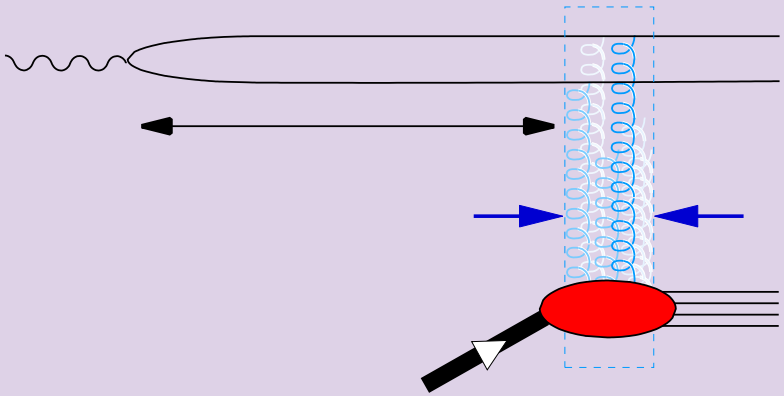

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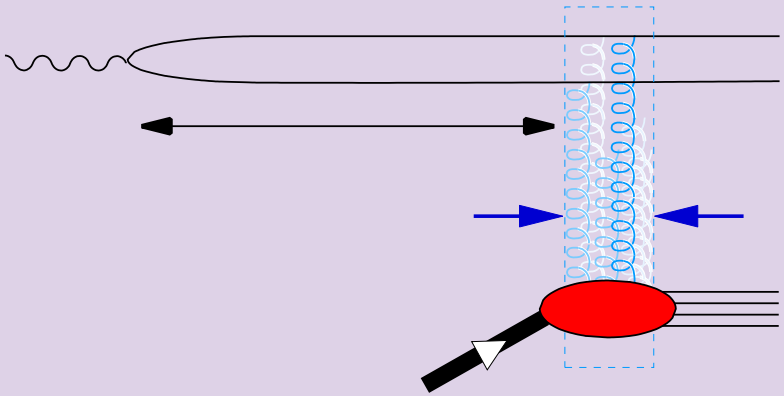

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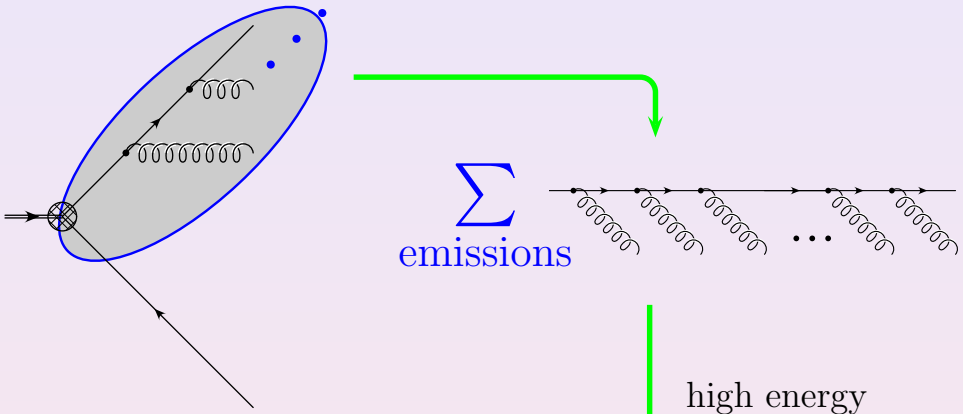
$$\sum_{\text{gluons}} \quad = \mathrm{P}\exp -ig \int dz_\mu A^\mu(z) \qquad = U_{\boldsymbol{x}}$$

Eikonal factors arise due to high energies



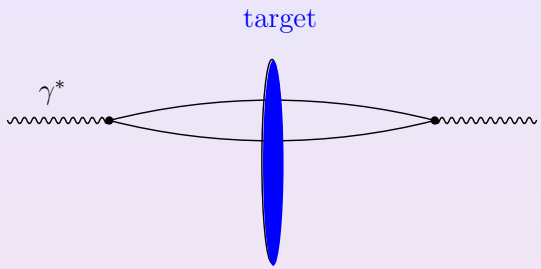
$$\mathrm{P}\exp -ig\int dz_\mu A^\mu(z)$$

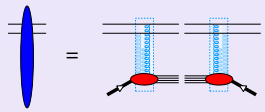
$U_{\boldsymbol{x}}$ @ small x (CGC)

$U_{\boldsymbol{p}}$ for jets

Cross section

$$\sigma_{\rm DIS}(Y, Q^2) = \mathrm{Im}$$





Cross section

energy, $\ln 1/x$

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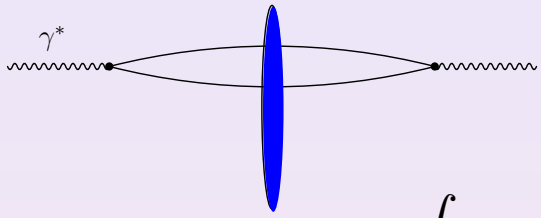

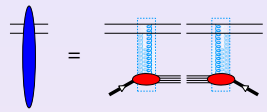

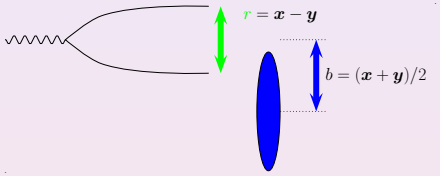

$$= \int d^2r \; |\psi^2|(r^2Q^2) \int d^2b \; \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \rangle (Y)$$

$\sigma_{\rm dipole}$

Cross section

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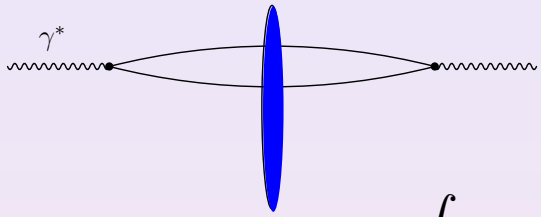

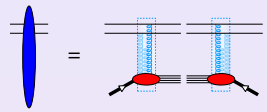

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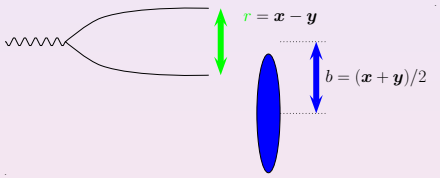

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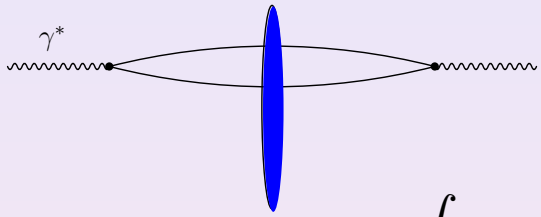

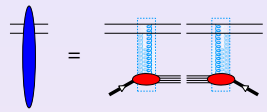

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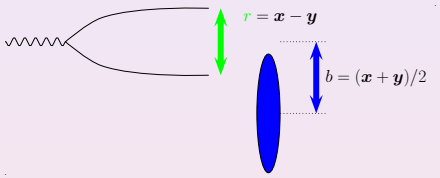

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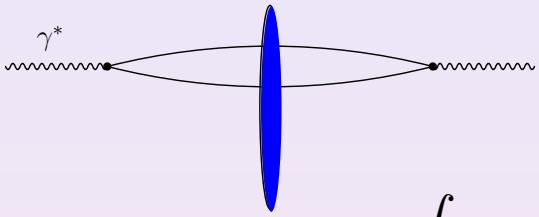

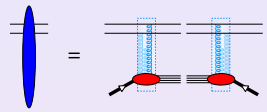

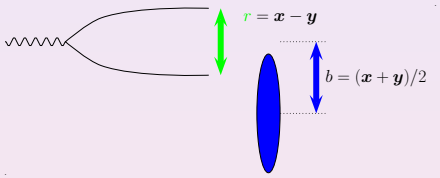

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target wavefunction: non-perturbative

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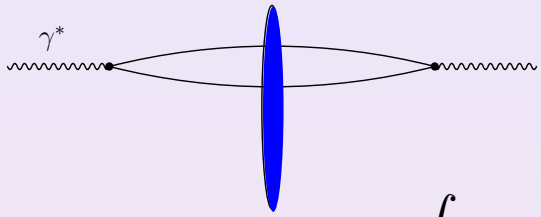

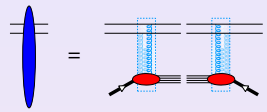

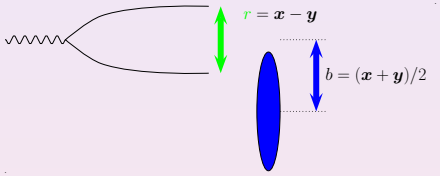

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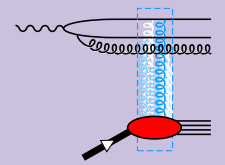

Cross section

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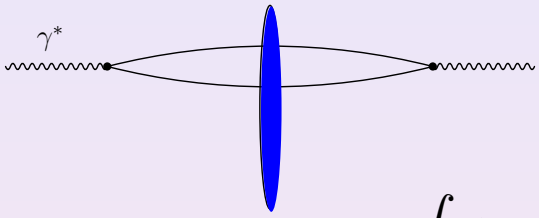
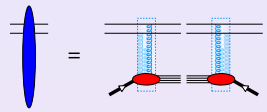
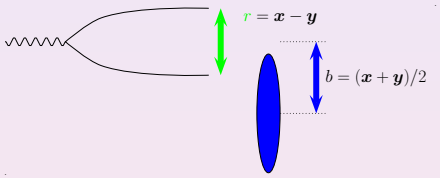




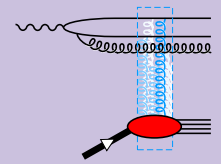


$$= \int d^2r \; |\psi^2|(r^2Q^2) \int d^2b \; \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \rangle (Y)$$

$\sigma_{\rm dipole}$

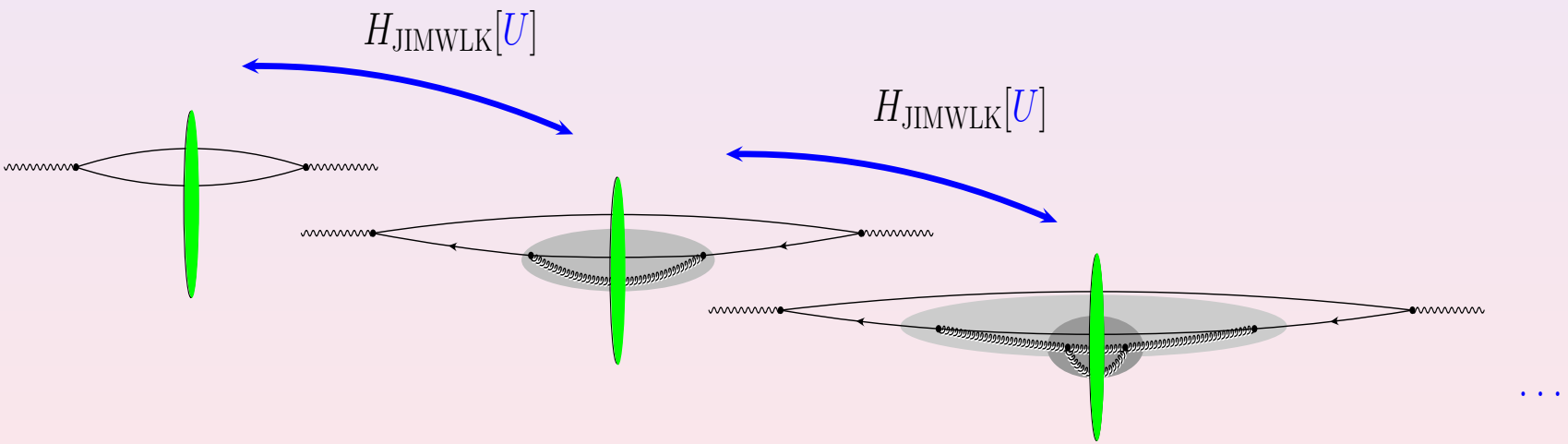
- $\sigma_{\rm dipole}$ contains $U_{\boldsymbol{x}}$
- $\langle \ldots \rangle (Y)$ hard!

target wavefunction: non-perturbative



- Bookkeeping device: $\langle \ldots \rangle (Y) = \int \hat{D}[U] \; \ldots \hat{Z}_Y[U]$

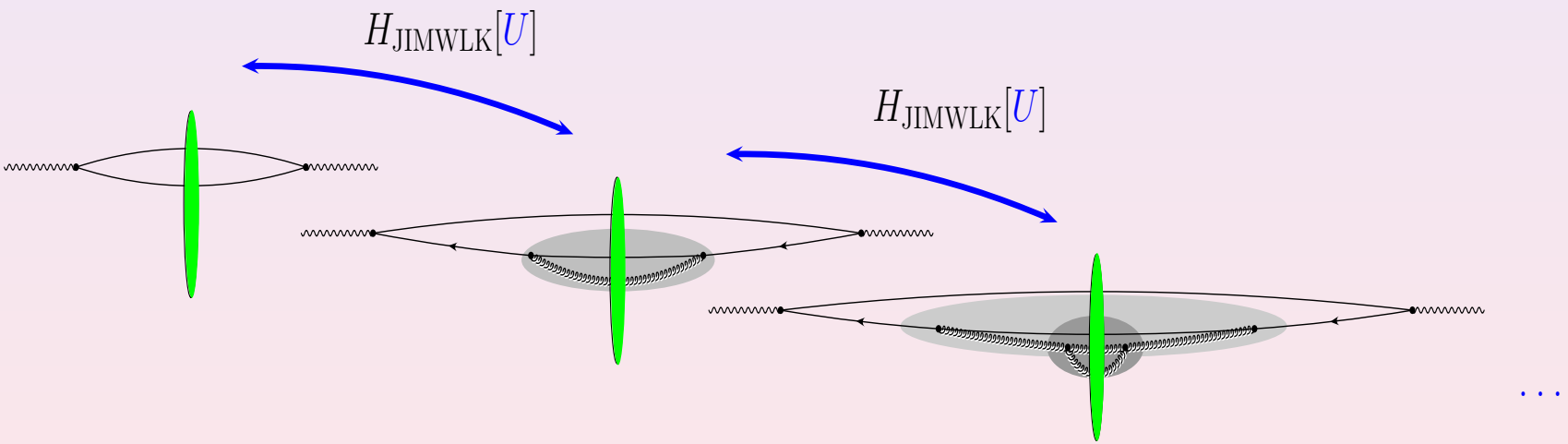
The JIMWLK evolution equation



The JIMWLK evolution equation

▸ explicit form Heribert Weigert **Nucl. Phys. A703**, 2002, 823

$$\frac{d}{dY} Z_Y[U] = -H_{\text{JIMWLK}}[U] \quad Z_Y[U]$$



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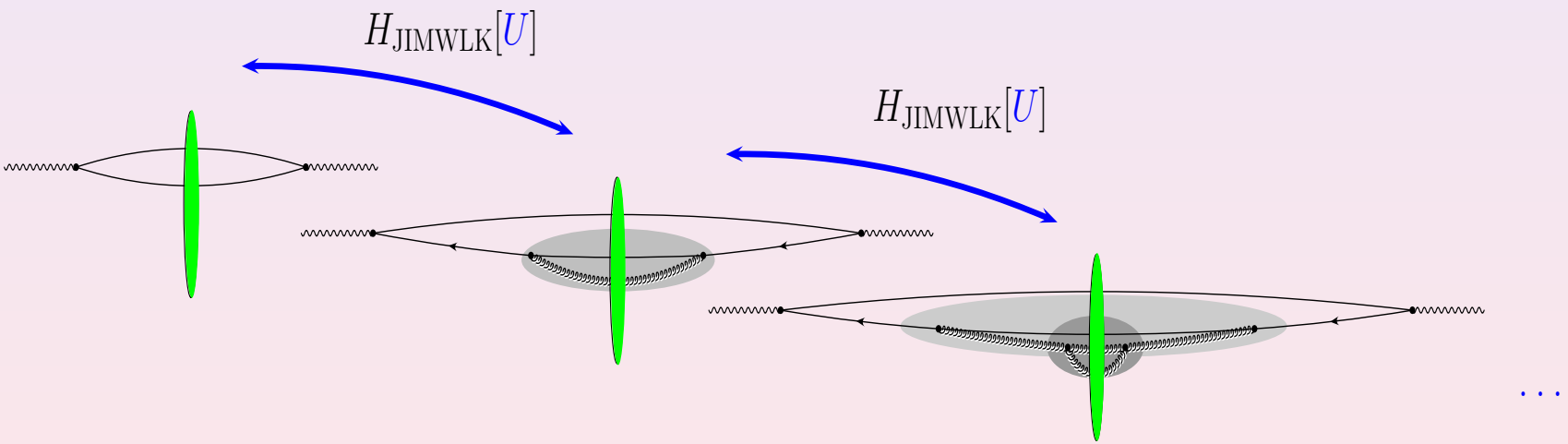
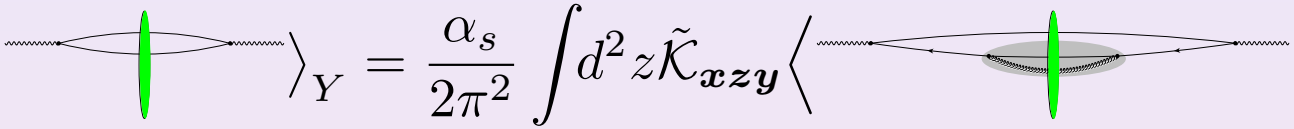


→ energy dependence of $\langle \ldots \rangle(Y)$

The Balitsky hierarchy

- JIMWLK $\Longleftrightarrow$ infinite tower of coupled evol. eqns.

$$\partial_Y \left\langle \quad \right\rangle_Y = \frac{\alpha_s}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{\boldsymbol{xzy}} \left\langle \quad - \quad \right\rangle_Y$$



The Balitsky hierarchy

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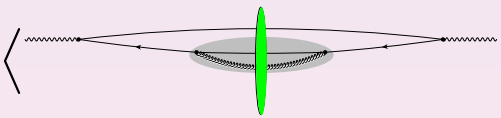
$$\partial_Y \left\langle \quad \right\rangle_Y = \ldots$$

The Balitsky hierarchy

- JIMWLK $\Longleftrightarrow$ infinite tower of coupled evol. eqns.

$$\partial_Y \left\langle \frac{\mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \right\rangle_Y = \frac{\alpha_s}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{\boldsymbol{xzy}} \left\langle \frac{\left[\tilde{U}_{\boldsymbol{z}}\right]^{ab} 2\ \mathrm{tr}(t^a U_{\boldsymbol{x}} t^b U^\dagger_{\boldsymbol{y}})}{N_c} - 2C_\mathrm{f} \frac{\mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \right\rangle_Y$$

$\partial_Y \Big\langle$  $\Big\rangle_Y = \ldots$

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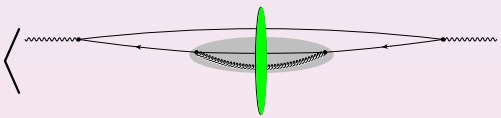


- N_c limit factorizes: $\Longrightarrow$ single closed nonlinear eqn. (BK)

$$\partial_Y S_{\boldsymbol{xy}} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{\boldsymbol{xzy}} \left(S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}} \right) \qquad S_{\boldsymbol{xy}} := \left\langle \frac{\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \right\rangle$$

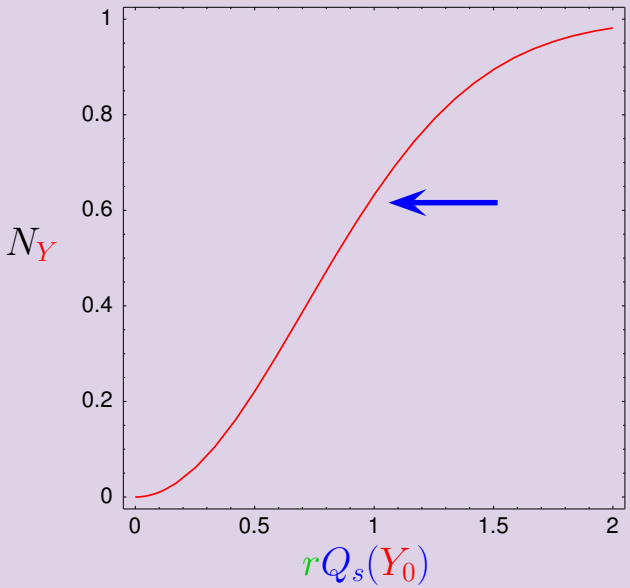
Saturation scale and cross section

$$\langle \ldots \rangle(Y) \quad \Rightarrow \quad \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U_{\boldsymbol{0}}^\dagger)}{N_c} \rangle(Y) \quad =: N_Y(\boldsymbol{r})$$

Saturation scale and cross section

$$\langle \ldots \rangle(Y) \quad \Rightarrow \quad \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U_{\boldsymbol{0}}^\dagger)}{N_c} \rangle(Y) \quad =: N_Y(\boldsymbol{r})$$

Correlation length shrinks



$$R_s(Y) \sim \frac{1}{Q_s(Y)}$$

$R_s(Y) \equiv$ correlation length
$Q_s(Y) \equiv$ saturation scale

Saturation scale and cross section

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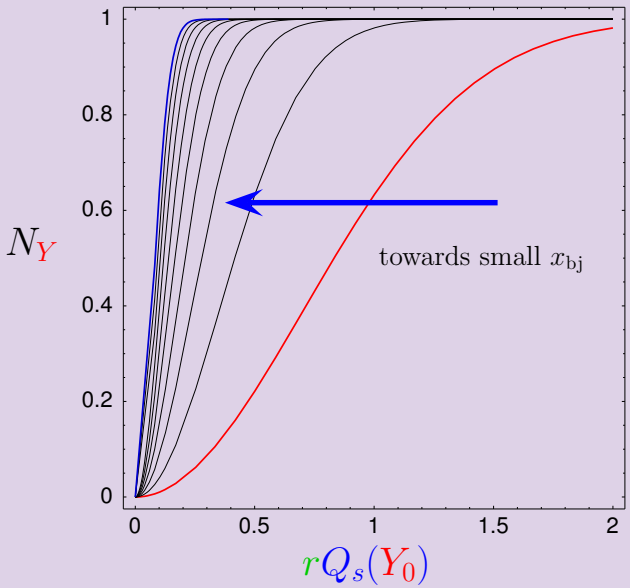

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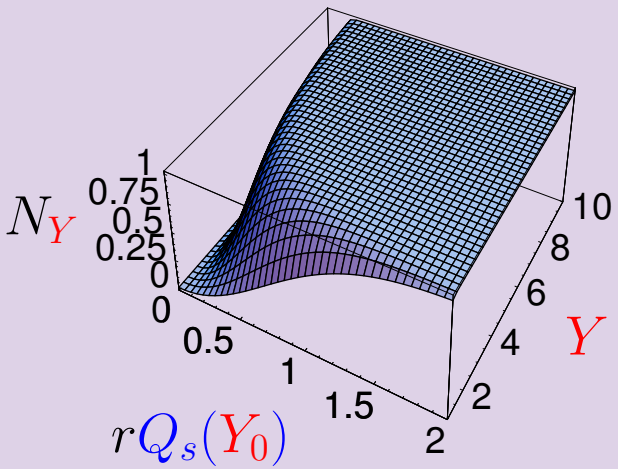
JIMWLK: IR safety and scaling

Activity above $\sim Q_s(Y)$



JIMWLK: IR safety and scaling

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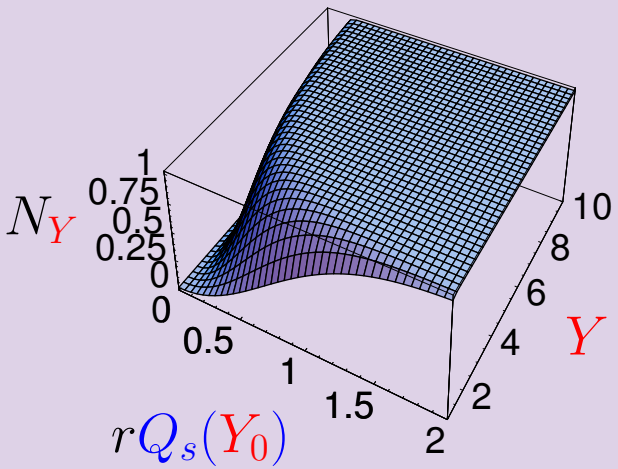
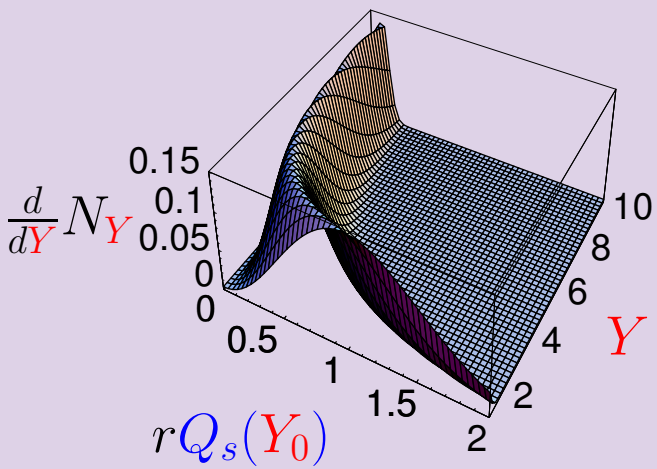
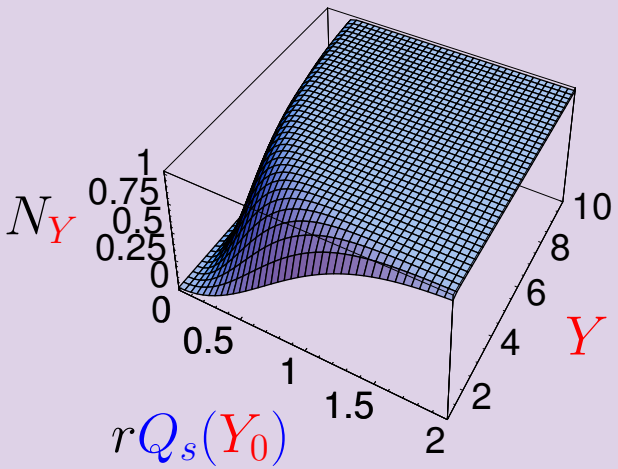


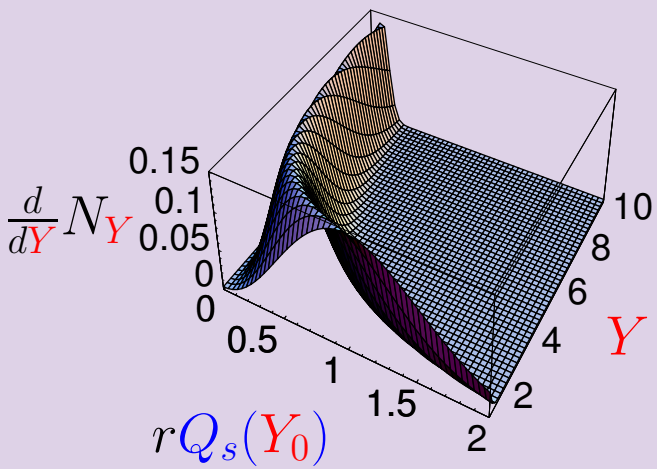


- activity follows $Q_s(Y)$

JIMWLK: IR safety and scaling

Activity above $\sim Q_s(Y)$





- activity follows $Q_s(Y)$
- IR safety

JIMWLK: IR safety and scaling

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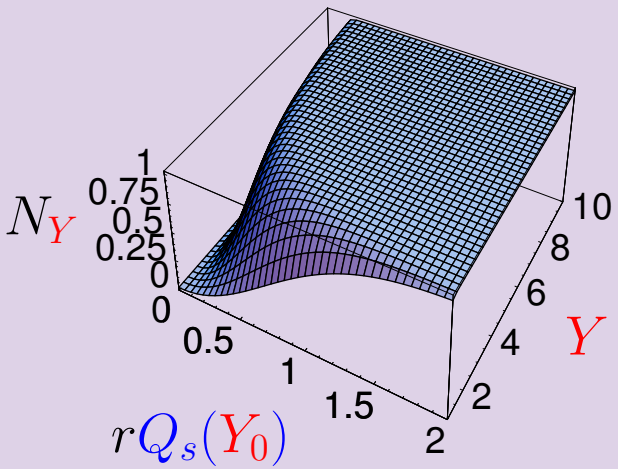
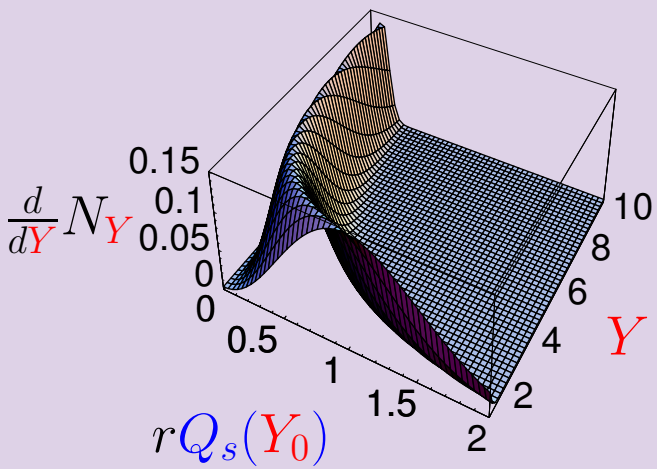
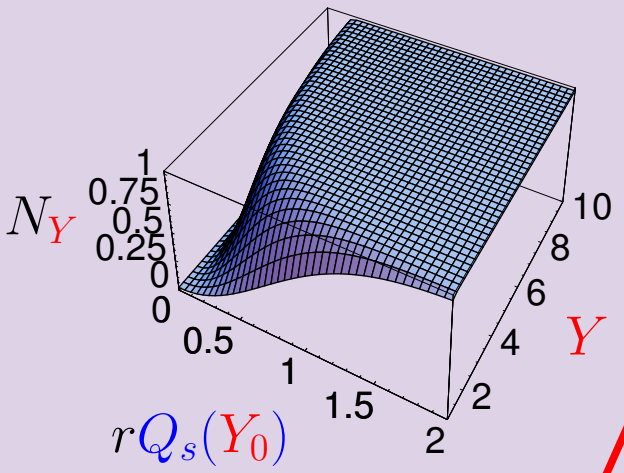




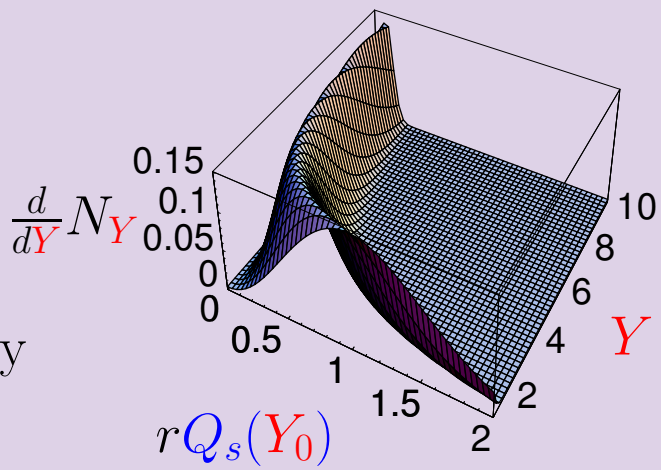
- activity follows $Q_s(Y)$
- IR safety
  perturbative ✓

JIMWLK: IR safety and scaling

Activity above $\sim Q_s(Y)$



scale $\boldsymbol{r}$ by $\frac{Q_s(Y)}{Q_s(Y_0)}$



- activity follows $Q_s(Y)$
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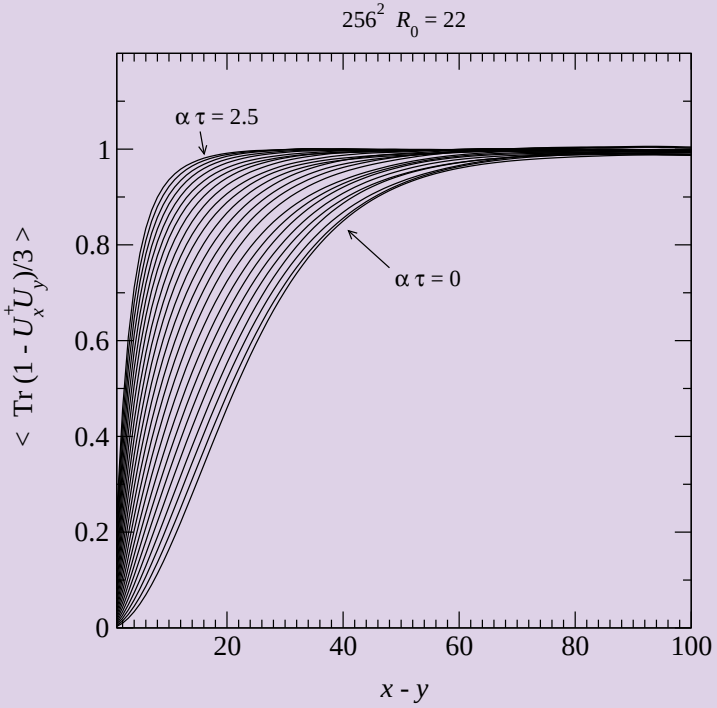
  perturbative ✓

From simulations:

scaling with $Q_s(Y)$
[persists @ running coupling]

JIMWLK: simulations

Simulations show scaling (Rummukainen & H.W)



JIMWLK: simulations

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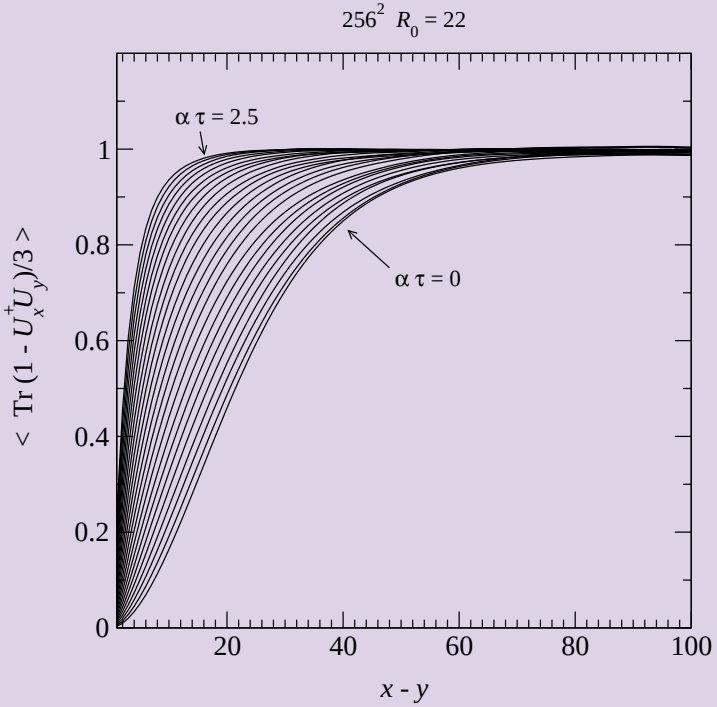

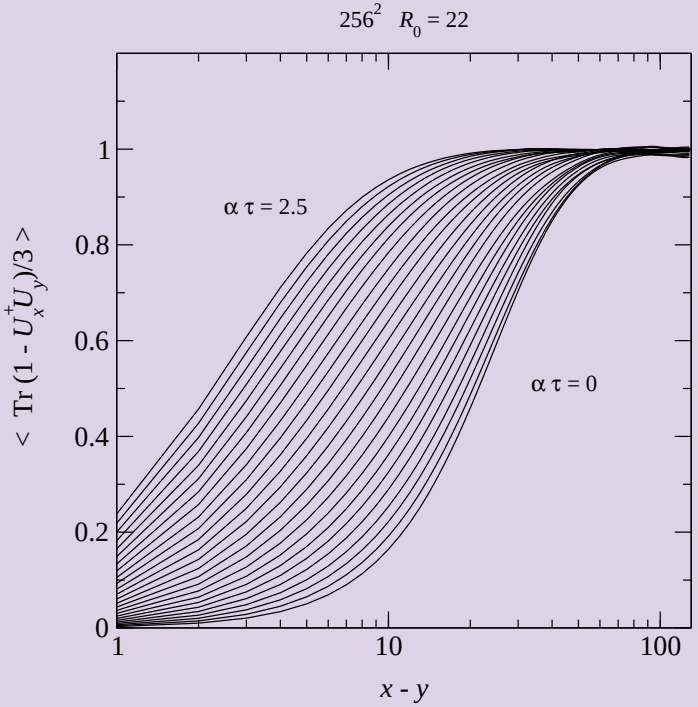

JIMWLK: simulations

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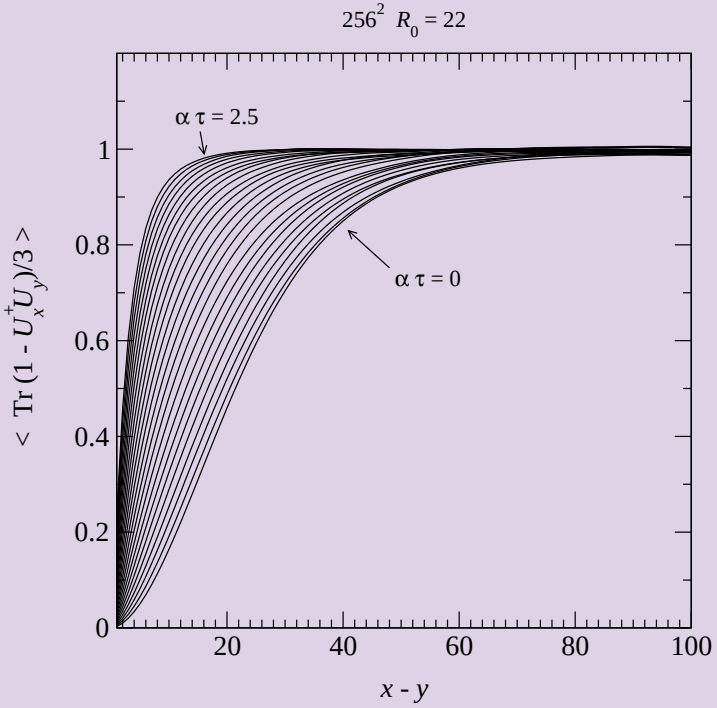
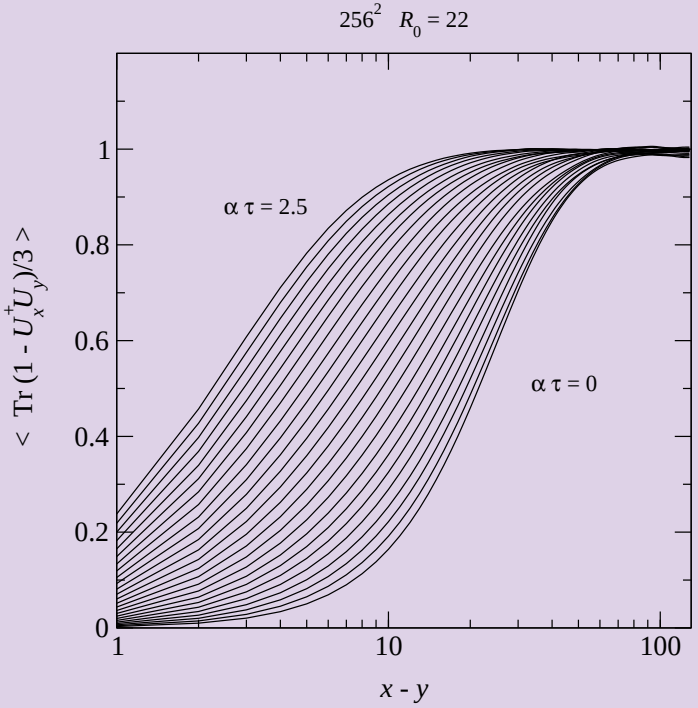


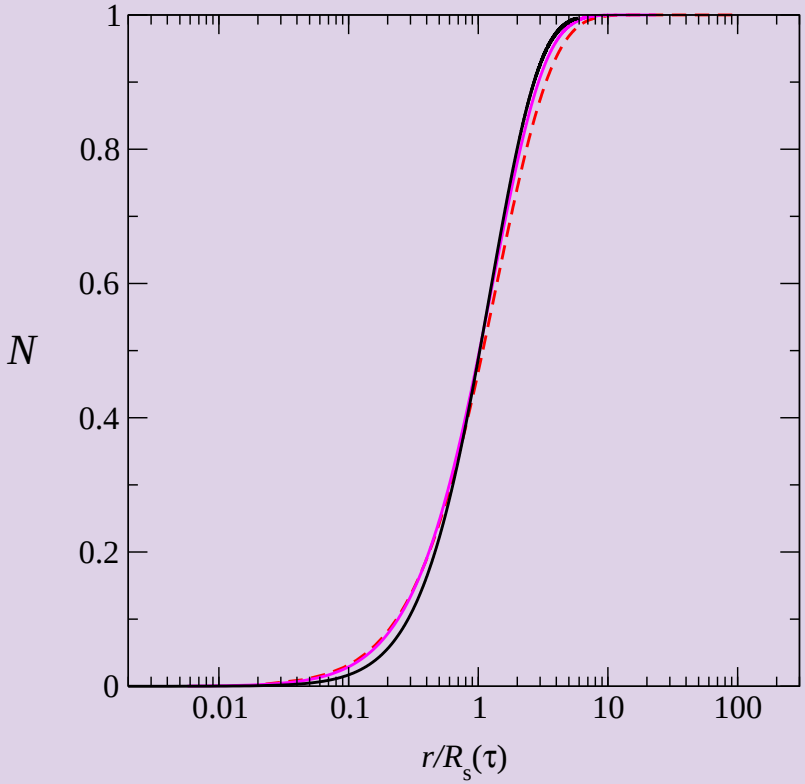


. . . in finite window: lattice artefacts

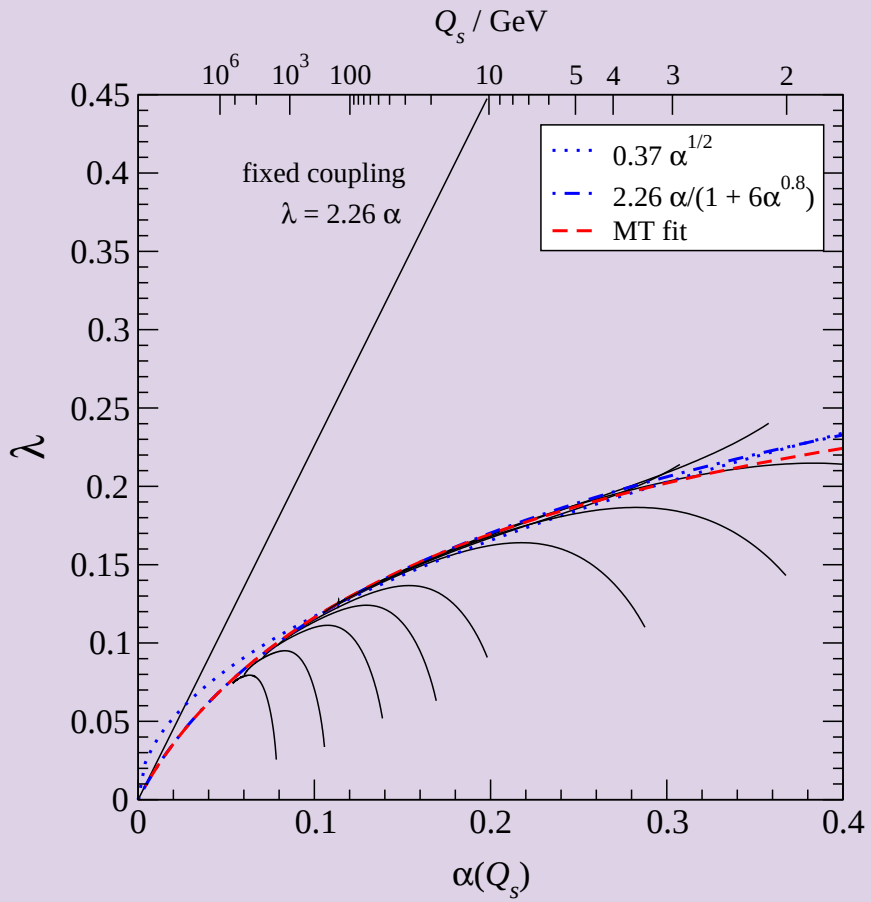
- $Q_s(Y)$ protects IR ✓
- check UV via $\lambda(Y) := \partial_Y \ln Q_s(Y)$

BK (parent dipole scheme): $\lambda(Y) = \partial_Y \ln Q_s(Y)$

Near scaling despite running coupling



$\lambda(Y) := \partial_Y \ln Q_s(Y)$



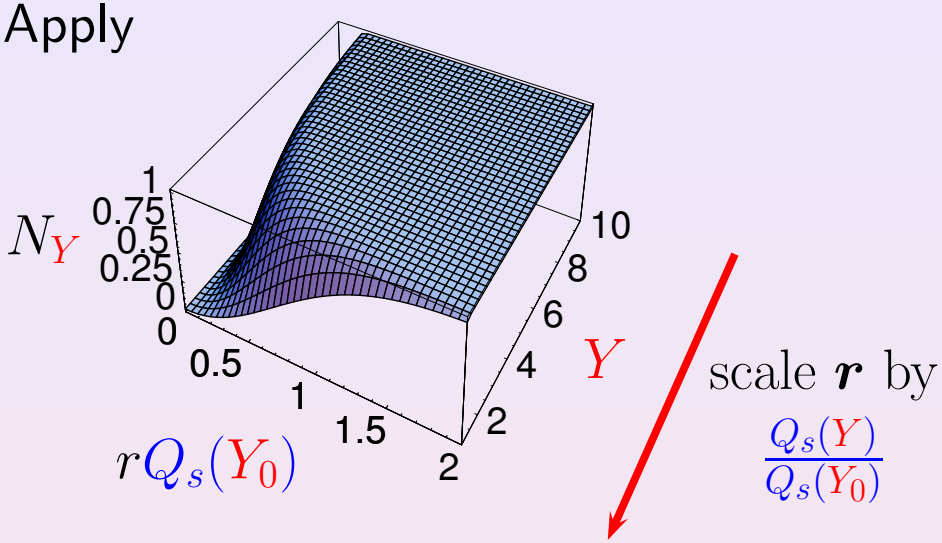
▸ Running coupling is essential

Outline

1. Motivation: gluons form the CGC
   - Current and planned collider experiments
   - Enhanced gluon production at high energies
   - CGC: why the name
2. JIMWLK evolution: properties of the CGC
   - Gluons in observables
   - The evolution equation
   - The saturation scale
3. Experiment
   - Geometric scaling @ HERA
   - Erasing the Cronin effect @ RHIC
4. Overview and outlook

Geometric scaling @ HERA

Apply

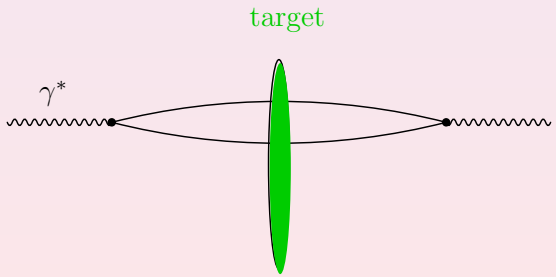


scale $\boldsymbol{r}$ by $\dfrac{Q_s(Y)}{Q_s(Y_0)}$

to Hera

energy, $\ln 1/x$

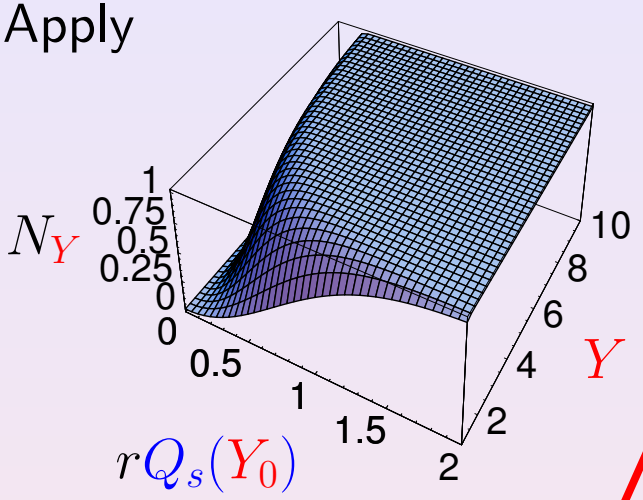
$\sigma_{\rm DIS}(Y, Q^2) = \mathrm{Im}$



Golec-Biernat, Wüsthoff; PRD **60** (1999) 114023 [hep-ph/9903358]

Geometric scaling @ HERA

Apply



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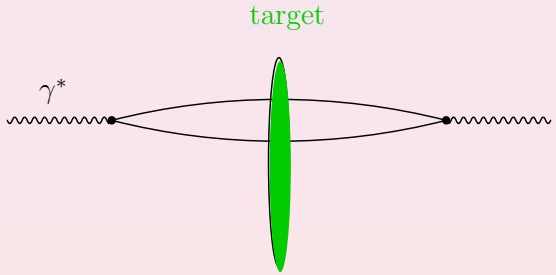
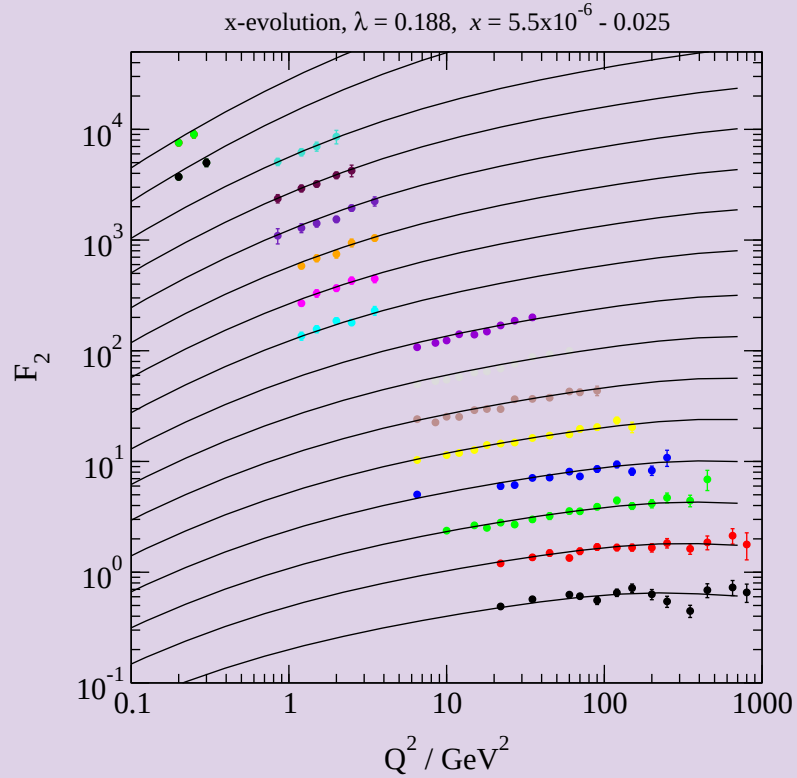


Golec-Biernat, Wüsthoff; PRD **60** (1999) 114023 [hep-ph/9903358]

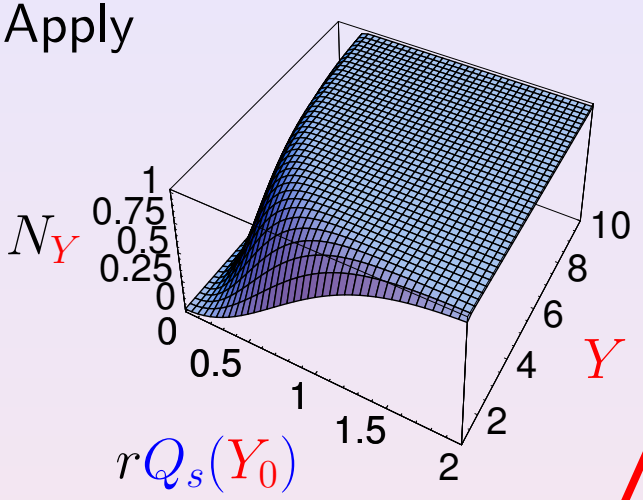
Phenomenological scaling fit to HERA:

$$\sigma(Y, Q^2) \sim F_2(Y, Q^2) \cdot Q^2$$



Geometric scaling @ HERA

Apply

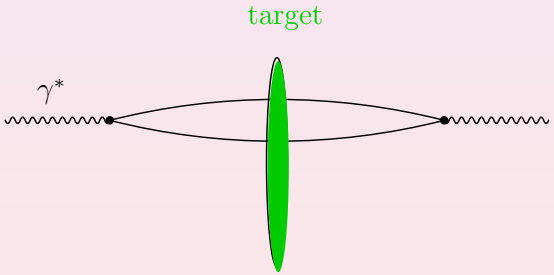


scale $\boldsymbol{r}$ by $\frac{Q_s(Y)}{Q_s(Y_0)}$

to Hera

energy, $\ln 1/x$

$\sigma_{\rm DIS}(Y, Q^2) = {\rm Im}$



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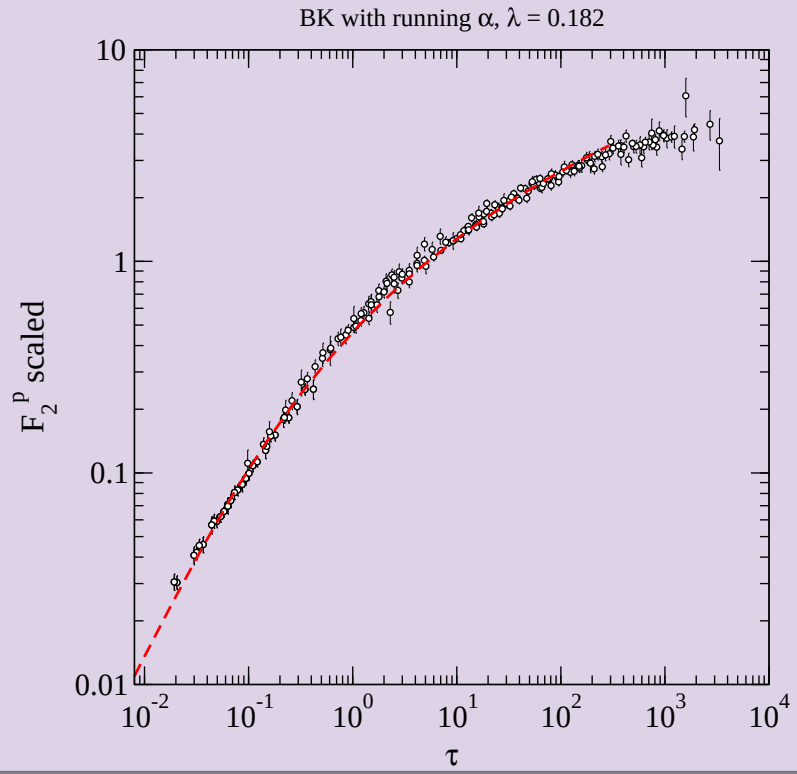
Phenomenological scaling fit to HERA:

$$\sigma(Y, Q^2) = \sigma(Y_0, \tau = Q^2 \frac{Q_s^2(Y_0)}{Q_s^2(Y)})$$



p Au: Cronin enhancement and evolution

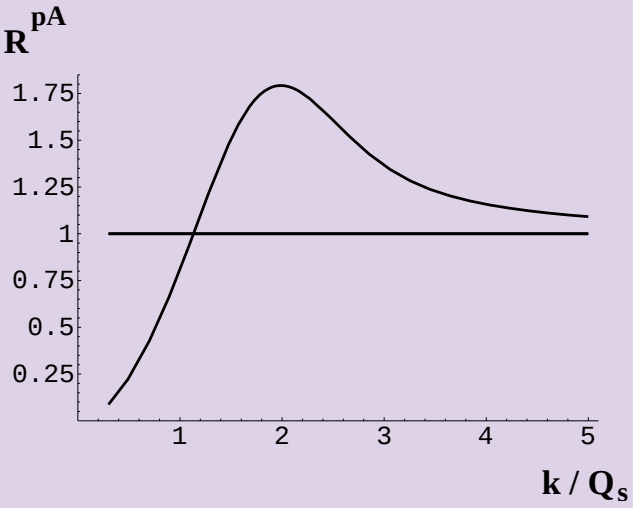
▸ A dep.

What is measured

$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k dY}}{A\frac{d\sigma^{pp}}{d^2k dY}}$$

Cronin effect

known from dilute systems
(fixed target experiments)



Evolution effects strongest where $Q_s(Y)$ largest

- at largest Y ➡ most forward rapidities (small angles)
- most central collisions

p Au: Cronin enhancement and evolution

▸ A dep.

What is measured

$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k dY}}{A\frac{d\sigma^{pp}}{d^2k dY}}$$

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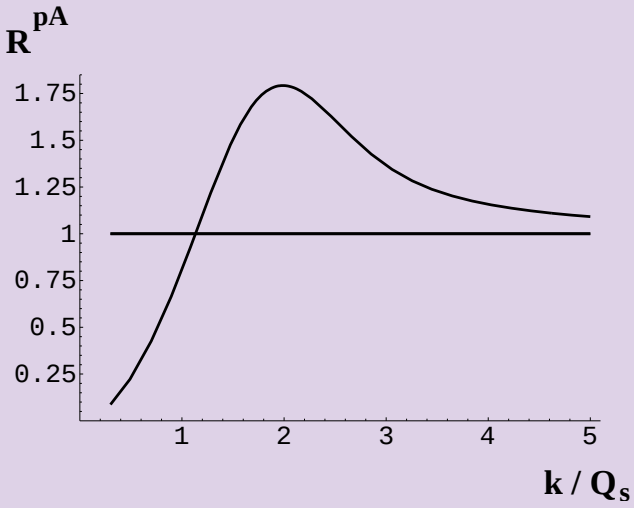
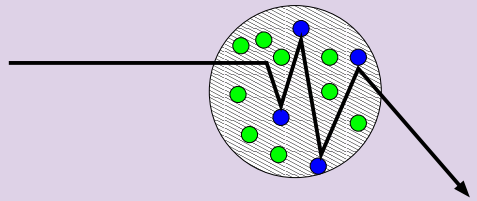


Intuitive explanation



$$p_t \sim Q_s(Y)$$

Evolution effects strongest where $Q_s(Y)$ largest

- at largest Y ➡ most forward rapidities (small angles)
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p Au: Cronin enhancement and evolution

▸ A dep.

What is measured

$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k dY}}{A\frac{d\sigma^{pp}}{d^2k dY}}$$

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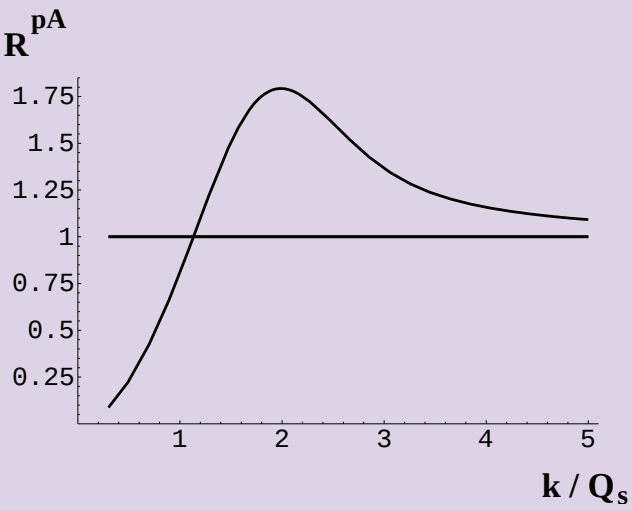
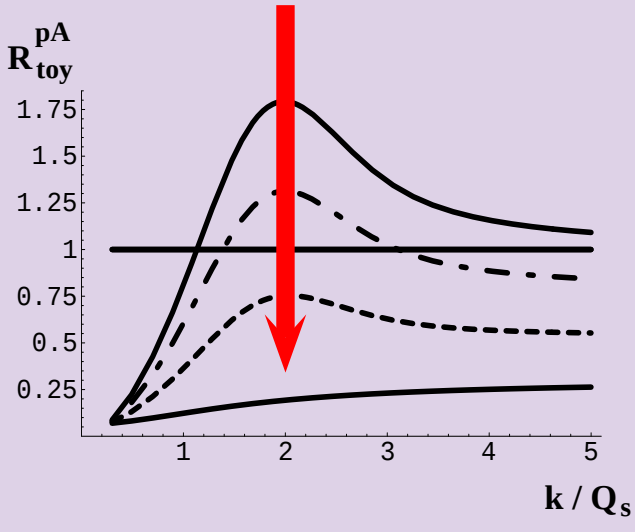


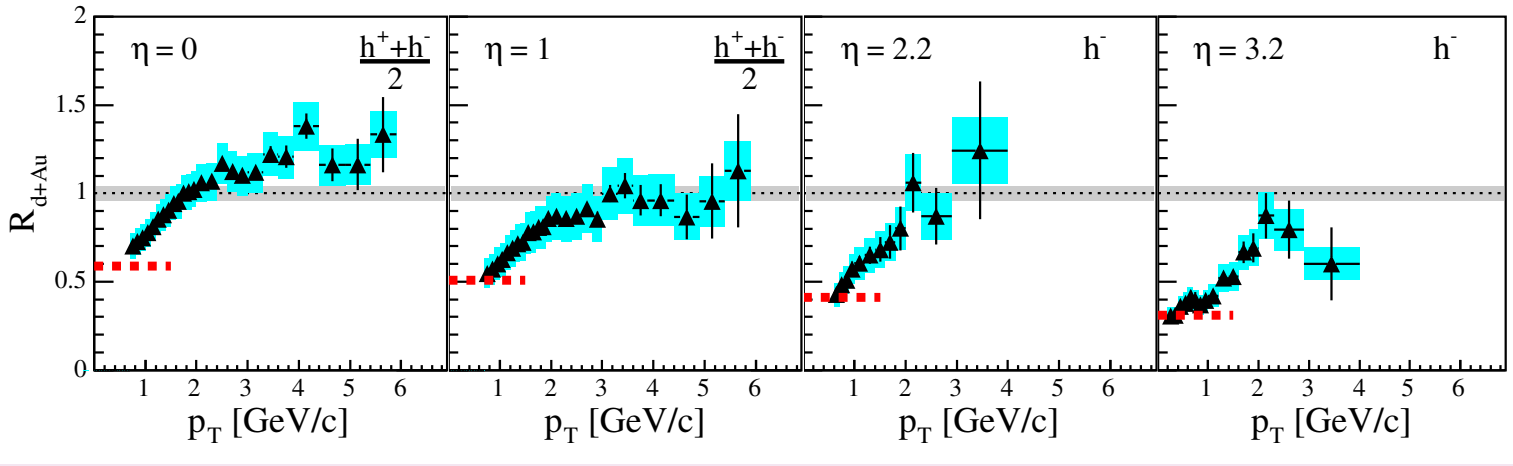
Effect of evolution

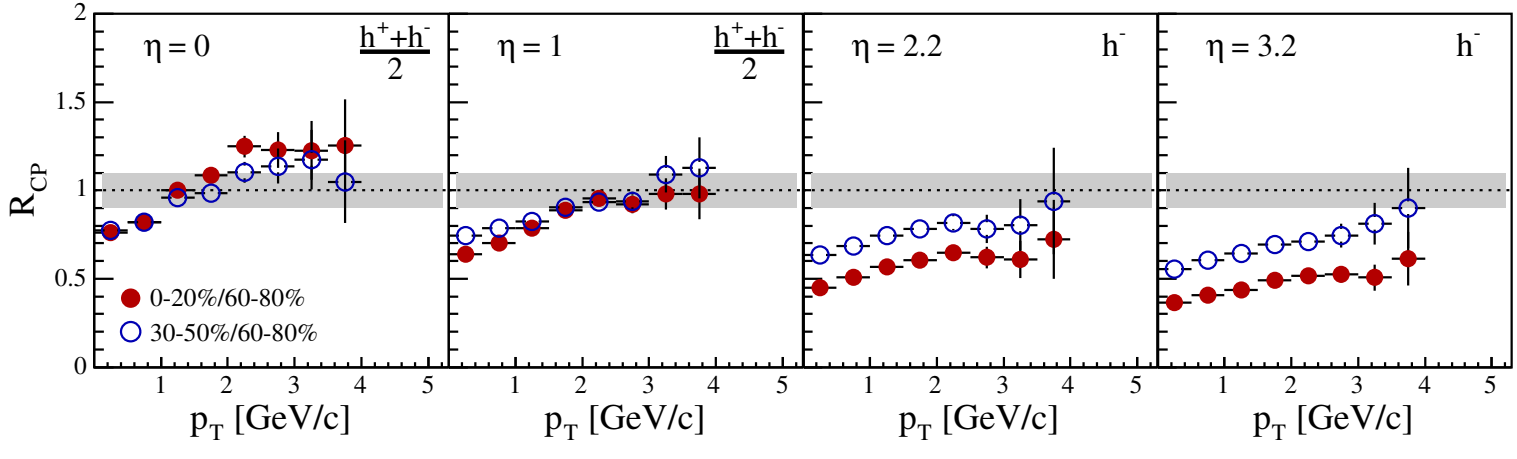


Evolution effects strongest where $Q_s(Y)$ largest

- at largest Y ➡ most forward rapidities (small angles)
- most central collisions

Erasing the Cronin effect on the parton level [BRAHMS]





Outline

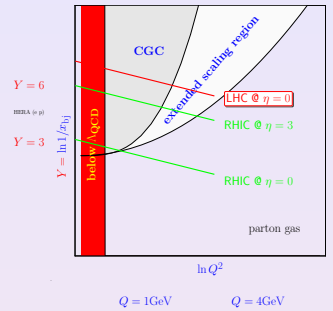
1. Motivation: gluons form the CGC
   - Current and planned collider experiments
   - Enhanced gluon production at high energies
   - CGC: why the name
2. JIMWLK evolution: properties of the CGC
   - Gluons in observables
   - The evolution equation
   - The saturation scale
3. Experiment
   - Geometric scaling @ HERA
   - Erasing the Cronin effect @ RHIC
4. Overview and outlook

Outlook: phenomena affected by the CGC

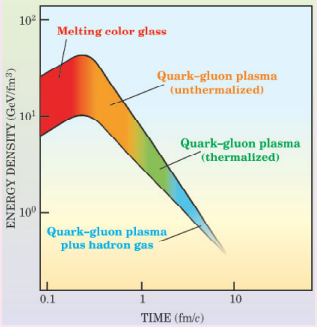
- CGC in $\gamma^* A$ ✓
  - saturation scale from JIMWLK ✓
  - geometric scaling in eA ✓
  - diffractive dissociation (rapidity gaps)

    [Levin & Kovchegov; H.W.& Hentschinski]
  - nonforward scattering, more differential observables

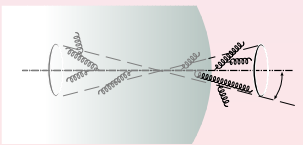
    [more detailed information, more stringent tests]



- CGC in heavy ion collisions (RHIC & LHC):
  - saturation scale & Cronin effect ✓
  - scales in initial conditions for QGP
  - saturation scale & particle multiplicities

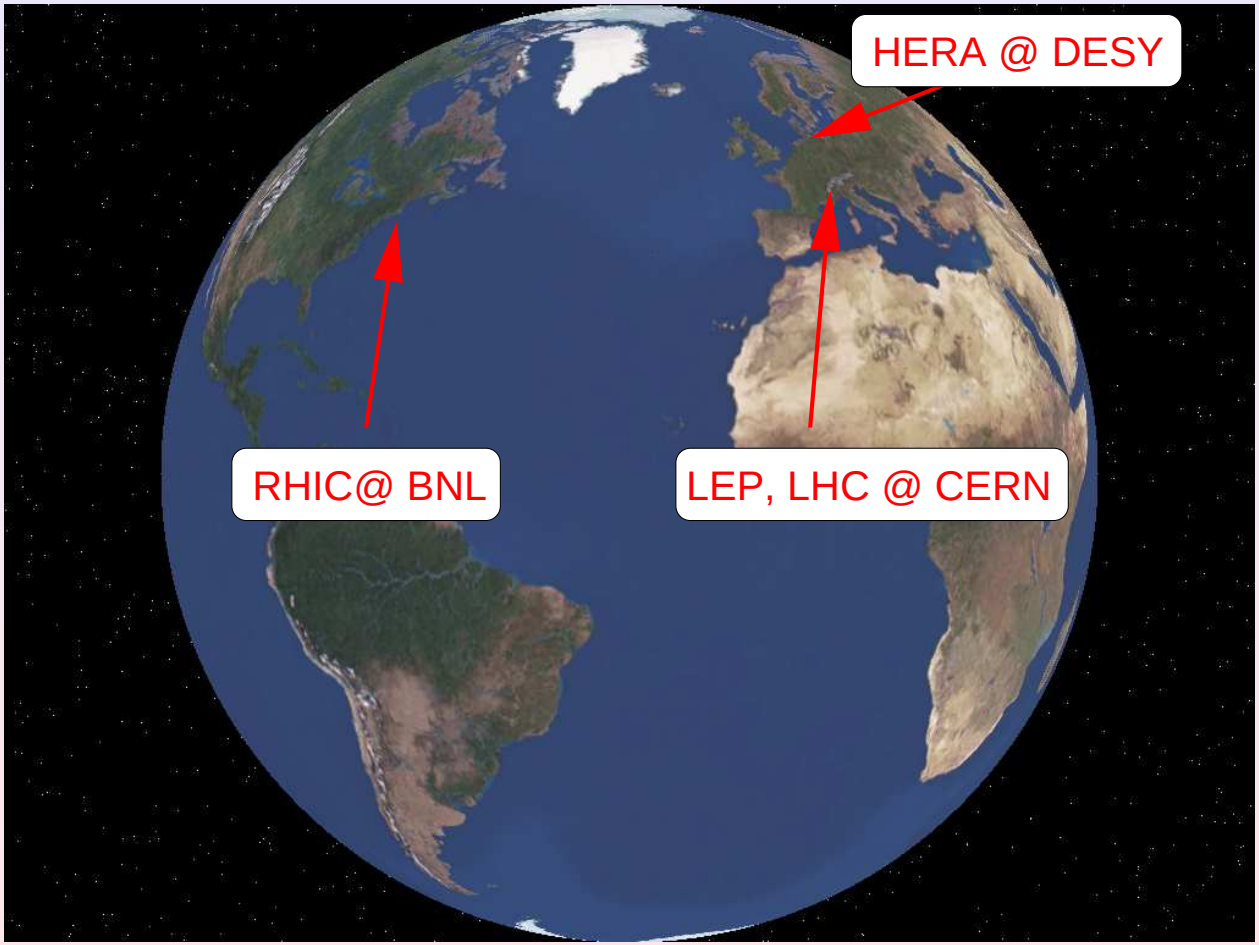


- JIMWLK like evolution for certain jet observables
  - "non-global" observables
  - → jets in medium @ LHC ?



- cosmic ray showers [extremely high energies]

The Color Glass Condensate, a birds eye view



CGC in experiments @:

- RHIC, HERA
- LHC
- EIC (dedicated!)

Main characteristic:

- correlation length $R_s(Y) \sim \frac{1}{Q_s(Y)}$
- Q_s-scaling: Y dependence via $Q_s(Y)$

Outline

5 The JIMWLK Hamiltonian

6 CGC @ LHC

7 From CGC to QGP

8 Running coupling is essential

9 A dependence in e A

10 JIMWLK like jet evolution

The JIMWLK Hamiltonian

back

$$H_{\text{JIMWLK}} = -\frac{1}{2}\frac{\alpha_s}{\pi^2}\,\mathcal{K}_{\boldsymbol{xzy}}\left[i\nabla^a_x i\nabla^a_y + i\bar{\nabla}^a_x i\bar{\nabla}^a_y + \tilde{U}^{ab}_z(i\bar{\nabla}^a_x i\nabla^b_y + i\nabla^a_x i\bar{\nabla}^b_y)\right]$$

$$\mathcal{K}_{\boldsymbol{xzy}} = \frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{z}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

[integration convention for x, z, y]

$i\nabla^a_{\boldsymbol{x}}$ and $i\bar{\nabla}^a_{\boldsymbol{x}}$ are functional derivatives:

$$i\nabla^a_{\boldsymbol{x}} := -[U_{\boldsymbol{x}}t^a]_{ji}\frac{\delta}{\delta U_{\boldsymbol{x},ij}} \qquad i\bar{\nabla}^a_{\boldsymbol{x}} := [t^a U_{\boldsymbol{x}}]_{ji}\frac{\delta}{\delta U_{\boldsymbol{x},ij}}$$

The JIMWLK Hamiltonian

back

$$H_{\text{JIMWLK}} = -\frac{1}{2}\frac{\alpha_s}{\pi^2}\,\mathcal{K}_{\boldsymbol{xzy}}\,\left[i\nabla^a_x i\nabla^a_y + i\bar\nabla^a_x i\bar\nabla^a_y + \tilde U^{ab}_z (i\bar\nabla^a_x i\nabla^b_y + i\nabla^a_x i\bar\nabla^b_y)\right]$$

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generate l. & r. inv vector fields, r & l rotations:

$$e^{-i\omega^a (i\nabla^a)} U = U e^{i\omega^a t^a} \qquad e^{-i\omega^a (i\bar\nabla^a)} U = e^{-i\omega^a t^a} U$$

reps of the algebras:

$$[i\nabla^a, i\nabla^b] = i f^{abc} i\nabla^c \qquad [i\bar\nabla^a, i\bar\nabla^b] = i f^{abc} i\bar\nabla^c \qquad [i\bar\nabla^a, i\nabla^b] = 0$$

The JIMWLK Hamiltonian

back

$$H_{\text{JIMWLK}} = -\frac{1}{2}\frac{\alpha_s}{\pi^2}\,\mathcal{K}_{\boldsymbol{xzy}}\left[i\nabla^a_x i\nabla^a_y + i\bar\nabla^a_x i\bar\nabla^a_y + \tilde U^{ab}_z (i\bar\nabla^a_x i\nabla^b_y + i\nabla^a_x i\bar\nabla^b_y)\right]$$

$$\mathcal{K}_{\boldsymbol{xzy}} = \frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{z}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

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physics content:

- $\tilde U^{ab}_z (i\bar\nabla^a_x i\nabla^b_y + i\nabla^a_x i\bar\nabla^b_y)$ — real emission
- $i\nabla^a_x i\nabla^a_y + i\bar\nabla^a_x i\bar\nabla^a_y$ — virt. correction
- real emission term ⟹ nonlinear evolution

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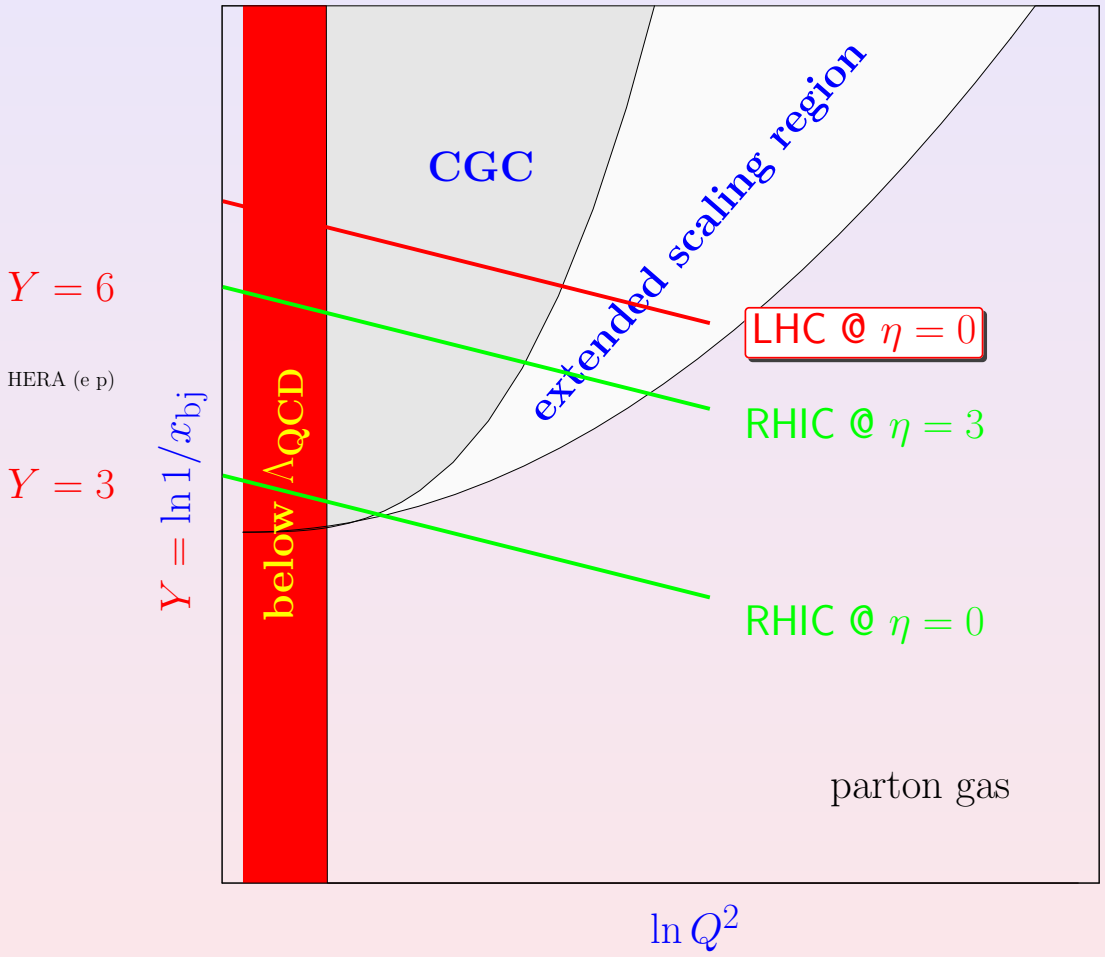
Colored Glass @ LHC with nuclei: YES



$Q = 1\text{GeV}$ $Q = 4\text{GeV}$

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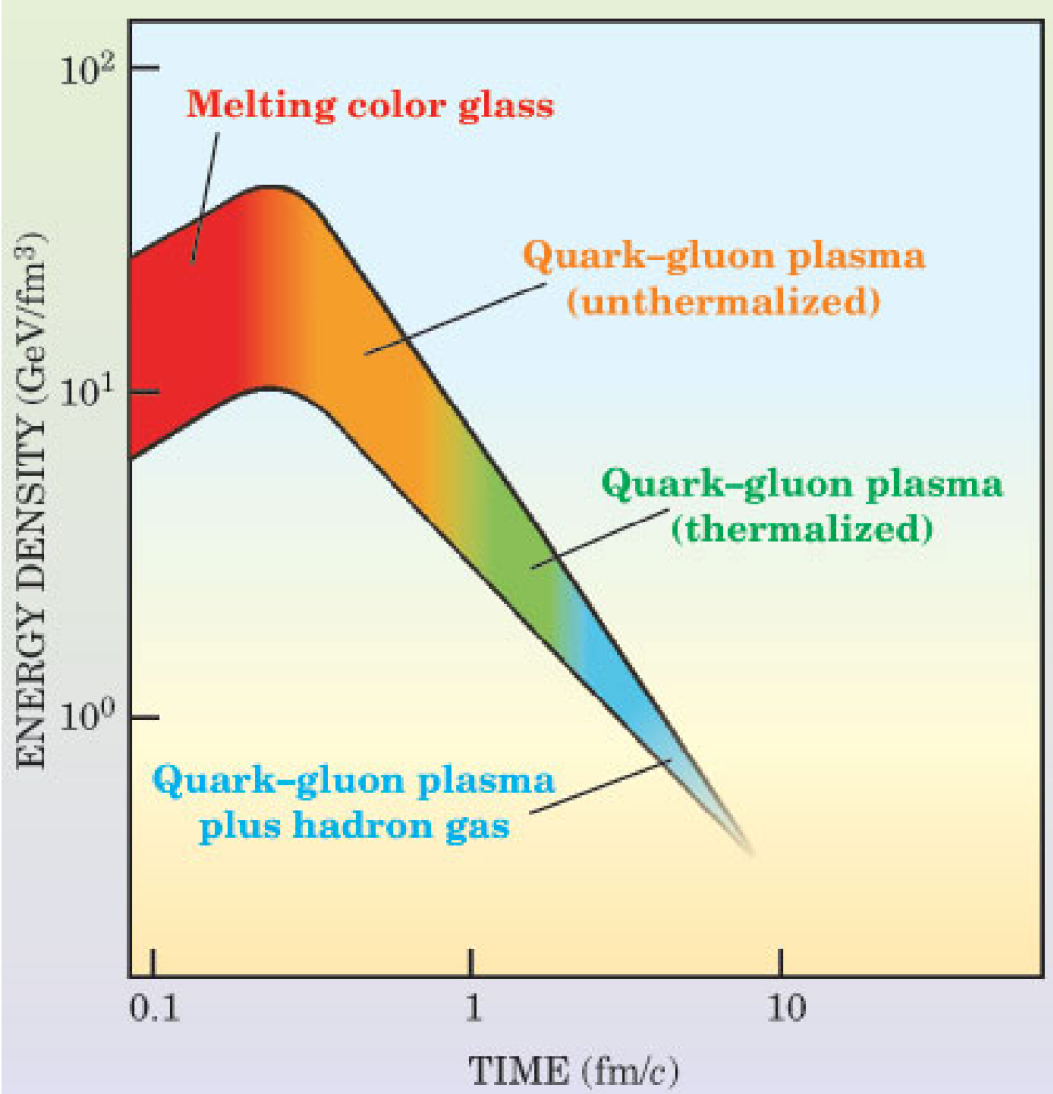
From Colored Glass to Quark Gluon Plasma



McLerran, Ludlam
Physics Today
Oct 2003

core of neutron star

nuclear matter

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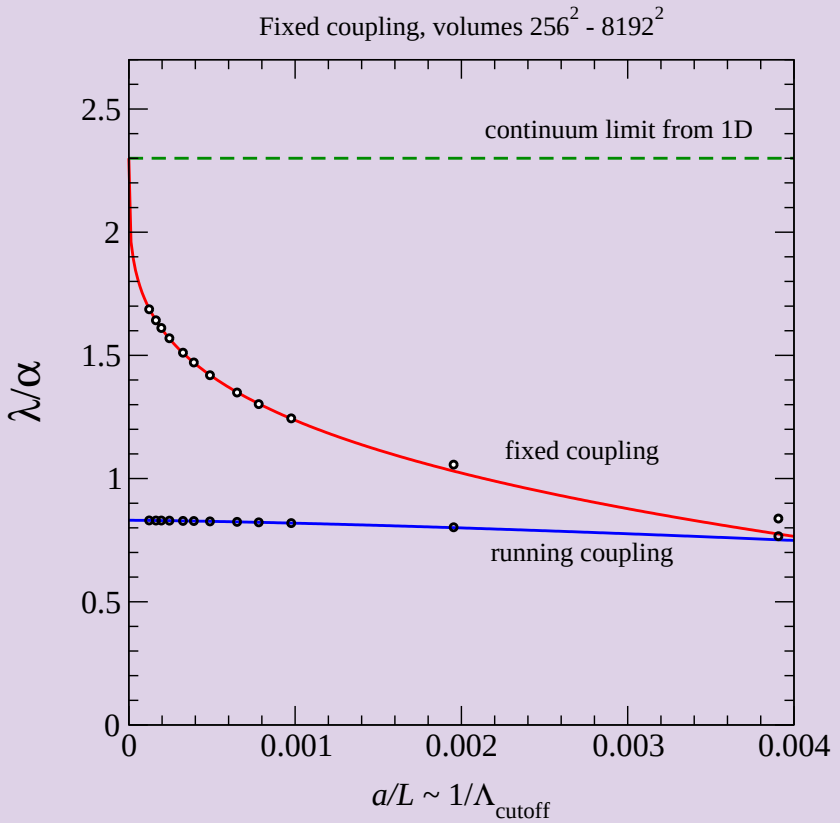
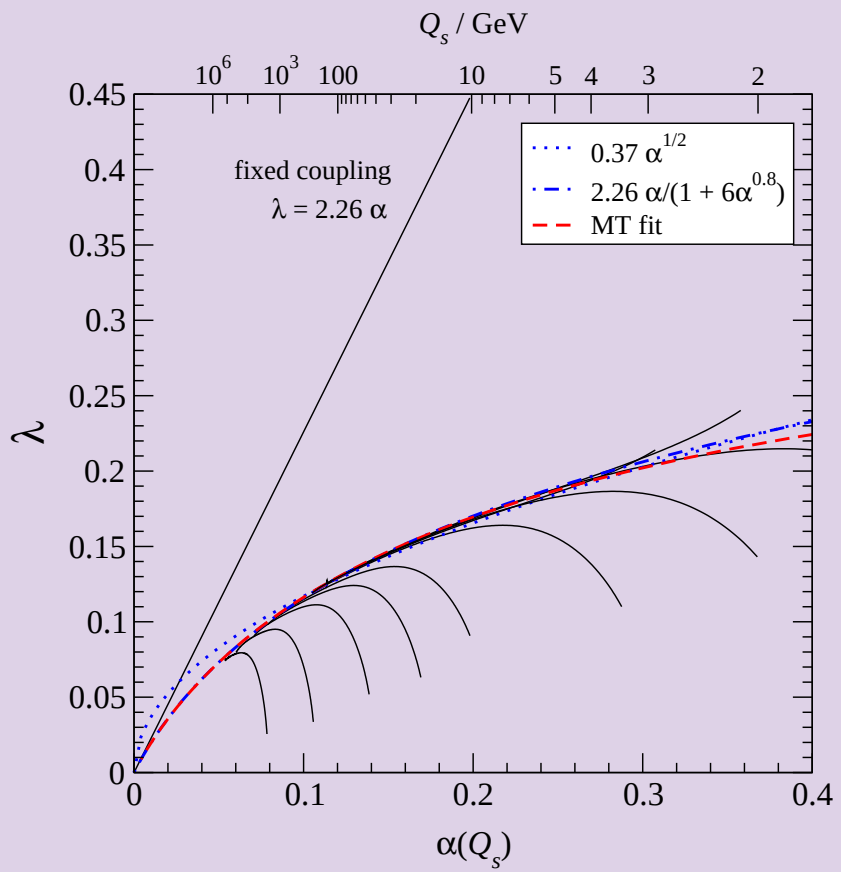
Running coupling is essential

phase space reduction



$\lambda(Y) := \partial_Y \ln Q_s(Y)$



back

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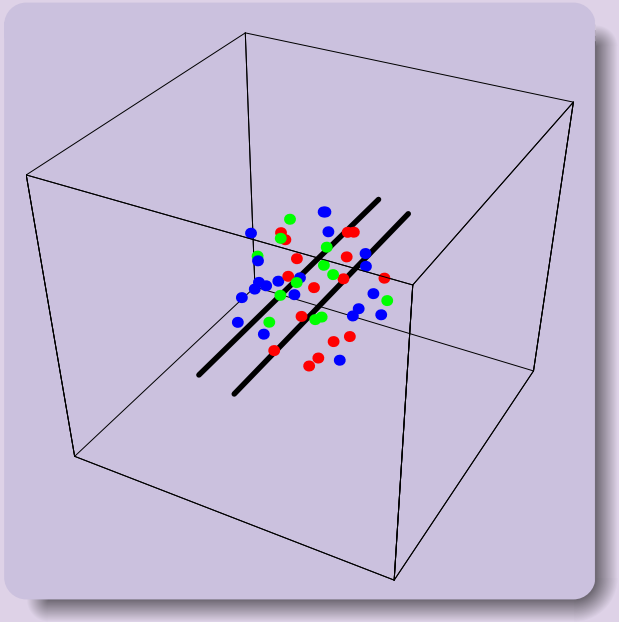
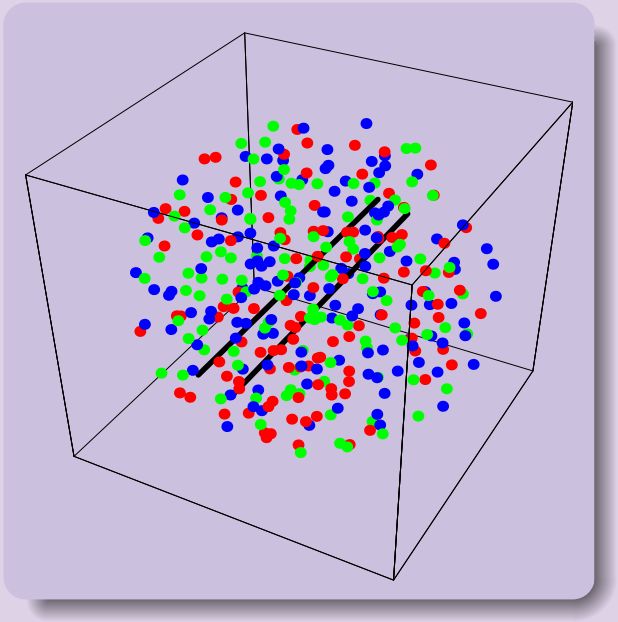
A dependence of $Q_s(Y)$ from BK

back

naive $A^{1/3}$ scaling

$$R \sim A^{1/3} \xrightarrow{\text{dilute}} (Q_s^A)^2 \sim (Q_s^p)^2 A^{1/3}$$





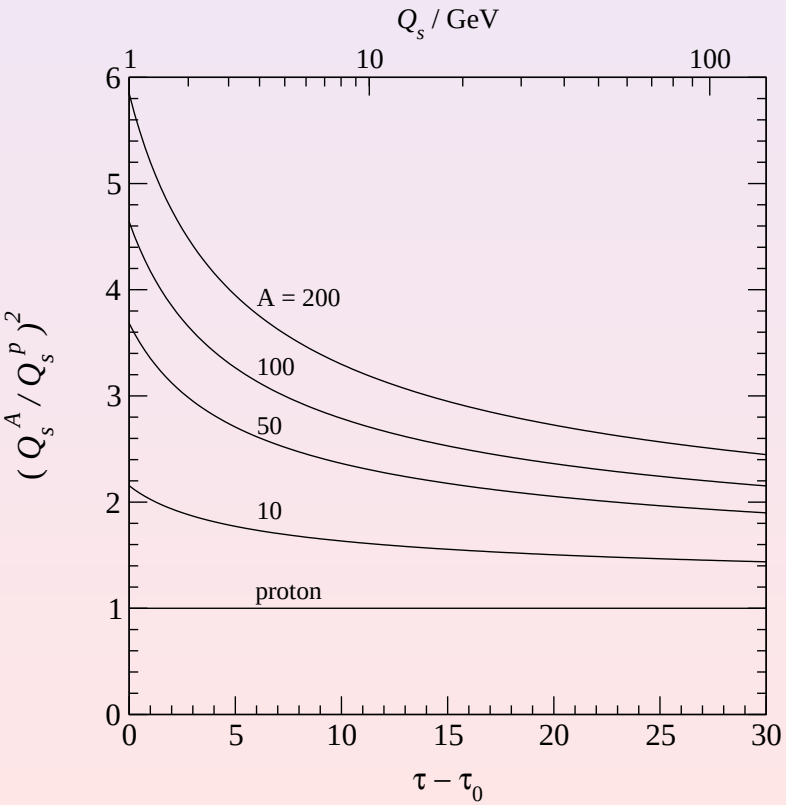
dilute $\Leftrightarrow (Q_s^A)^2(Y_0) \sim (Q_s^p)^2 A^{1/3}(Y_0)$

A dependence of $Q_s(Y)$ from BK

back

rule of tumb estimate: $n \to 1/2$

$$\frac{Q_s^A(Y)^2}{\Lambda^2_{\text{QCD}}} = \exp\left\{(n+1)2c\left(\frac{\pi}{\beta_0}\right)^n (Y - Y_0) + \left[\ln\left(\frac{A^{1/3} Q_s(Y_0)^2}{\Lambda^2_{\text{QCD}}}\right)\right]^{n+1}\right\}^{\frac{1}{n+1}}$$



- $\frac{Q^A{}_s(Y)^2}{Q^p{}_s(Y)^2} \xrightarrow{Y\to\infty} 1$ slowly
- $A = 50$: 13% loss from $Y = 0 \ldots 5$

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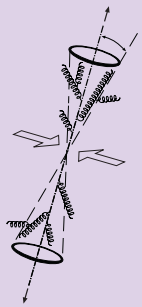
Non-global (exclusive) jet observables

back

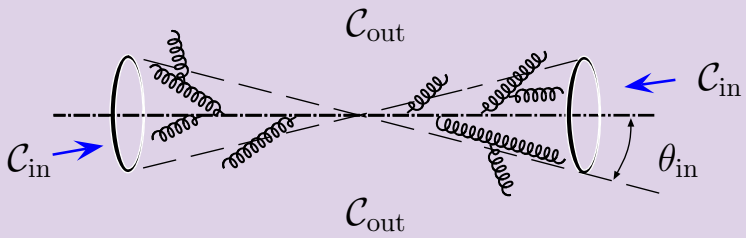
Already seen in $e^+e^- \longrightarrow$ jets @ total energy E



Non-global (exclusive) jet observables

back

Already seen in $e^+e^- \longrightarrow$ jets @ total energy E



Non-global (exclusive) jet observables

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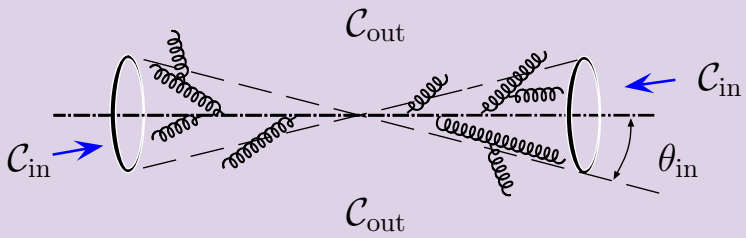
Already seen in $e^+e^- \longrightarrow$ jets @ total energy E

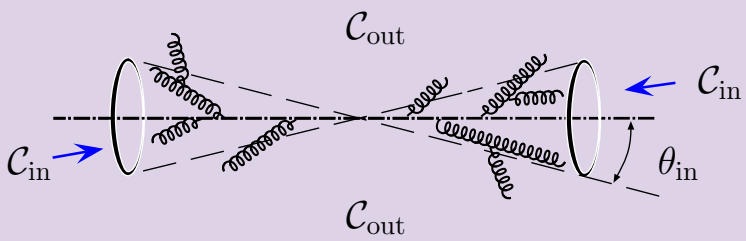


- fix geometry
- measure soft rad. into $\mathcal{C}_{\text{out}}$ only
- require $\sum E_{\text{soft}} < E_{\text{out}}$
- evolution equation in $\ln(E/E_{\text{out}})$

Non-global (exclusive) jet observables

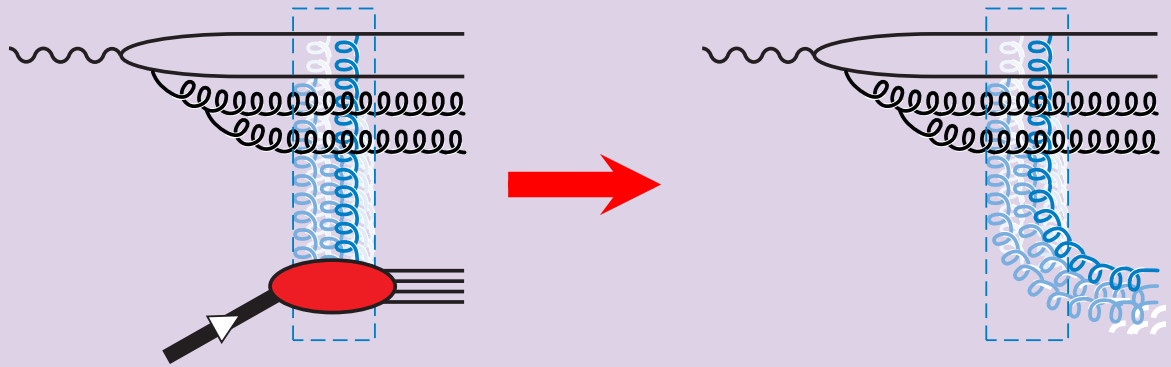
back

Already seen in $e^+e^- \to$ jets @ total energy E



- fix geometry
- measure soft rad. into $\mathcal{C}_{\text{out}}$ only
- require $\sum E_{\text{soft}} < E_{\text{out}}$
- evolution equation in $\ln(E/E_{\text{out}})$

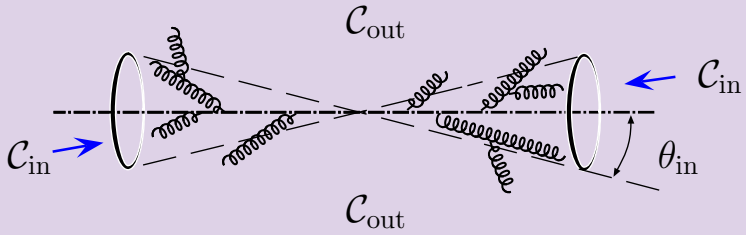
Analogy with CGC amplitudes:



Non-global (exclusive) jet observables

back

Already seen in $e^+e^- \to$ jets @ total energy E



- fix geometry
- measure soft rad. into $\mathcal{C}_{\rm out}$ only
- require $\sum E_{\rm soft} < E_{\rm out}$
- evolution equation in $\ln(E/E_{\rm out})$

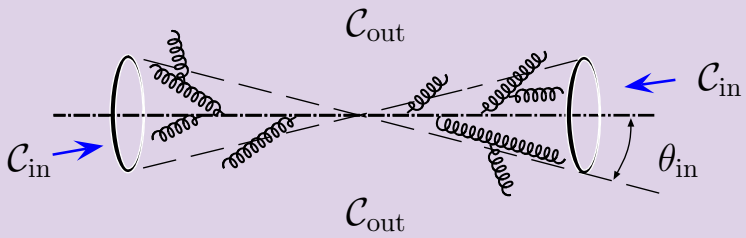
recall JIMWLK/BK:

$$\partial_Y S_{\boldsymbol{xy}} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{\boldsymbol{xzy}} \Big(S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}} \Big) \qquad S_{\boldsymbol{xy}} := \Big\langle \frac{\mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \Big\rangle$$

Non-global (exclusive) jet observables

back

Already seen in $e^+e^- \to$ jets @ total energy E



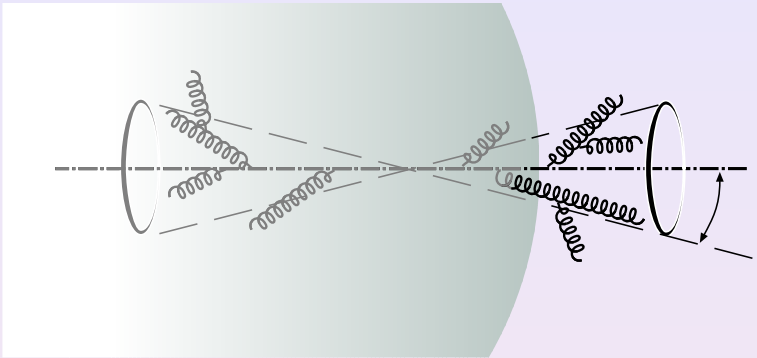
- fix geometry
- measure soft rad. into $\mathcal{C}_{\text{out}}$ only
- require $\sum E_{\text{soft}} < E_{\text{out}}$
- evolution equation in $\ln(E/E_{\text{out}})$

JIMWLK like evol. eqns. (BMS, H.W.)

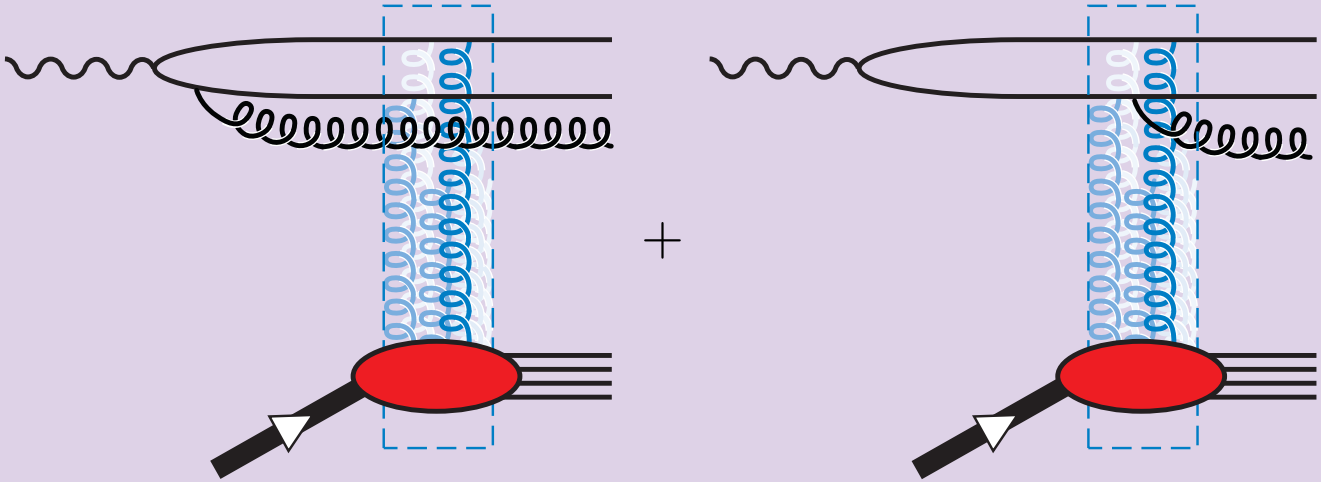
$$\partial_{\ln(\frac{E}{E_{\text{out}}})} S_{pq} = \int_{\mathcal{C}_{\text{in}}} \frac{d^2\Omega_k}{4\pi}\, \bar{\alpha}_s w_{pq}(k)\, [\, S_{pk}(E) \cdot S_{kq}(E) - S_{pq}(E)\,] - \int_{\mathcal{C}_{\text{out}}} \frac{d^2\Omega_k}{4\pi}\, \bar{\alpha}_s w_{pq}(k)\, S_{pq}(E)$$

Jets in a medium

back



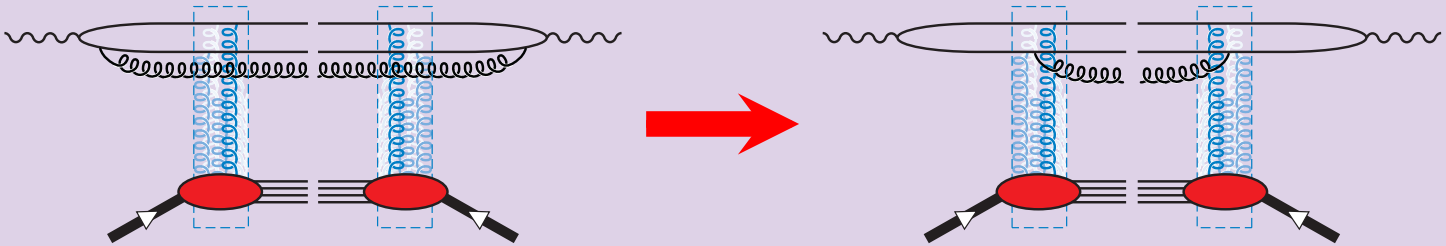
Already for global observables: difference to small x



Jets in a medium

back

Evolution equations change, driven by



$$\int_{\mathcal{C}_{\text{in}}} \frac{d^2\Omega_k}{4\pi}\, w_{pq}(k)\, \frac{\left[\tilde{U}_k\right]^{ab} 2\ \text{tr}(t^a U_p t^b U_q^\dagger)}{N_c}$$

$$\int_{\mathcal{C}_{\text{in}}} \frac{d^2\Omega_k}{4\pi} \int_0^\infty \frac{dx_0 dy_0}{p_0 q_0} e^{ip.kx_0/p_0 - iq.ky_0/q_0}\, p.q\,(k^0)^2$$

$$\times\ \frac{\left[\tilde{U}_k\right]^{ab}_{x_0,y_0} 2\ \text{tr}(t^a [U_p]_{x_0} t^b [U_q^\dagger]_{y_0})}{N_c} + \ldots$$