Jet Energy Loss & High P_T Photon Production in Hot QGP

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McGill

with S.Turbide, C.Gale and G.Moore

Why high energy photons?

- $\alpha_{\text{EM}}/\alpha_s \sim O(3\,\%) \Longrightarrow$ Photons carry un-modified information
  - Sensitive to high temperature
  - Sensitive to the modification of jet properties

Many sources of high p_T photons

- Prompt photons
- Thermal photon radiation from QGP
- Photons from jets $(*)$
- Jet fragmentation $(*)$

Jet quenching important for the above two $(*)$'s

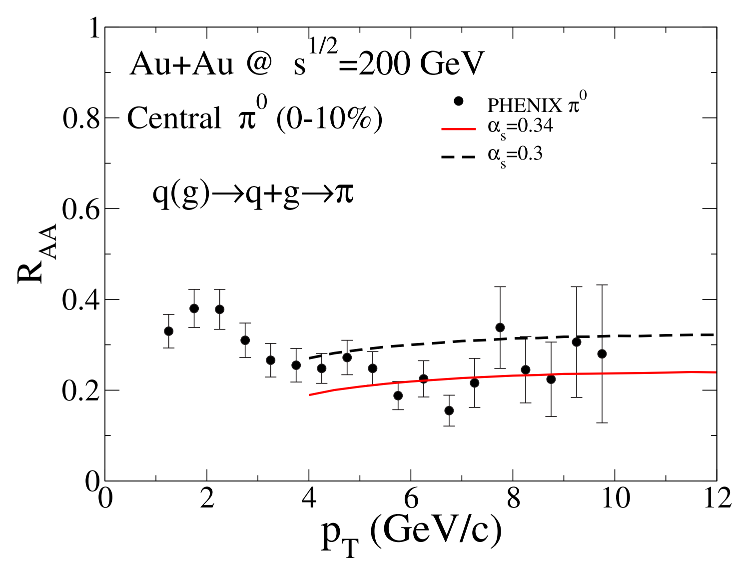
$\Longrightarrow$ Another evidence for jet quenching and QGP

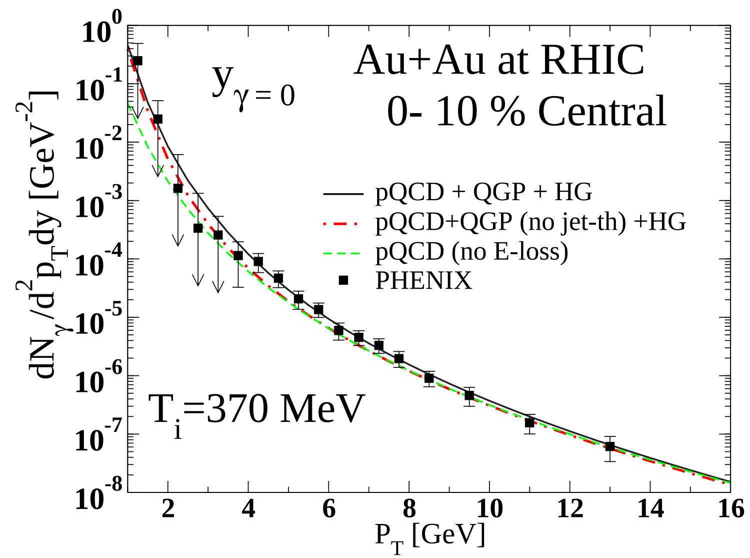
$\Longrightarrow$ The γ production must be consistent with Jet quenching

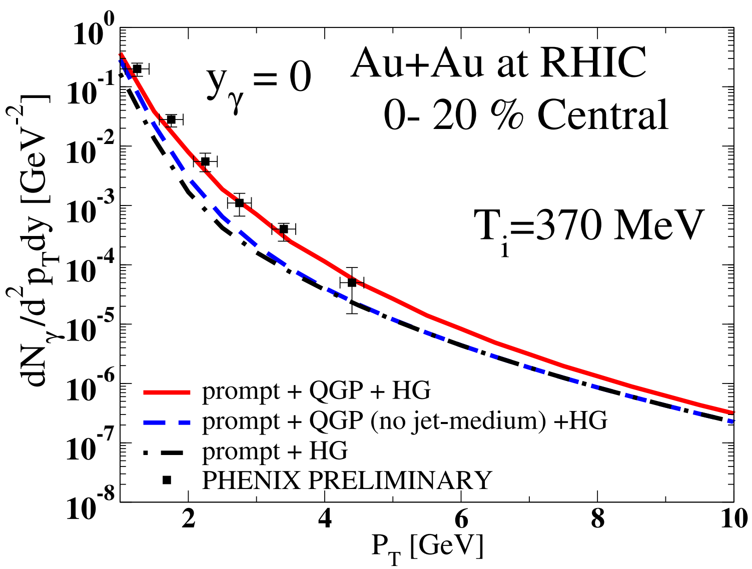
Strategy

- Jet & Prompt photon calculation by a proven numerical method (courtesy of P. Aurenche)
- Calculate full leading order in α_s:
  - Jet energy loss rate (AMY)
  - Thermal photon production rate (AMY)
  - γ bremsstrahlung from jets (AMY)
  - Jet-photon conversion rate
- Embed jets in a hydrodynamically evolving medium.
- Jet evolution by solving the rate equations for both hard quarks and gluons $\Longrightarrow$ Jets fragment outside

Where we want to go:







S.J. and G.Moore, PRC71:034901,2005

S.Turbide, C.Gale, S.J. and G.Moore, PRC72:014906,2005

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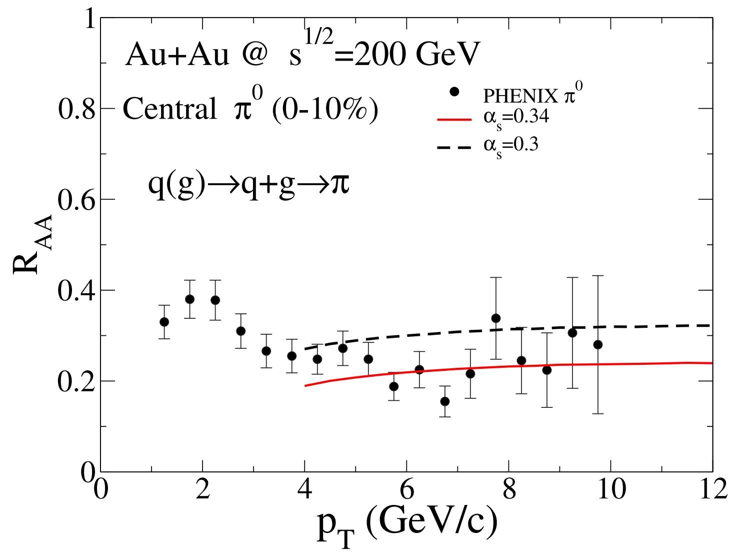
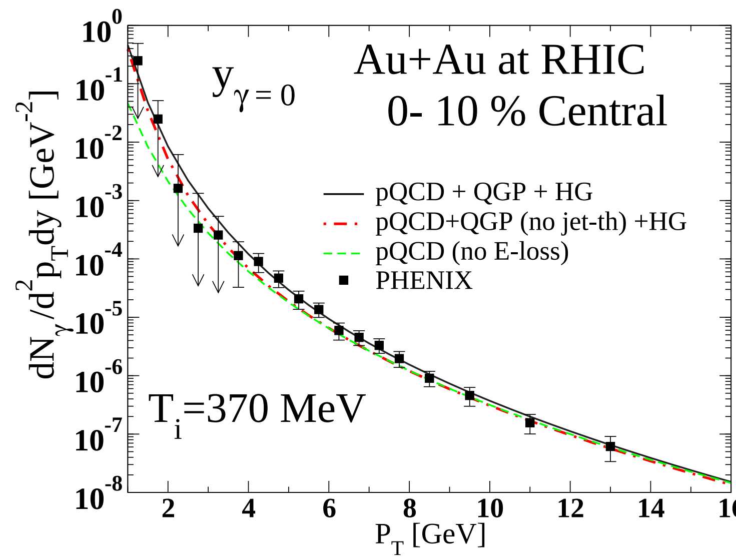
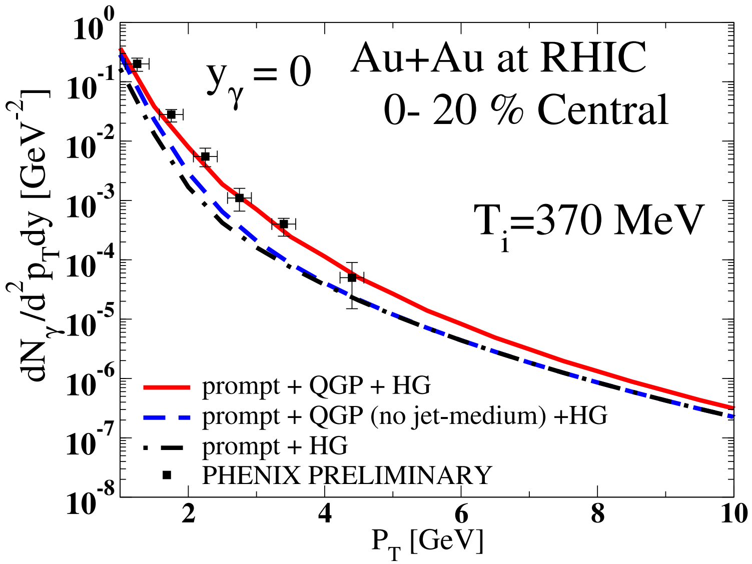






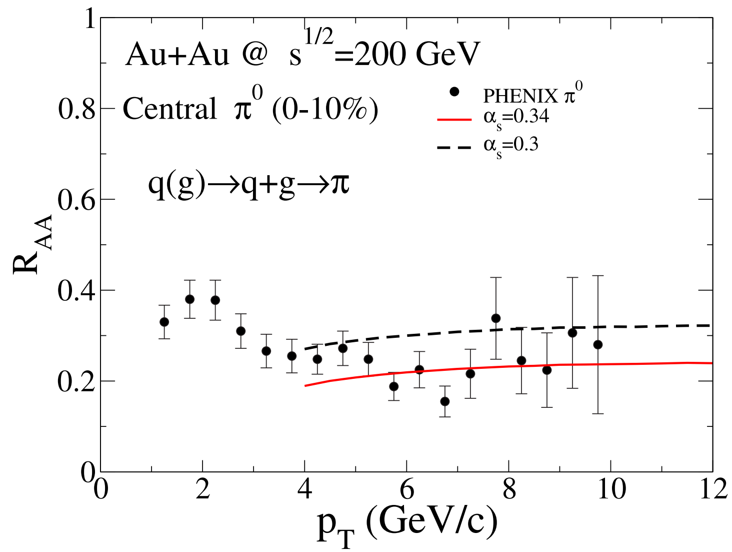
Fix α_s and T.

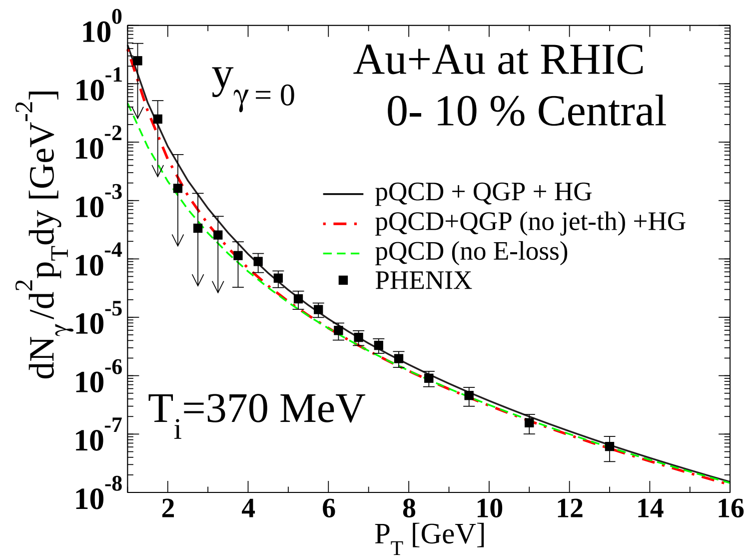
Achieve good description.

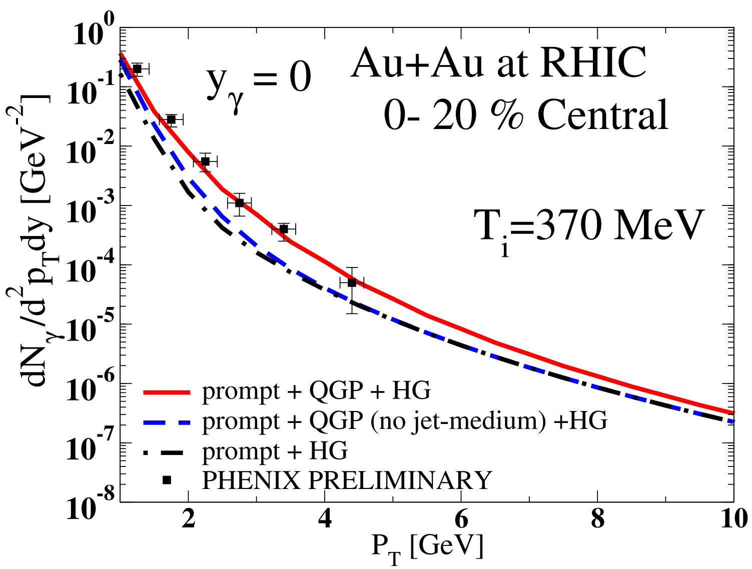
S.J. and G.Moore, PRC71:034901,2005

S.Turbide, C.Gale, S.J. and G.Moore, PRC72:014906,2005

Where we want to go:







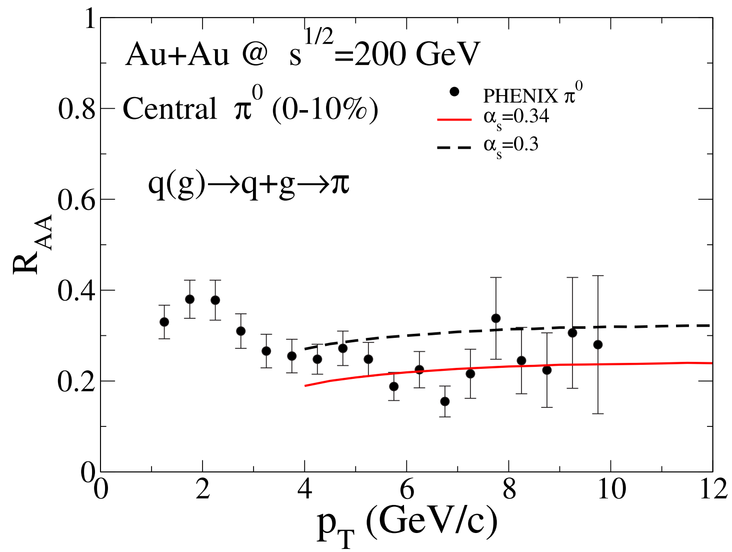
↑

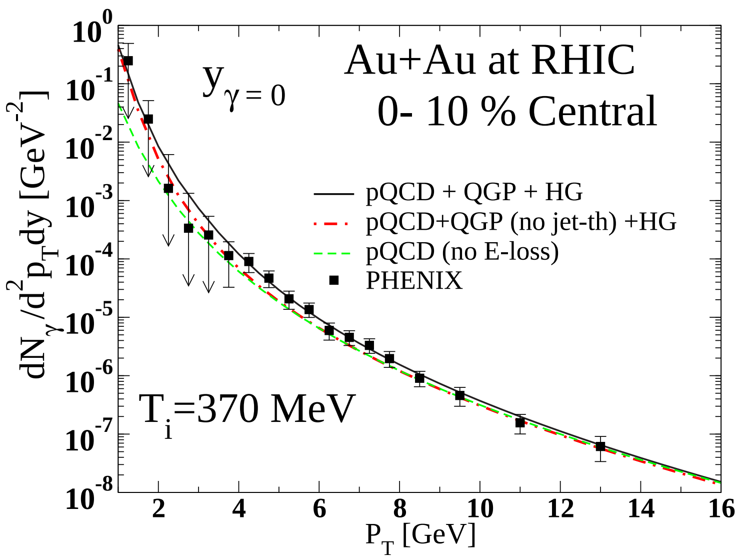
With the fixed parameters

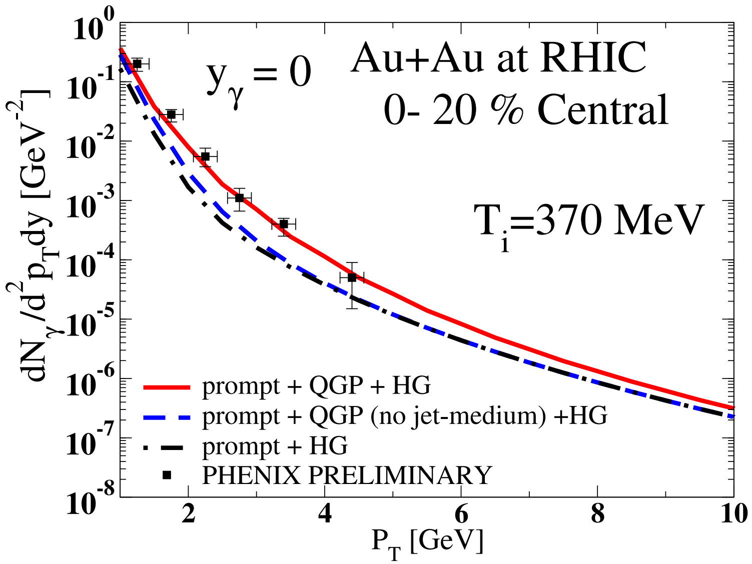
S.J. and G.Moore, PRC71:034901,2005

S.Turbide, C.Gale, S.J. and G.Moore, PRC72:014906,2005

Where we want to go:







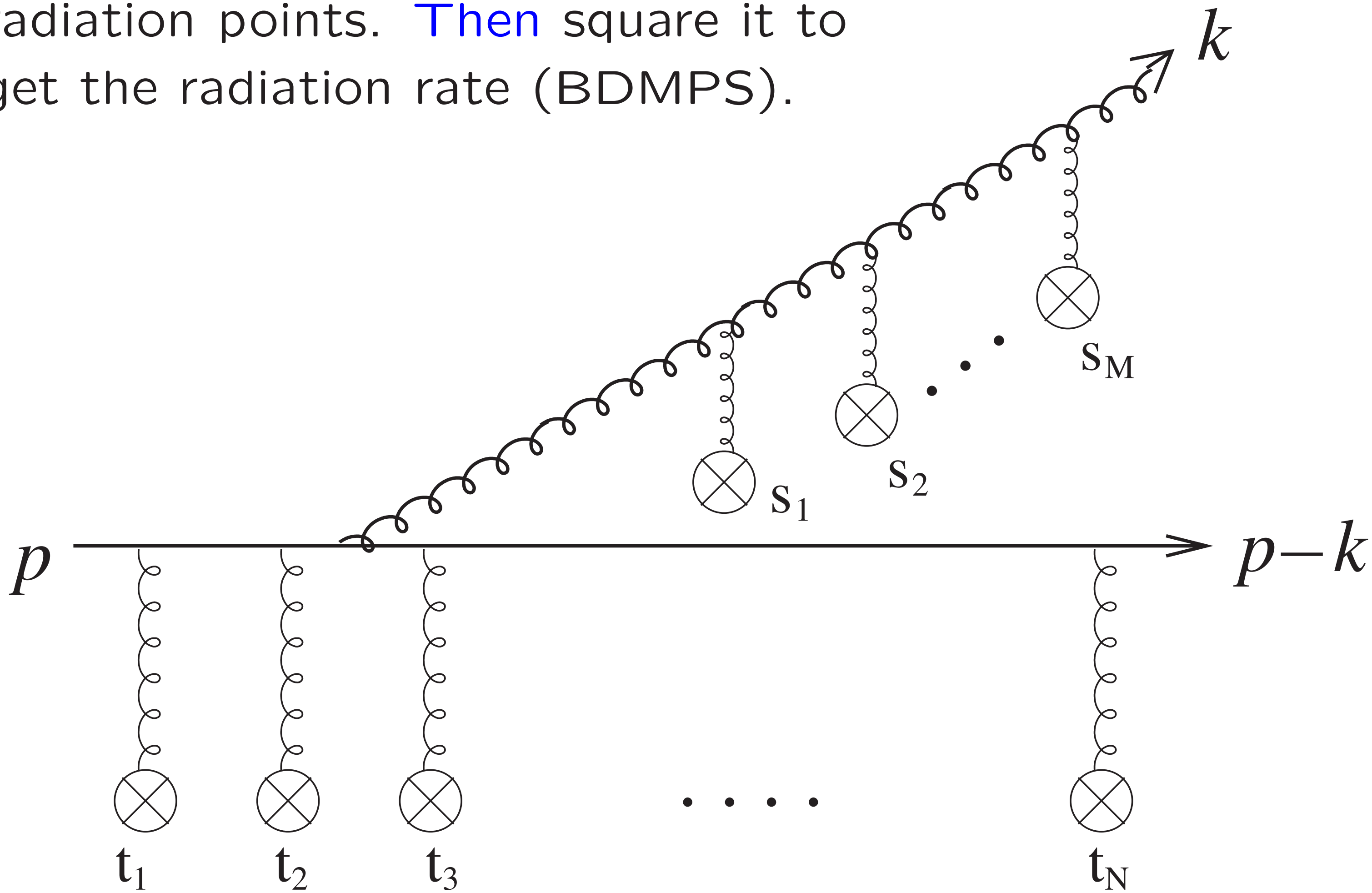
← **Prediction**

S.J. and G.Moore, PRC71:034901,2005

S.Turbide, C.Gale, S.J. and G.Moore, PRC72:014906,2005

Gluon Radiation – Jet Quenching

Amplitude to radiate: Need to sum over all N and all M and all possible radiation points. Then square it to get the radiation rate (BDMPS).

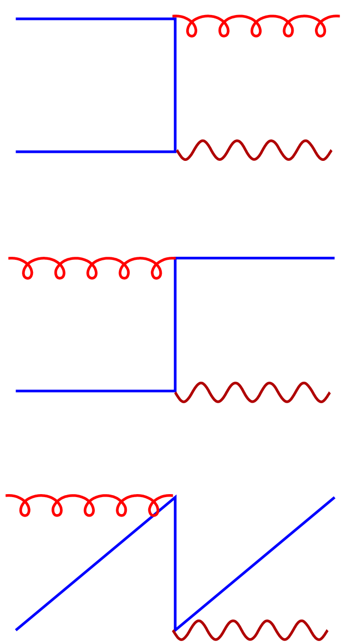


Why is this so hard?

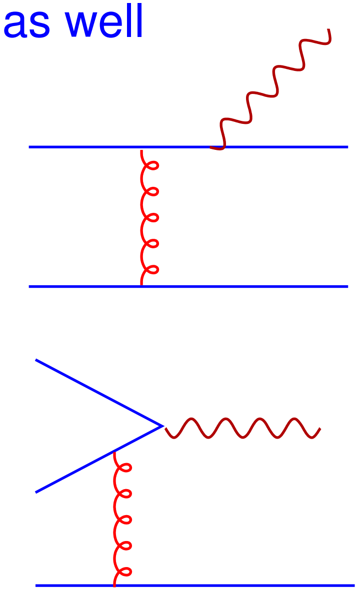
- Collinear enhancement in photon & gluon radiations

  Aurenche, Gelis, Kobes and Zaraket, PRD58:085003,1998, Arnold, Moore and Yaffe (AMY), JHEP 0206:030,2002; JHEP 0112:009,2001; JHEP 0111:057,2001

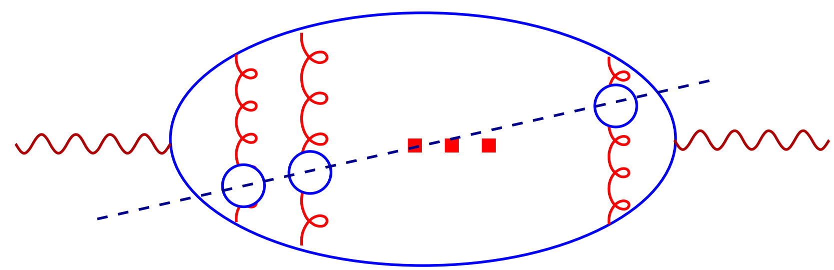
Leading order



Collinear enhancement makes these leading order as well

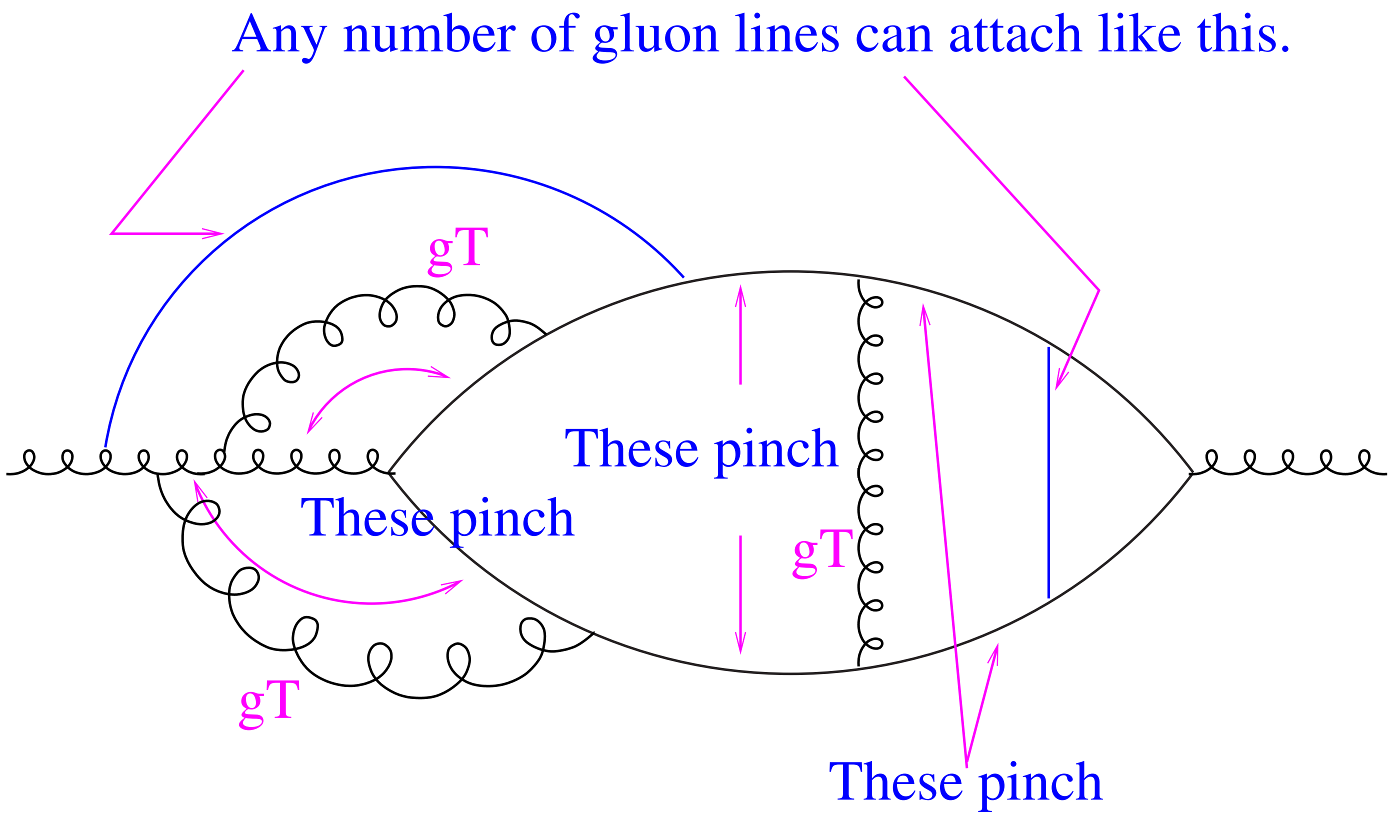


Need to resum all these, too (AMY)



○ : Hard Thermal Loop

Gluon ladder diagrams



Adding one more rung = O(1).
Need to resum.

SD equation

$$
\begin{aligned}
2\mathbf{h} &= i\delta E(\mathbf{h},p,k)\mathbf{F}(\mathbf{h}) + g^2 \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \times \\
&\quad \times \Big\{ (C_s - C_\mathsf{A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} - k\,\mathbf{q}_\perp)] \\
&\quad\quad + (C_\mathsf{A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} + p\,\mathbf{q}_\perp)] \\
&\quad\quad + (C_\mathsf{A}/2)[\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h} - (p-k)\,\mathbf{q}_\perp)] \Big\}, \\
\delta E(\mathbf{h},p,k) &= \frac{\mathbf{h}^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p}.
\end{aligned}
$$

Here m^2 are the medium induced thermal masses, equal to $m_D^2/2$ for a gluon and $C_f g_\mathsf{s}^2 T^2/4 = g_\mathsf{s}^2 T^2/3$ for a quark. For the case of $g \to qq$, the $(C_s - C_\mathsf{A}/2)$ term is the one with $\mathbf{F}(\mathbf{h} - p\,\mathbf{q}_\perp)$ rather than $\mathbf{F}(\mathbf{h} - k\,\mathbf{q}_\perp)$.

Gluon Radiation Rate

$$
\begin{aligned}
\frac{d\Gamma_g(p,k)}{dk\,dt} \;=\; & \frac{C_s g_{\rm s}^2}{16\pi p^7}\frac{1}{1\pm e^{-k/T}}\frac{1}{1\pm e^{-(p-k)/T}} \times \\
& \times \left\{ \begin{array}{cc} \frac{1+(1-x)^2}{x^3(1-x)^2} & q\to qg \\ N_{\rm f}\frac{x^2+(1-x)^2}{x^2(1-x)^2} & g\to qq \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g\to gg \end{array} \right\} \\
& \times \int \frac{d^2\mathbf{h}}{(2\pi)^2} 2\mathbf{h}\cdot {\rm Re}\,\mathbf{F}(\mathbf{h},p,k)\,,
\end{aligned}
$$

where $x \equiv k/p$ is the momentum fraction in the gluon (or either quark, for the case $g \to qq$).

$\mathbf{h} \equiv \mathbf{p} \times \mathbf{k}$: 2-D vector. $O(gT^2)$

Time evolution equation

$$\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} + 2P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt},$$

$$\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dkdt} + P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} - P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt} + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(k-p/2)\right),$$

- k integrals range: $(-\infty,\infty)$.
- $k < 0$: Absorption of thermal gluons.
- $k > p$: annihilation against and antiquark of energy $(k-p)$.
- $\Theta(k-p/2)$: To prevent double counting of final states.

More on the evolution:

Poisson ansatz (BDMS)

$$P(p) = \int d\epsilon \, D(\epsilon, p) \, P_0(p + \epsilon)$$

where

$$D(\epsilon, p) \; = \; e^{-\int d\omega \frac{dI}{d\omega}(p,\omega)} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \, \frac{dI}{d\omega_i}(p, \omega_i) \right] \delta \left(\epsilon - \sum_{i=1}^{n} \omega_i \right)$$

Now let

$$\frac{dI}{d\omega}(p, \omega) \; = \; \int_{t_0}^{t} dt' \, \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(p, \omega, t')$$

Can show that the Poisson ansatz solves:

$$\frac{dP(p,t)}{dt} = \int d\omega \, \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(\omega) P(p+\omega,t) - P(p,t)\int d\omega \, \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(\omega)$$

We solve:

$$\begin{aligned}
\frac{dP_{q\bar{q}}(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,k)}{dkdt} - P_{q\bar{q}}(p)\frac{d\Gamma^q_{qg}(p,k)}{dkdt} \\
&\quad + 2P_g(p+k)\frac{d\Gamma^g_{q\bar{q}}(p+k,k)}{dkdt}, \\
\frac{dP_g(p)}{dt} &= \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma^q_{qg}(p+k,p)}{dkdt} + P_g(p+k)\frac{d\Gamma^g_{gg}(p+k,k)}{dkdt} \\
&\quad - P_g(p)\left(\frac{d\Gamma^g_{q\bar{q}}(p,k)}{dkdt} + \frac{d\Gamma^g_{gg}(p,k)}{dkdt}\Theta(k-p/2)\right),
\end{aligned}$$

What are we doing?

– "Better" BDMPS

- Full leading order α_s momentum space calculation of the emission + absorption rate in fully dynamic thermal medium. Includes
  - Bremsstrahlung
  - Pair annihilation
  - Absorption from the medium
  - Thermal dispersion corrections
  - Correct and smooth transition from Bethe-Heitler to LPM
- Solve Fokker-Planck equation for the *distribution* instead of Poisson ansatz. Includes nuclear geometry and can accomo-date expansion scenarios.

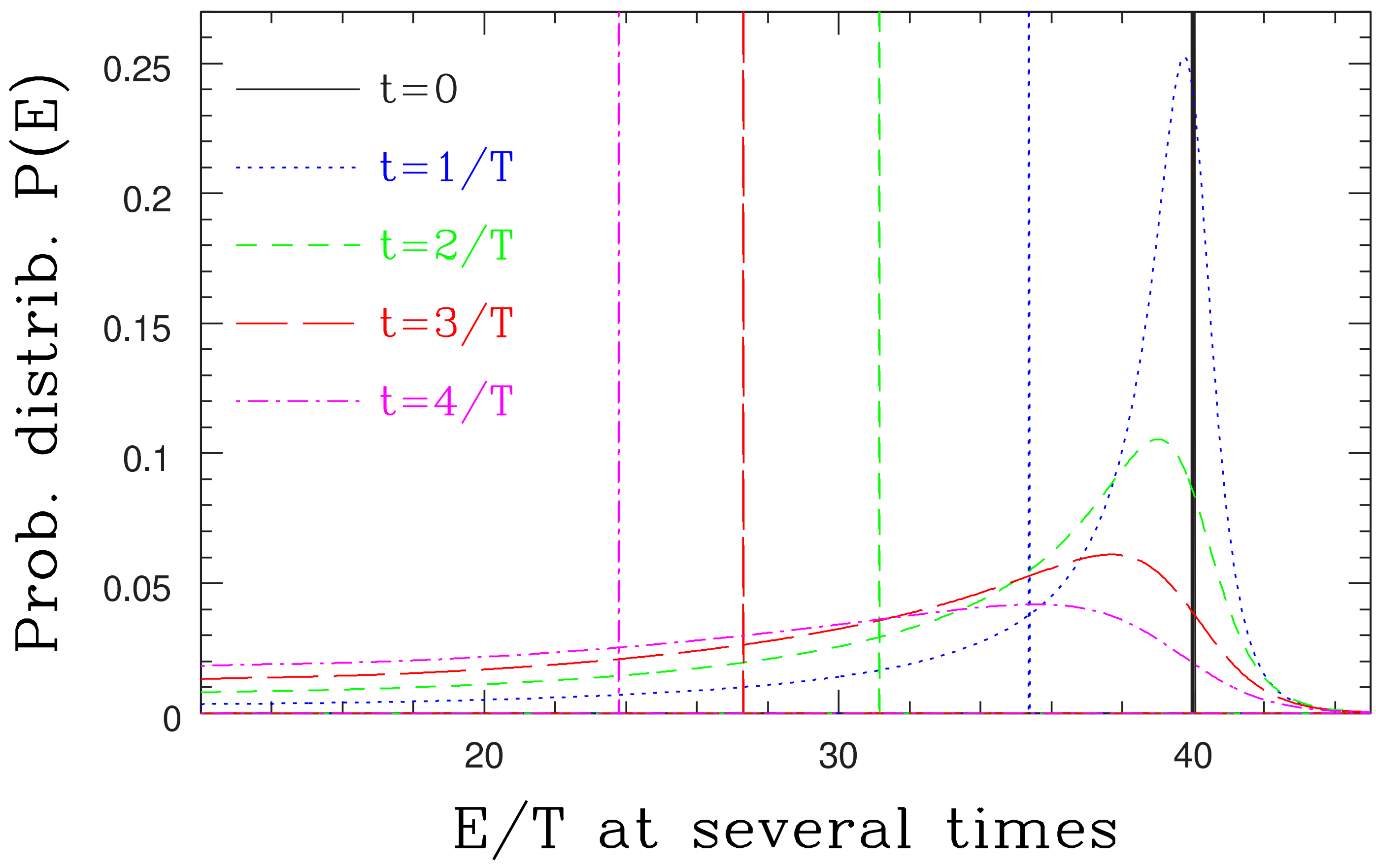
Validity

- Caveat: Weak coupling limit. $g \ll 1$.
- $\tau_{\text{coh}} \ll L$
- $\tau_{\text{coh}} \ll (d \ln T(x)/dx)^{-1}$
- One must distinguish what's important for ΔE and R_{AA} (BDMPS, JM).
  - R_{AA} dominated by many soft emissions.
    - Fully treated in AMY.
  - ΔE dominated by rare hard emissions.
    - $k > E_{\text{fact}}$ is not fully treated in AMY. But not important for R_{AA}.
- Rough estimates (Bounds for the emitted energy):

$E_{\text{LPM}} \sim T \sim 300\,\text{MeV}$,

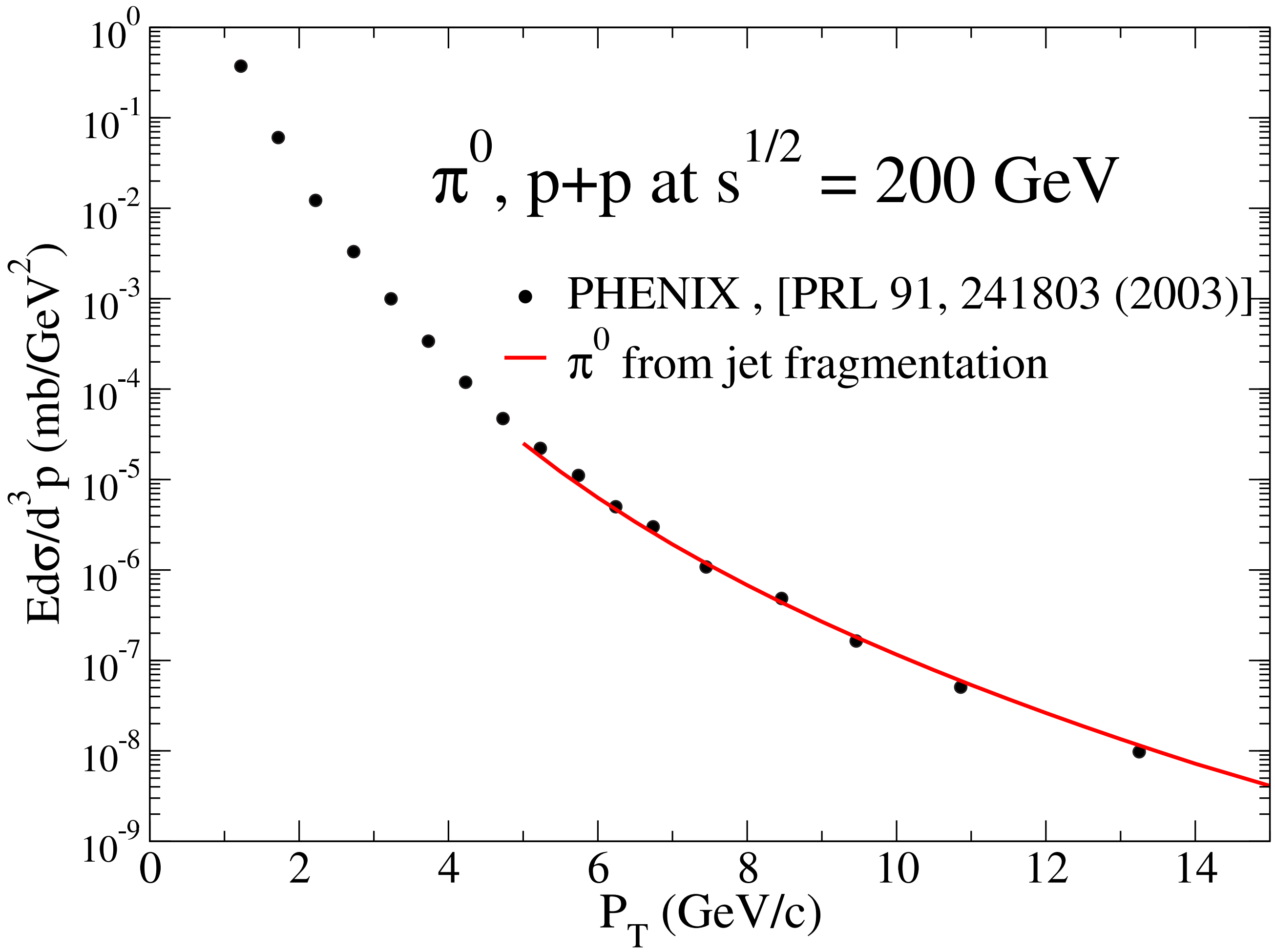
$E_{\text{fact}} \approx (0.3\,\text{GeV}) \times (L/\lambda)^2 \approx 7.5 - 30\,\text{GeV}$ for $L/\lambda = 5 - 10$

Results – δ-function Evolution



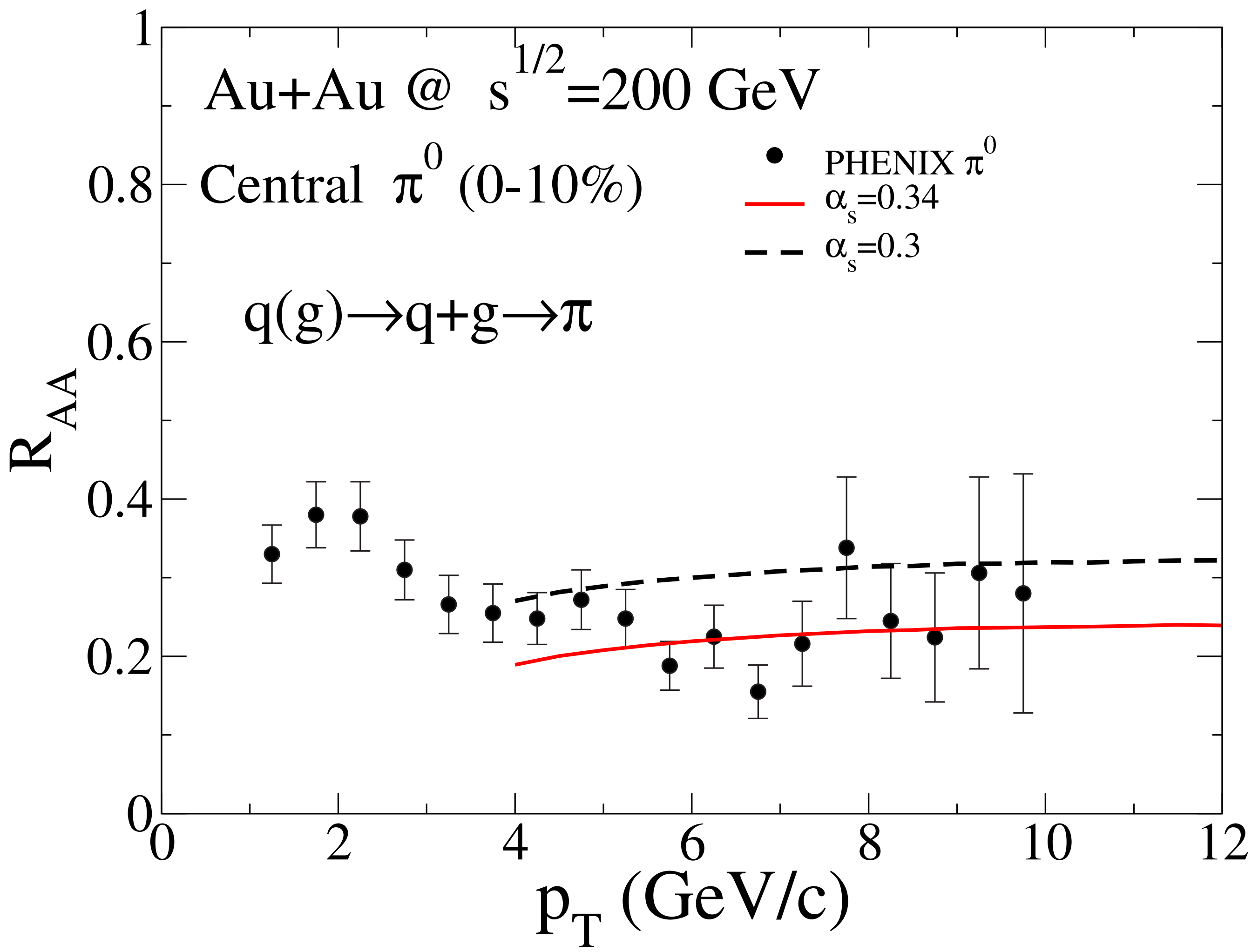
Lesson: Use the full distribution function.

Baseline calculation – PP π^0



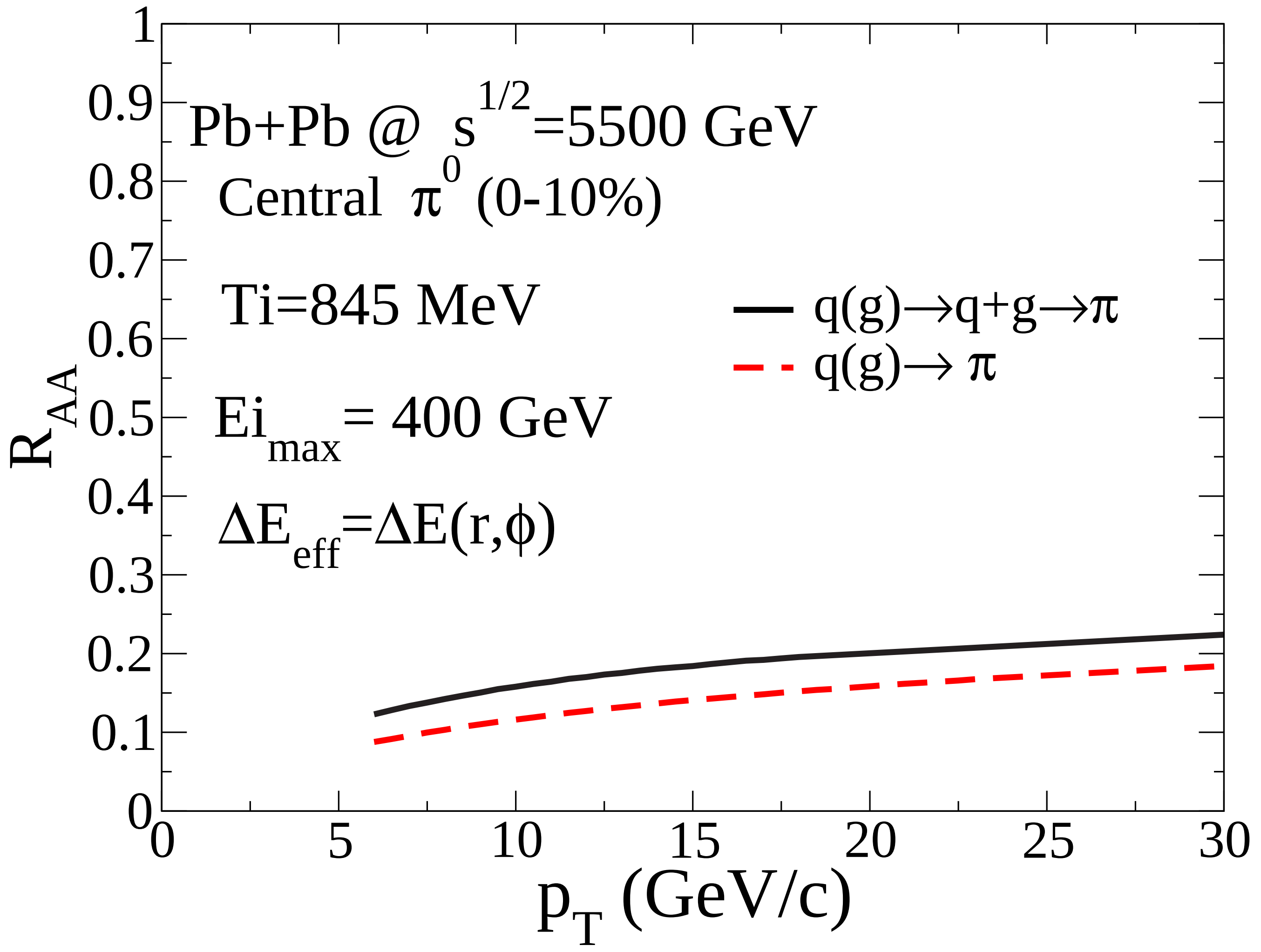
Using P.Aurenche et al.'s program.

Nuclear Modification Factor



$T_i = 370\,\text{MeV}$, $dN/dy = 1260$. 1-D Bjorken expansion included.

Nuclear Modification Factor (LHC)



$dN/dy = 5620.$

Where are we now?

- Jet quenching under control in the hadronic part.

Where are we going? – γ production

- Need:
  - Thermal photon radiation rate (AMY)
  - Jet bremsstrahlung rate (AMY)
  - Jet-photon conversion rate (Fries, Mueller, Srivastava)

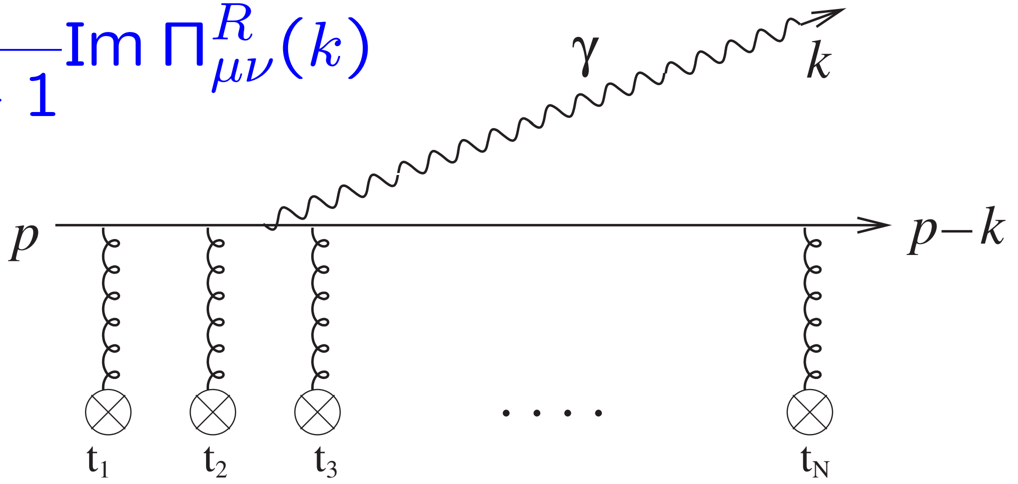
Photon Radiation

Arnold, Moore and Yaffe (AMY), JHEP 0206:030,2002; JHEP 0112:009,2001; JHEP 0111:057,2001

Thermal Radiation rate:

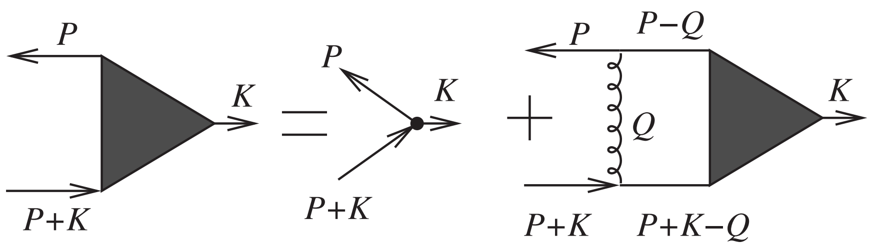
$$\omega \frac{dR}{d^3k} = -\frac{g^{\mu\nu}}{(2\pi)^3} \frac{1}{e^{\omega/T} - 1} \text{Im}\,\Pi^R_{\mu\nu}(k)$$

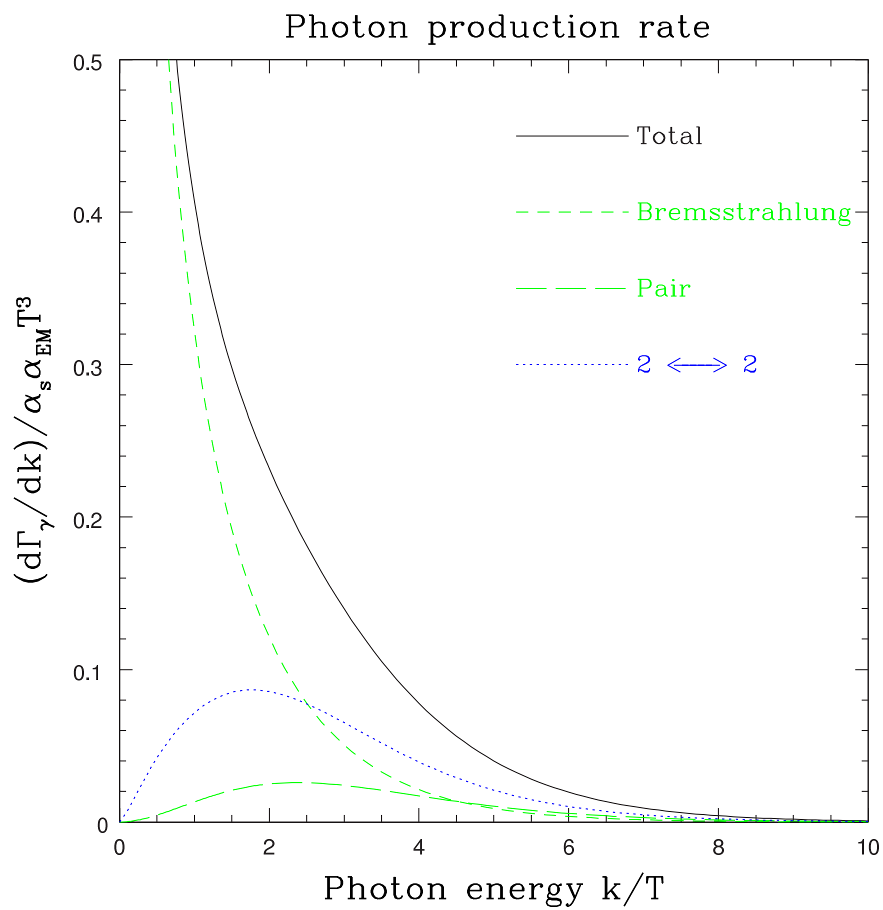
Physical process:



Need to sum over the scattering centers and the radiation points.

Schwinger-Dyson Equation

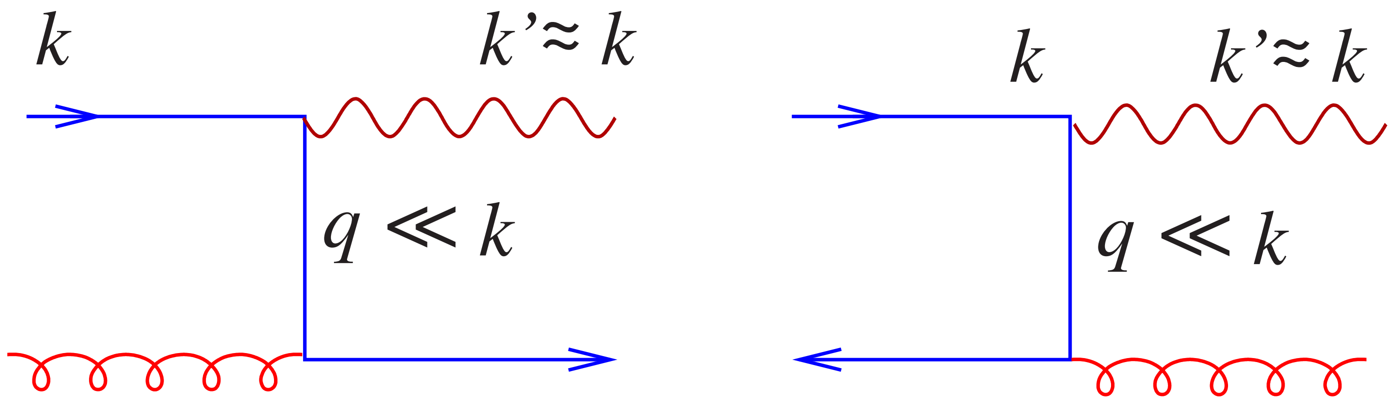




Arnold, Moore and Yaffe, JHEP 0112 (2001) 009

The same formalism can be used to calculate thermal radiation ($P \sim T$) and bremsstrahlung from jets ($P \gg T$).

Jet-Thermal Conversion



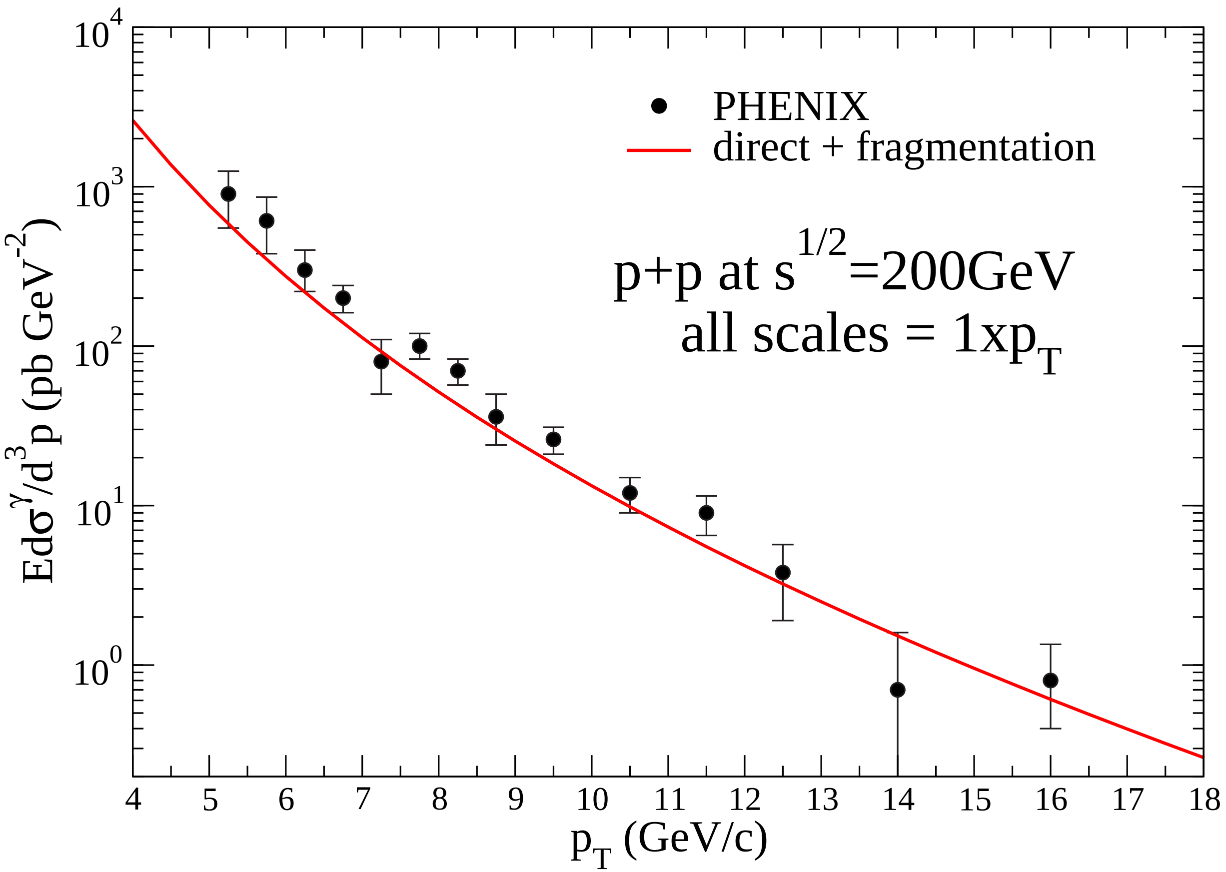
Fries, Mueller, Srivastava (nucl-th/0208001)

$$\frac{dR}{dyd^2p_T} = \sum_f \left(\frac{e_f}{e}\right)^2 \frac{T^2\alpha\alpha_s}{8\pi^2}[f_q(\overrightarrow{p_\gamma}) + f_{\bar{q}}(\overrightarrow{p_\gamma})] \left[2\ln\left(\frac{4E_\gamma T}{m^2}\right) - C\right]$$

with $C \approx 2.33$.

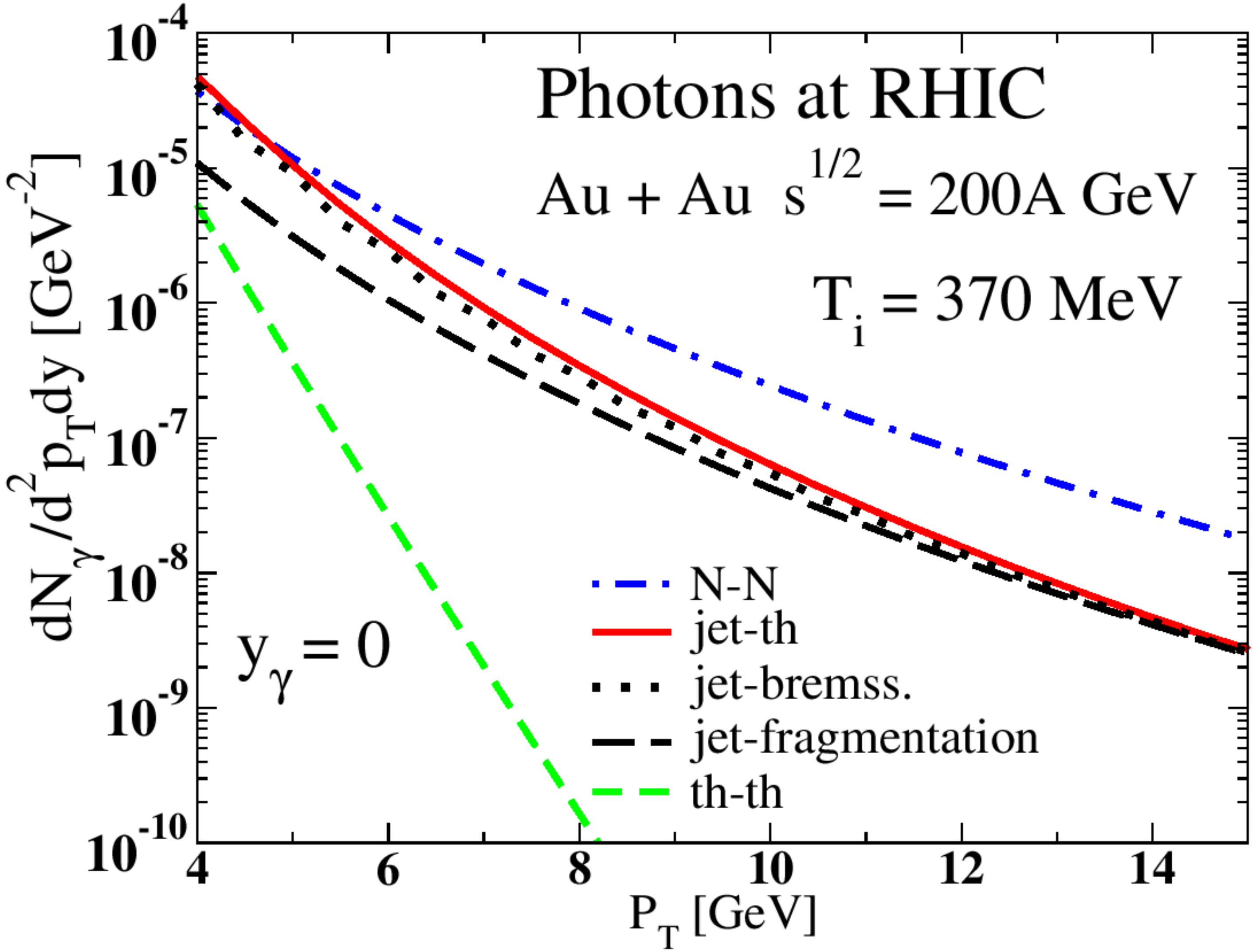
Putting everything together...

γ – Baseline

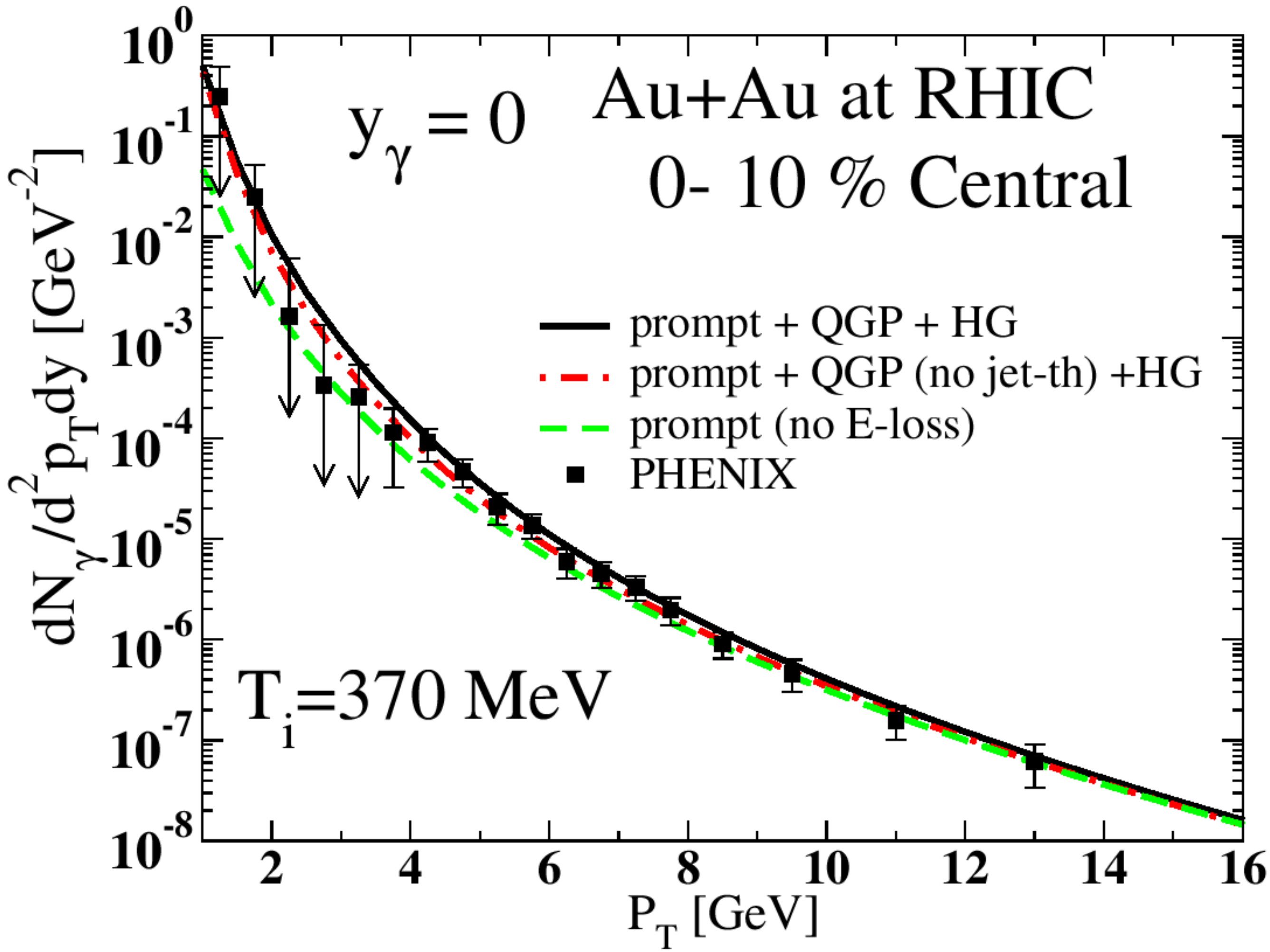


P. Aurenche's pQCD program.

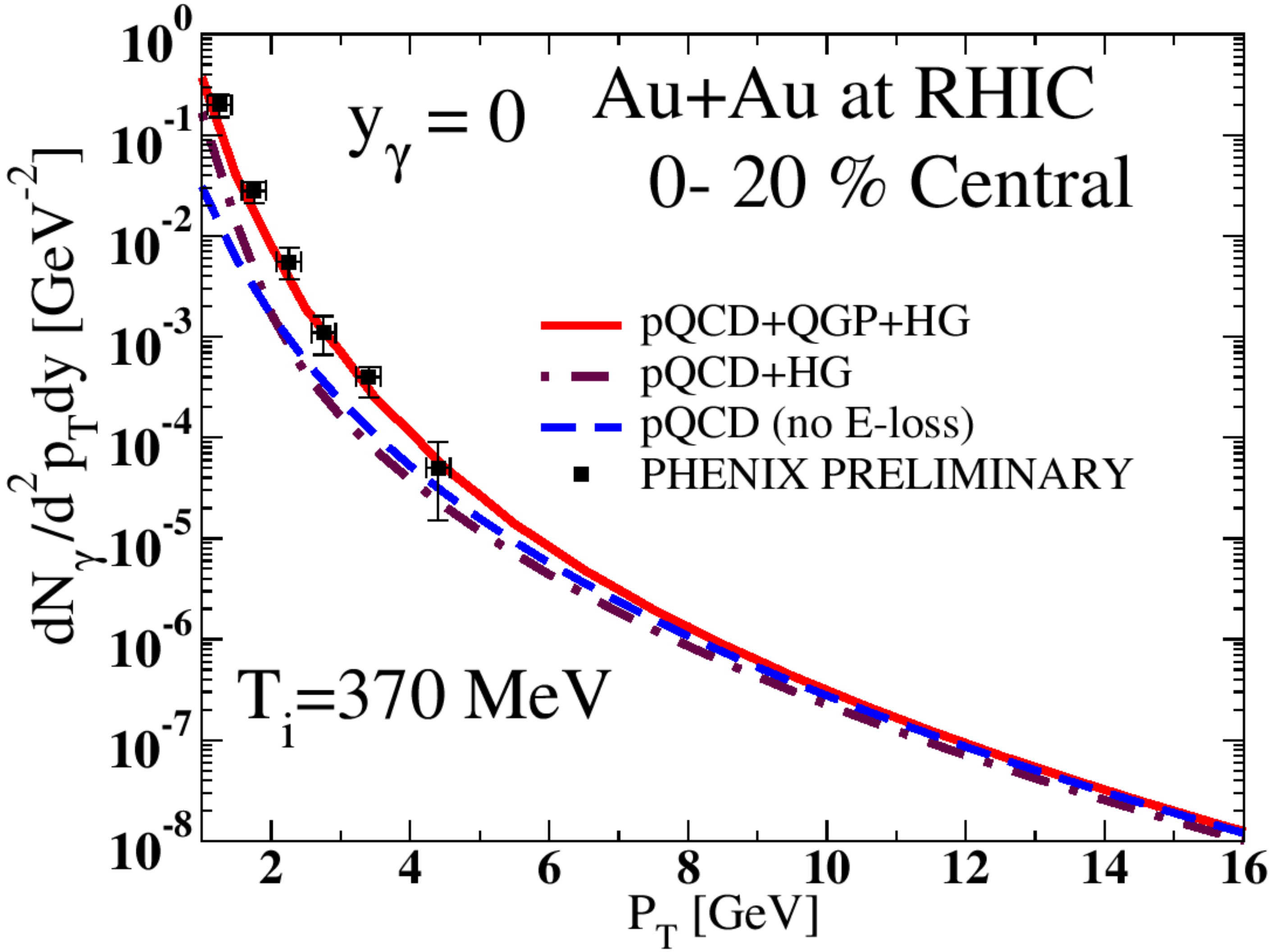
γ − Composition



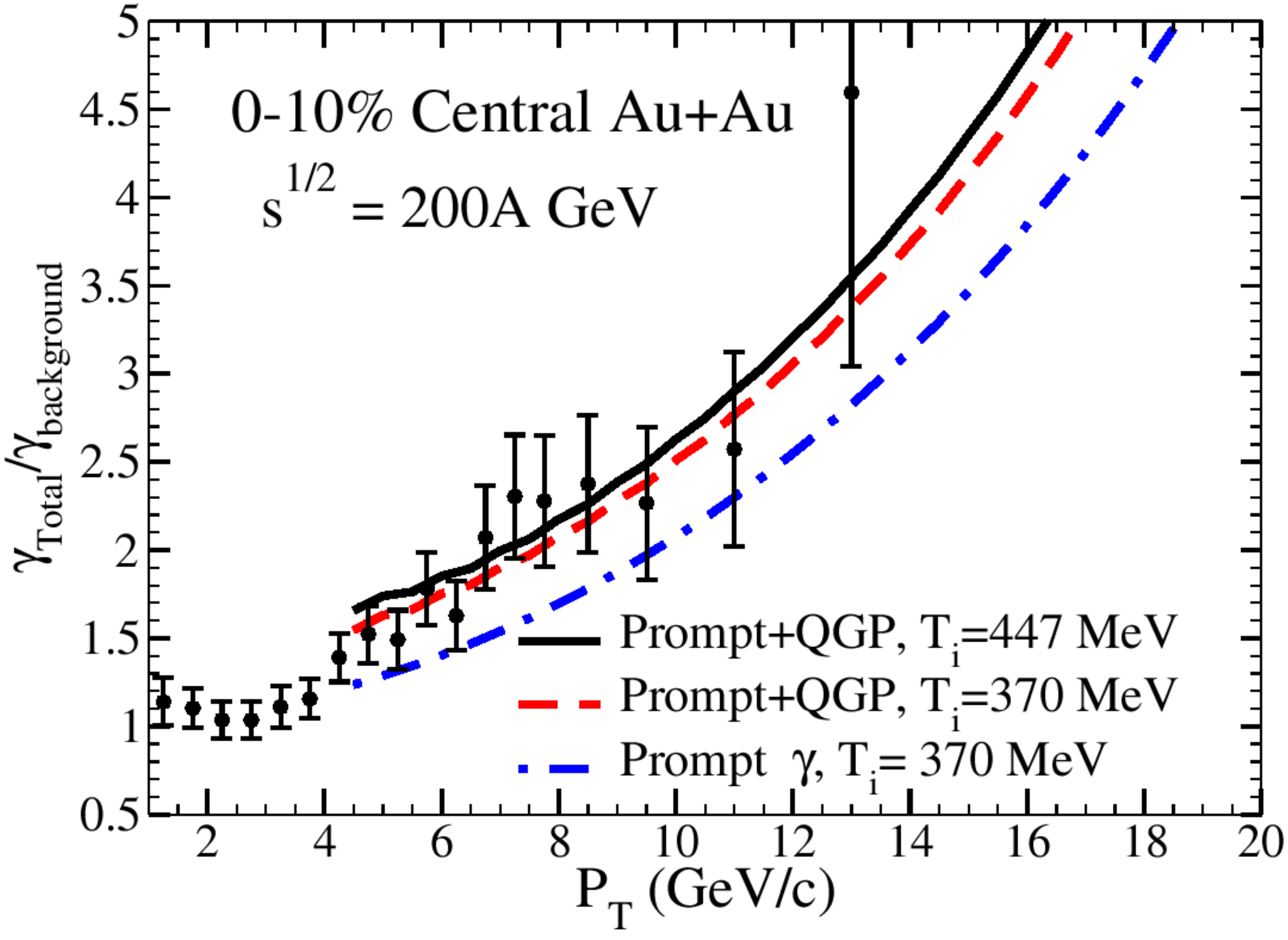
γ – Our Calc vs. PHENIX Data



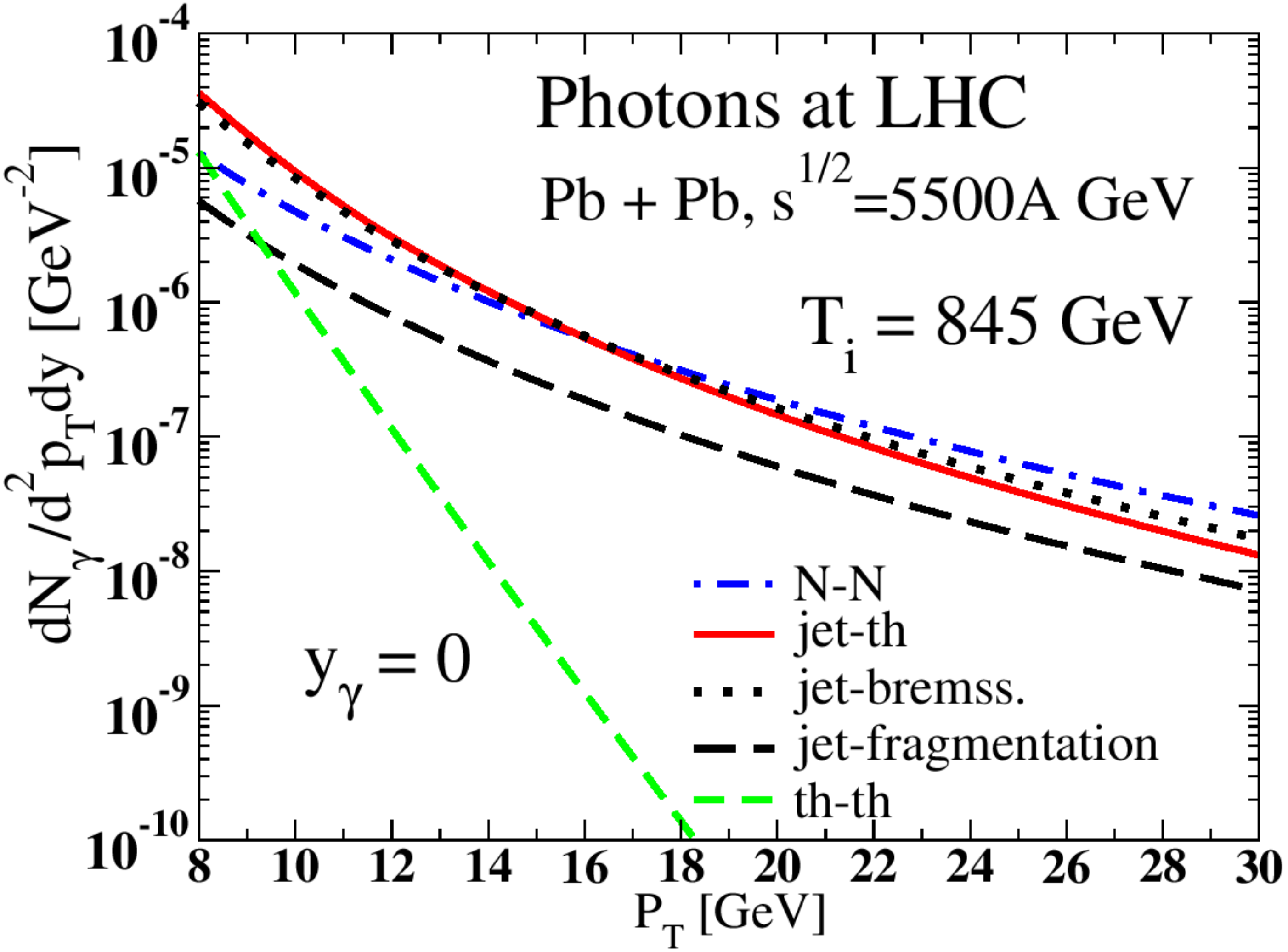
γ – Our Prediction vs. Data



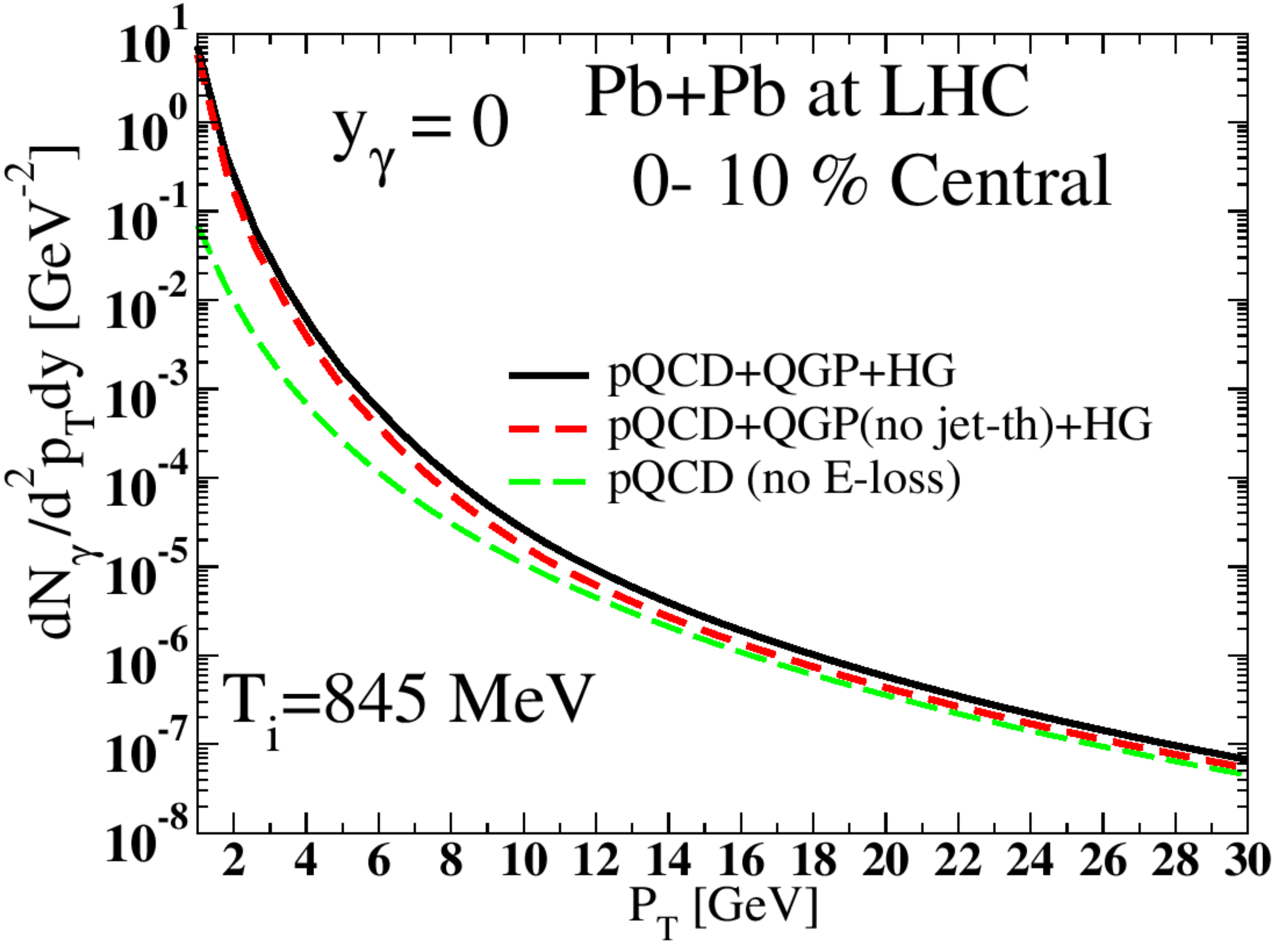
γ/γ Ratio – PHENIX



γ – Composition – LHC



γ − LHC prediction



Conclusions and Caveats

- Calculated and used the full leading order Hot QCD radiation rates for both gluons and photons.
- Important to use the full momentum distribution at any given time, not just dE/dx.
- Geometry and 1-D expansion included.
- Good description of existing data – pions and photons.
- For photons, jet-thermal interaction is crucial.
- LHC predictions – Should be better since pQCD should work better there.

- Calculations consistent in the $g \ll 1$ limit for momenta $T < p$. Yet for quantitative calculations, we needed $\alpha_s \approx 1/3$ or $g \approx 2$! So in reality, one must sum all diagrams, not just pinching part of the ladder diagrams!
  - At this leading order, α_s is an overall factor. So one might hope that the structure of the solution is OK.
  - Right now, this is best we can do with perturbative calculation.
- Elastic scatterings should be incorporated.

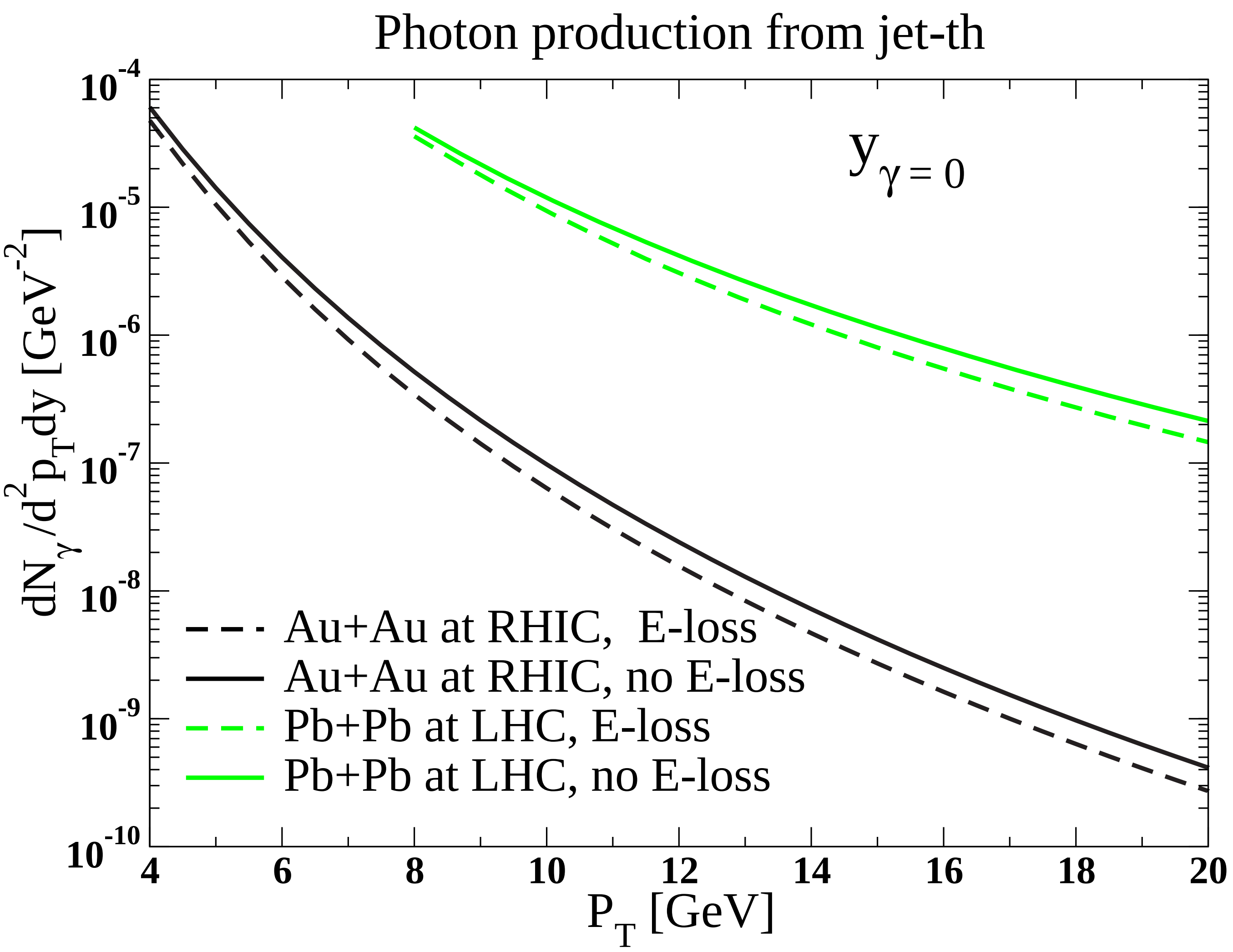
- Do better job with the medium evolution? – Need 3-D hydro code. Working on it.

- What about jet correlations? – Need to keep track of the evolution of the joint probability function of two jet energies. Much harder than single particle distributions! Will work on it.

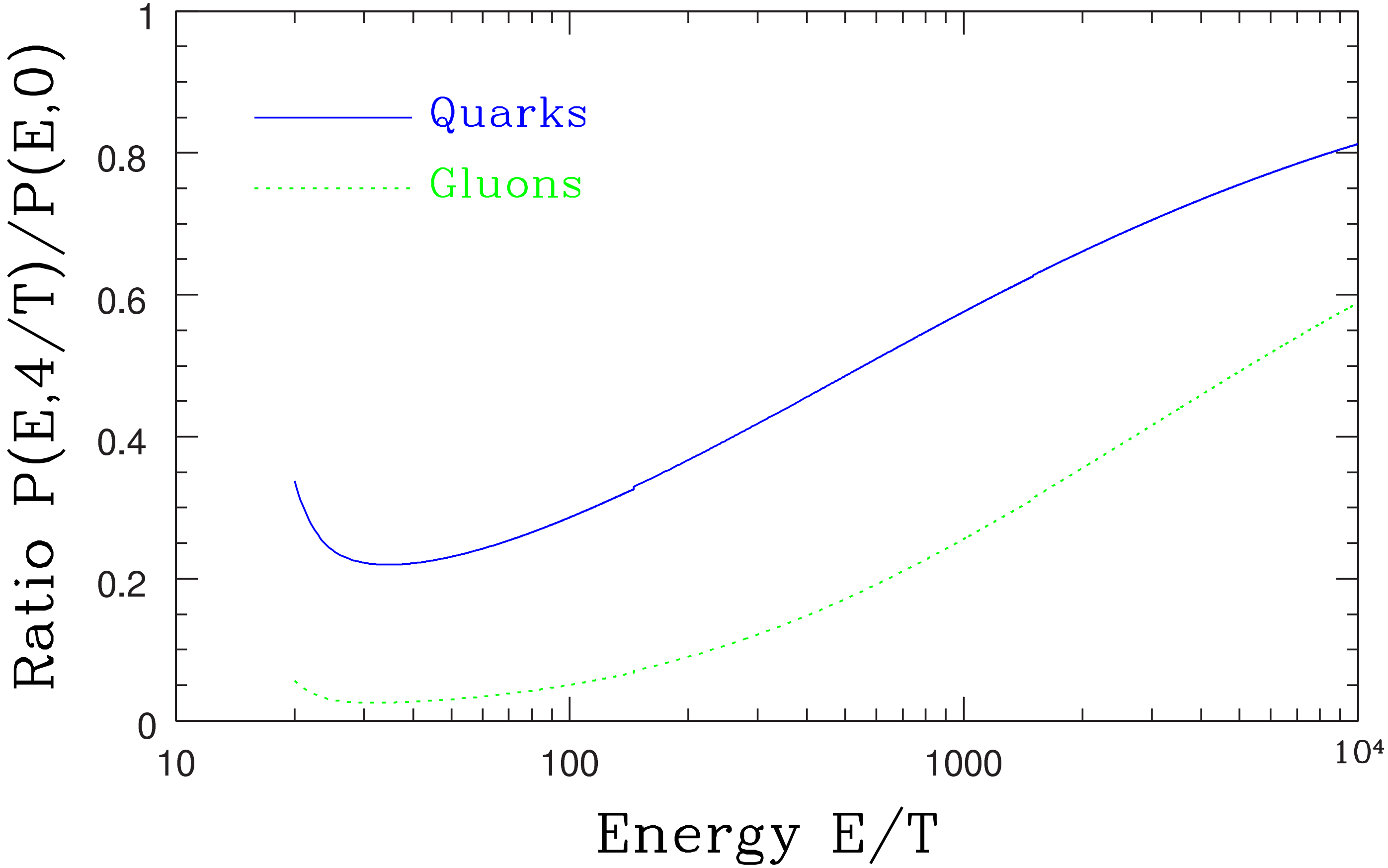
- Most energy-loss calculations these days do get R_{AA} right. Is there an experimental way to distinguish?
  – Photon bremsstrahlung + jet-photon conversion should be able to distinguish different scenarios. How to fish that out of all others is another matter.

Backup slides

γ – Effect of parton energy loss



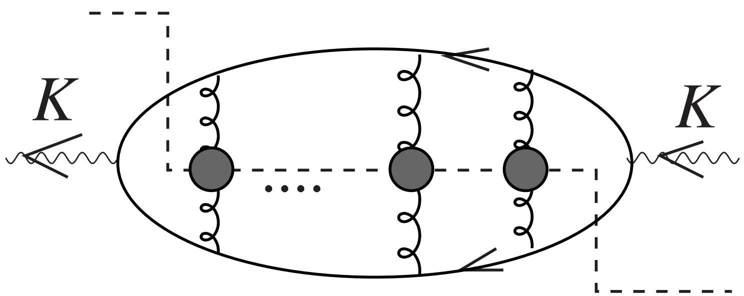
Parton distribution ratios

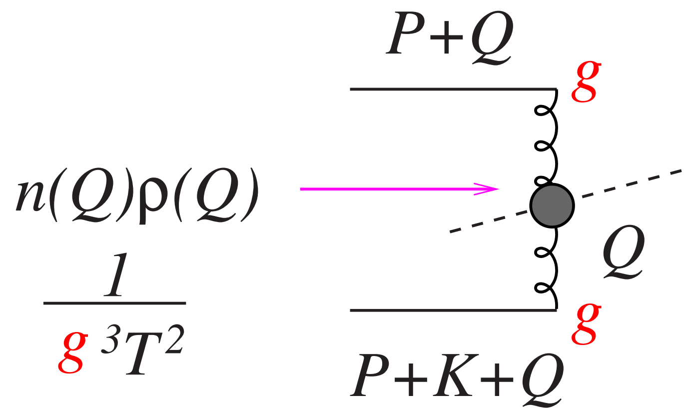


This is for illustration only.

Extreme LPM limit is reached, but only at very high energies.

Ladder Diagrams for γ





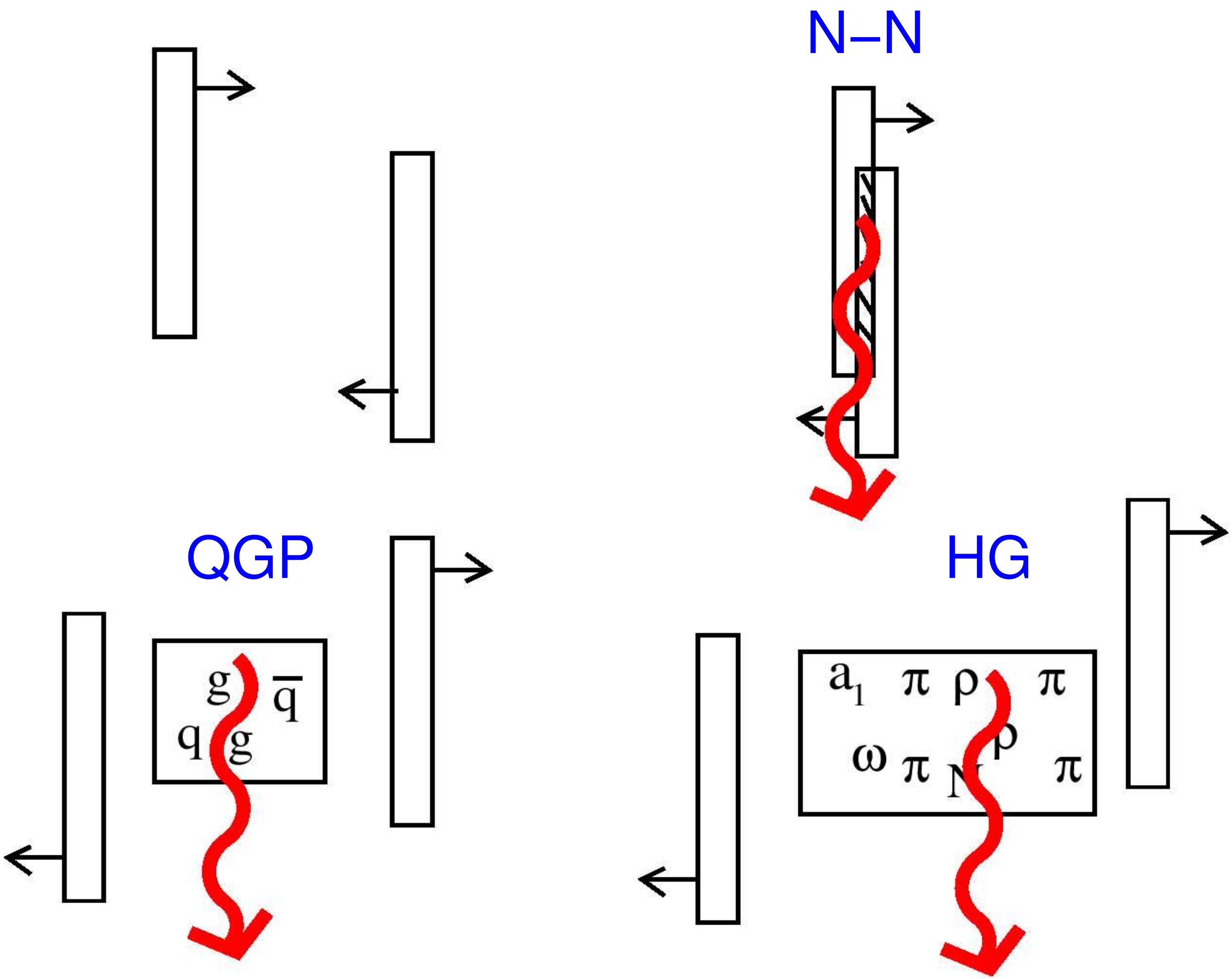
Pinching pole enhancement: $1/g^2$

Collinear phase space: $d^3Q \sim g^3T^3$

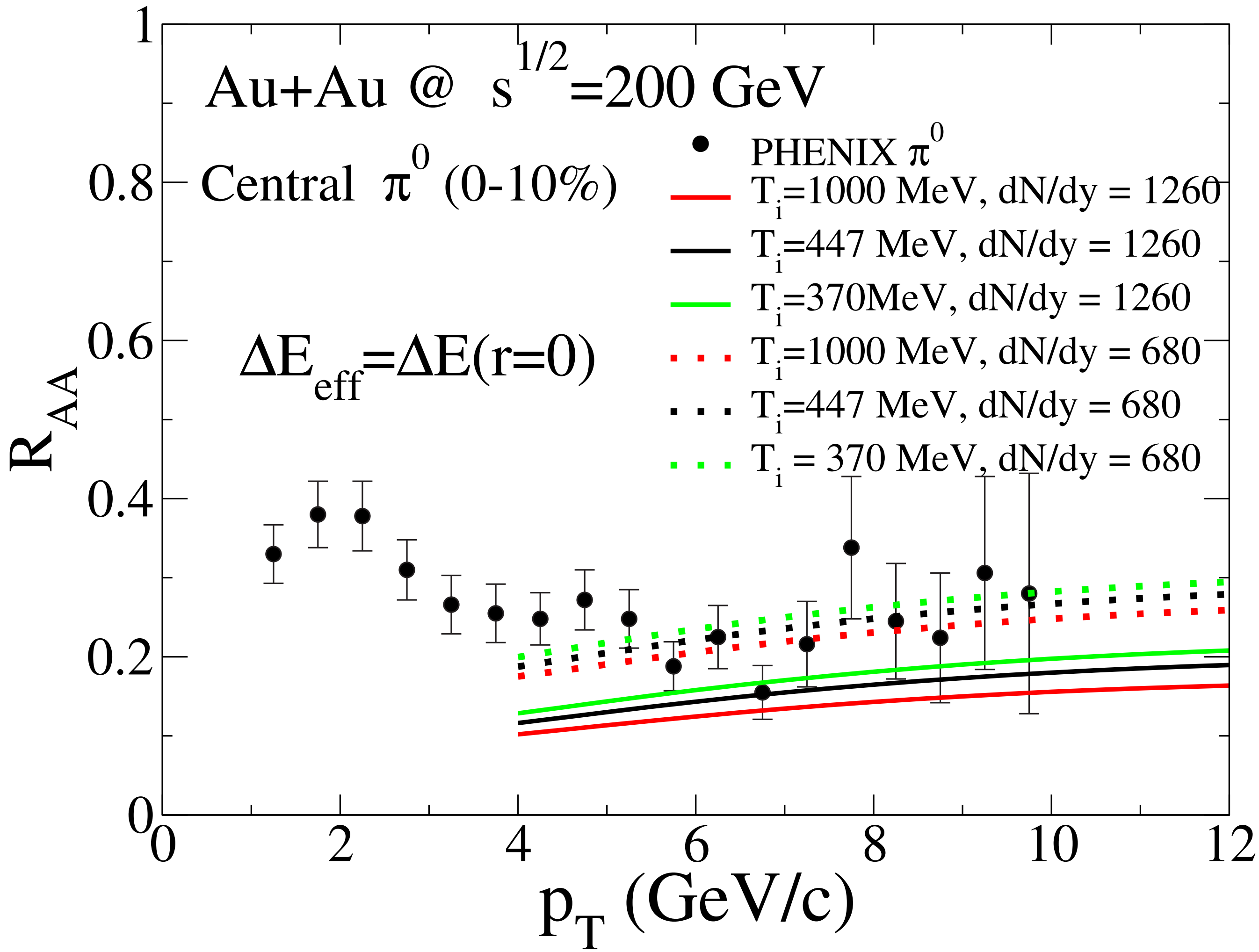
Total cost of adding one more rung: $O(1)$

Must resum.

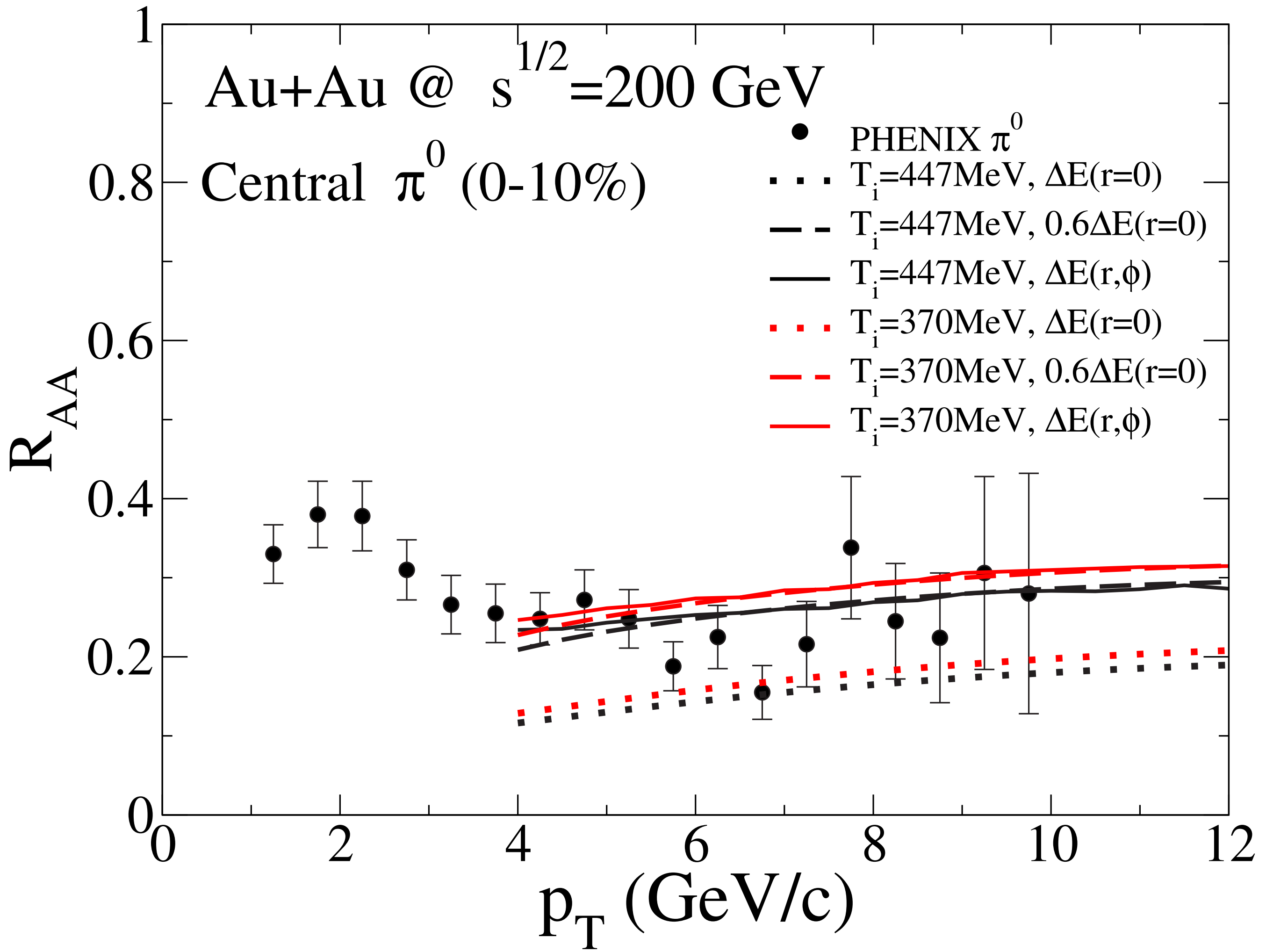
Graphically,



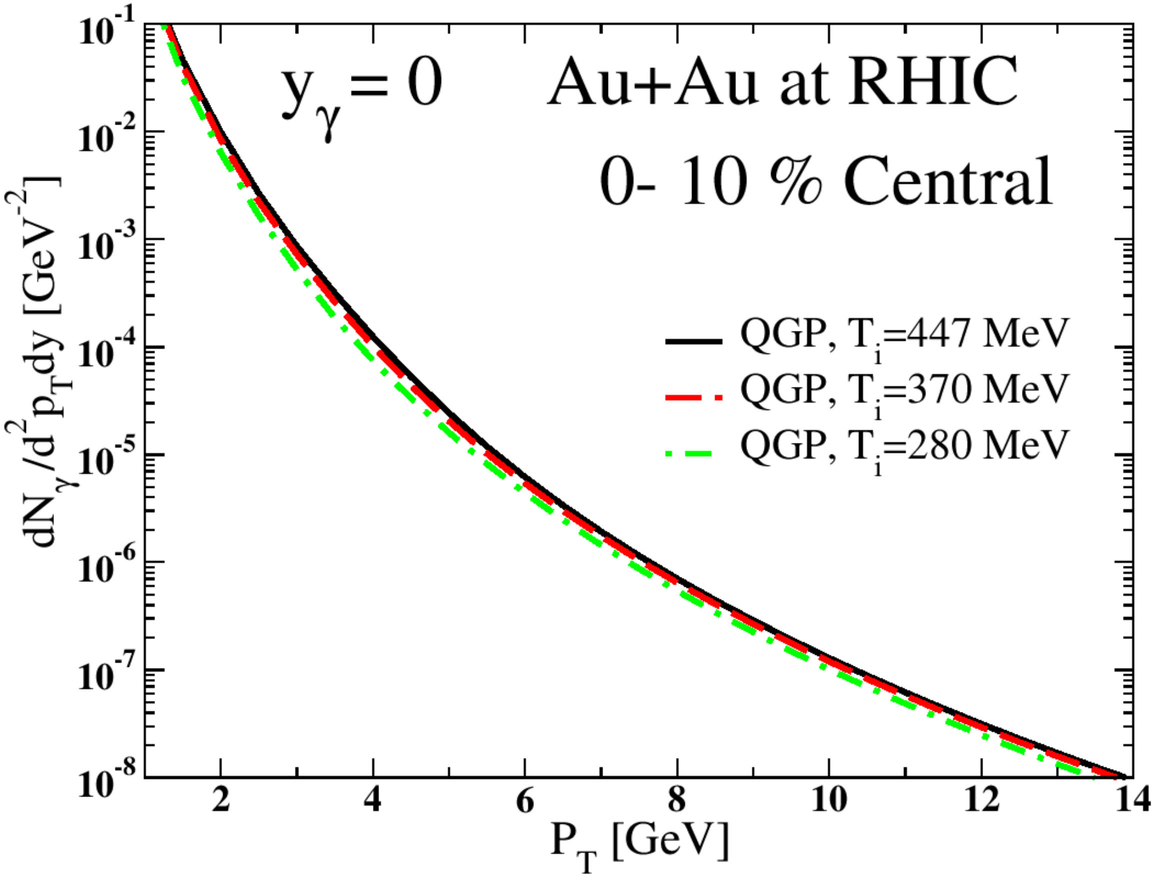
Temperature dependence 1



Temperature dependence 2



Temperature dependence 3



Production of γ during QGP phase.

How do we get there?

- Calculate local radiation/absorption rate within the leading order Hot QCD – Following Arnold, Moore and Yaffe (AMY).

- Use Boltzmann-like kinetic equation to evolve parton momentum distributions.

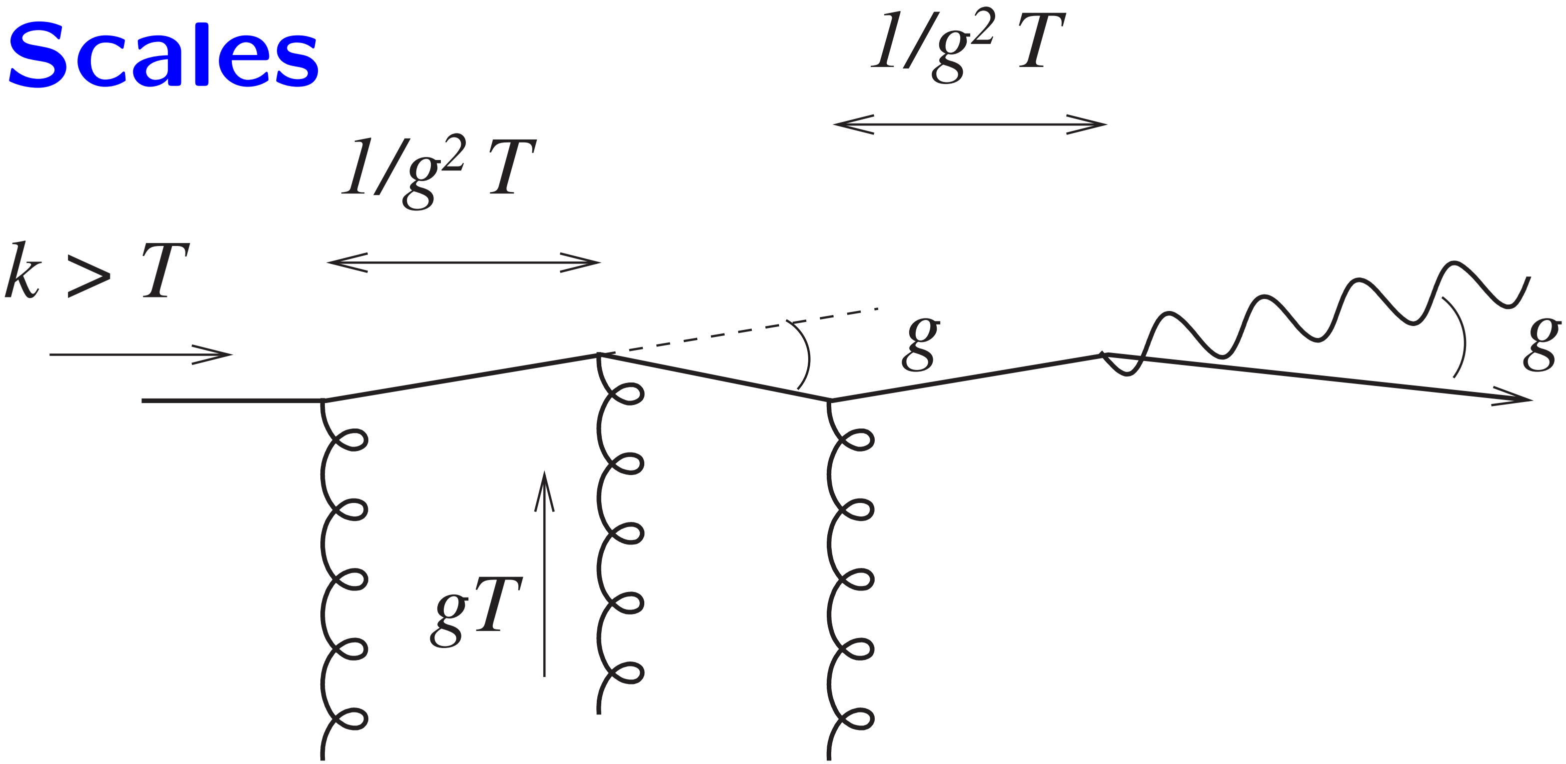
(Major) Differences with others

- Medium is dynamic. – Absorption, $q\bar{q}$ annihilation included.

- Rates are good for all $p_T > T$.

- We solve the time evolution equation.

Scales



Reason to resum:

1. Deflection angle is g. Hence the transverse speed is g.
2. gT kick makes the size of the parton $1/gT$.
3. It takes $(1/gT)/g = 1/g^2T$ to get separated.
4. But that's just the mean free path for the next gT kick!
5. Another way of saying LPM matters.

After much analysis, simplification: Final result

$$2\mathbf{p}_\perp = i\delta E\,\mathbf{f}(\mathbf{p}_\perp; p_{||}, \mathbf{k}) + g^2 C_R \int_Q 2\pi\delta(q^0 - q_{||}) \frac{m_D^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + m_D^2)} \times \Big[\mathbf{f}(\mathbf{p}_\perp; p_{||}, \mathbf{k}) - \mathbf{f}(\mathbf{p}_\perp - \mathbf{q}_\perp; p_{||}, \mathbf{k})\Big]$$

with $\delta E = k^0 + E_p - E_{p+k}$

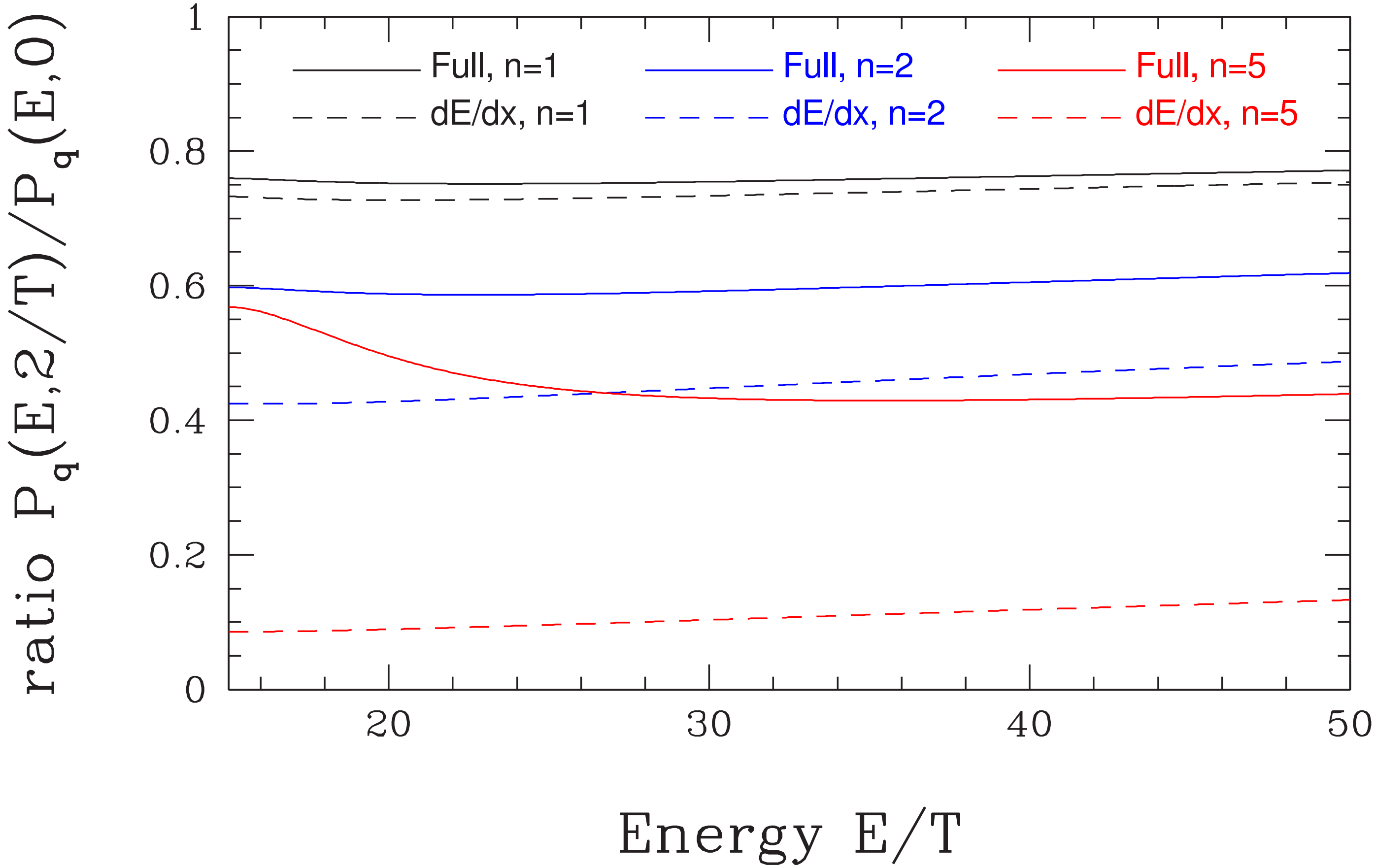
Photon Radiation Rate

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{d_F q^2 \alpha_{EM}}{4\pi^2 k} \int_{-\infty}^{\infty} \frac{dp_{||}}{2\pi} \int \frac{d\mathbf{p}_\perp}{(2\pi)^2} \left| \mathcal{J}_{p_{||} \leftarrow p_{||}+k} \right|^2 \times \mathrm{Re} \left\{ 2\mathbf{p}_\perp \cdot \mathbf{f}(\mathbf{p}_\perp; p_{||}, \mathbf{k}) \theta(p_{||}) \right\}$$

with

$$\left| \mathcal{J}_{p_{||} \leftarrow p_{||}+k} \right|^2 = \frac{n_f(k+p_{||})[1 + n_b(p_{||})]}{2[p_{||}(p_{||}+k)]^2} \left[p_{||}^2 + (p_{||}+k)^2 \right]$$

Parton distribution ratios



This is for illustration only. Using $P(p,0) = 1/(p^2 + p_0^2)^n$.

Lesson: Using just dE/dx is dangerous!

Pion Production

$$\frac{dN_{AA}}{dyd^2\mathbf{p}_T} = \frac{\langle N_{\mathsf{coll}}\rangle}{\sigma_{in}} \sum_{a,b,c,d} \int dx_a dx_b\, g_A(x_a,Q) g_A(x_b,Q) \times K_{\mathsf{jet}} \frac{d\sigma_{a+b\to c+d}}{dt} \frac{\tilde{D}_{\pi^0/c}(z,Q)}{\pi z}$$

with

$$\tilde{D}_{\pi^0/c}(z,Q) = \int d^2r_\perp \mathcal{P}(\mathbf{r}_\perp) \tilde{D}_{\pi^0/c}(z,Q,\mathbf{r}_\perp,\mathbf{n})$$

and

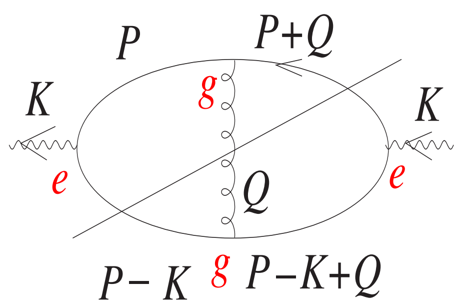
$$\tilde{D}_{\pi^0/c}(z,Q,\mathbf{r}_\perp,\mathbf{n}) = \int dp_f \frac{z'}{z} \Big(P_{qq/c}(p_f;p_i;\Delta t)\, D_{\pi^0/c}(z',Q) + P_{g/c}(p_f;p_i;\Delta t)\, D_{\pi^0/c}(z',Q) \Big)$$

with $z = p_T/p_i$ and $z' = p_T/p_f$.

Δt determined by the location of the production $\mathbf{r}$ and the direction of the jet $\mathbf{n}$.

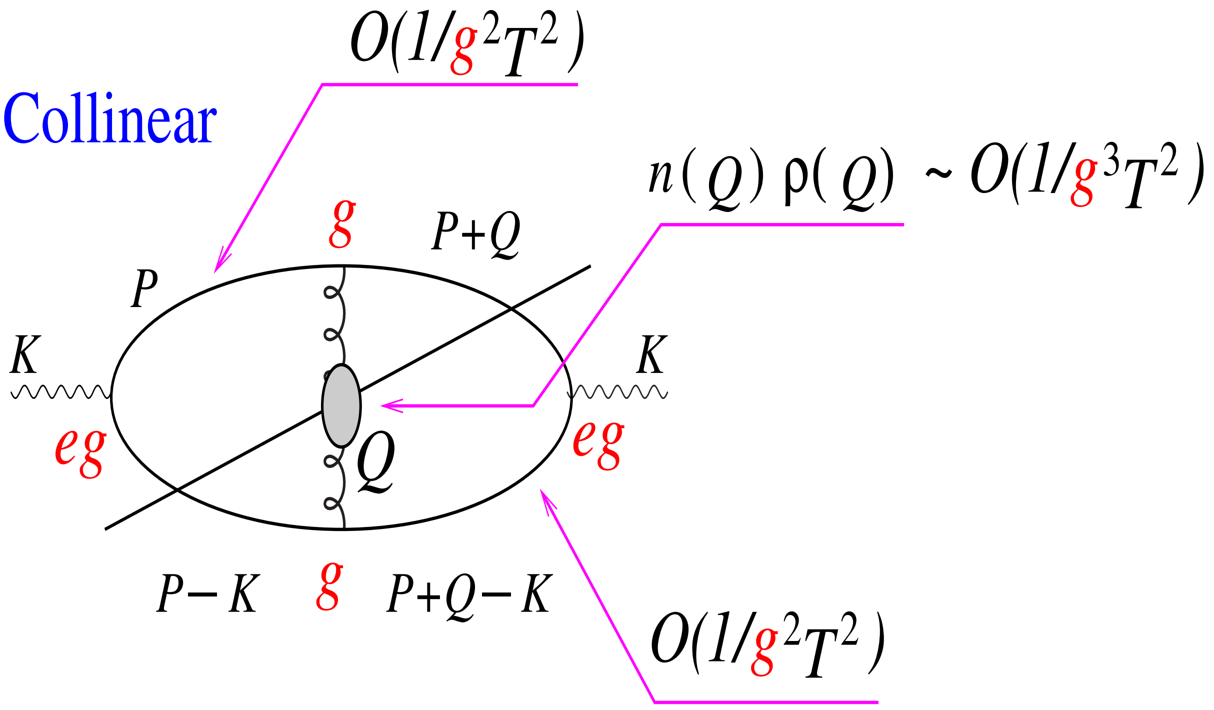
Two loop Diagram for γ

Non−collinear



Total count: $O(e^2 g^2)$

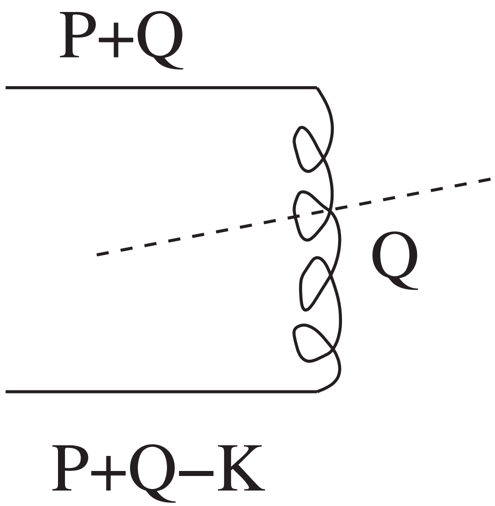
Collinear



Collinear phase space $d^2P_\perp \approx O(g^2T^2)$

Total count: $O(e^2 g^2)$ $d^3Q \approx O(g^3T^3)$

Adding a rung



Phase space integral:

$$\int d^4Q\,\delta((P+Q)^2-m^2)\frac{1}{(P+Q-K)^2-m^2} \propto \int \frac{d^3Q}{E_{P+Q}}\frac{1}{K^\mu(P+Q)_\mu} \tag{1}$$

We have

$$K^\mu(P+Q)_\mu = |K|(E_{P+Q}-(P+Q)_L) = |K|\left(\frac{m^2+(P+Q)_\perp^2}{(P+Q)_L}\right) \tag{2}$$

As long as $P_\perp = O(gT)$ and $Q = O(gT)$, this is $O(g^2T^2)$ regardless whether

$P \sim T$ or $P \gg T$ since $|K|/P_L = O(1)$.

$$\int \frac{d^3Q}{E_{P+Q}} \frac{1}{K^\mu (P+Q)_\mu} = O(gT/K) \tag{3}$$

The vertices are now g^2P^2 and the cut rung is $1/g^3T^2$ as usual. Hence, altogether,

$$\text{one rung} = O(P^2/KT) \tag{4}$$

Are we being sensible?

- For the rate, we correctly handle:
  - Thermal dispersion corrections for all species in the plasma
  - Real moving plasma particles to scatter from, rather than idealized static scatterers
  - LPM effect–transition between BH and strong LPM handled in smooth and correct way. NOTE that the LPM effect is often numerically not that large, and treating it as large can cause large errors
  - Stimulated emission and absorption processes are included
- However the rate is determined in a fixed-temperature, extensive medium.
- Then we use the rate in a Fokker-Planck equation.

Sensible?

- Our approach: Valid for $l_{\mathrm{coh}} \ll T/(dT/dx)$
- For gluon radiation, $k \ll p$ has smaller coherence time.
- Which is more important? Multiple soft radiation (AMY does that correctly) or rare hard radiation (AMY is incomplete for this)?
  - For jet energy loss, multiple soft radiation is more important because the initial spectrum is steeply falling.