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Factorization of Heavy Quarkonium Production

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Phys. Lett. B (2005), Phys. Rev. D (2005),
and in preparation

Outline

- ☐ **Heavy quarkonium**
- ☐ **Existing production models and their successes**
- ☐ **Surprises – experimentally**
- ☐ **Surprises – theoretically**
- ☐ **Factorization of heavy quarkonium production**
- ☐ **Lessons from pQCD factorization of light hadrons**
- ☐ **Summary and Outlook**

Good probes for a dense medium

- ❑ **Basic requirements:**
  - ❖ **Cleanly measurable experimentally**
  - ❖ **Reliably calculable theoretically**

- ❑ **Necessary conditions:**
  - ❖ **Sensitive to the scales and properties of strong interacting matter – low momentum scale (a few hundred MeV)**
  - ❖ **Large momentum transfer to ensure pQCD calculation**

    ⟹ **a hard probe sensitive to low momentum physics**

- ❑ **Potentially good probes:**
  - ❖ **Have two observed scales (one hard and one soft)**
  - ❖ **Have one observed hard scale and a steeply falling distribution**

Quarkonium could be a good probe

- ☐ **It has two intrinsic scales:**
  - ❖ **Heavy quark mass:**

Heavy quark pairs are produced at a distance scale much less than fm

$$\Delta r \sim \frac{1}{2m_Q} \leq 0.1\text{ fm} \quad \text{(for a charm-quark pair)}$$

$$\leq 0.025\text{ fm} \quad \text{(for a b-quark pair)}$$

PQCD is expected to work for the production of heavy quarks

- ❖ **Quarkonium binding energy:**

$$\frac{\left|M^2 - 4m_Q^2\right|}{4m_Q^2} \ll 1$$

for both charm and bottom quarkonia

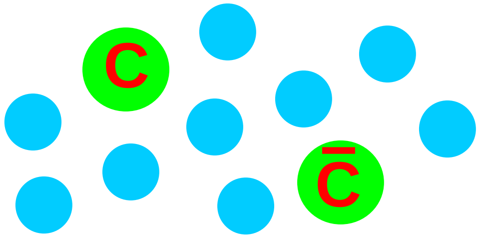
The transition from a heavy quark pair to a quarkonium should be sensitive to the soft physics

J/ψ Suppression in QGP

- **Heavy quarkonium provides a non-relativistic system, potentially, very similar to a QED bound state**

Charm: $\frac{v^2}{c^2} \sim \frac{k_Q^2}{m_Q^2} \sim \frac{\left|M^2 - 4m_c^2\right|}{4m_c^2} \sim 0.3$ **Bottom:** $\frac{v^2}{c^2} \sim 0.1$

- **Color screening in QGP suppresses the formation of J/ψ**



Potential: $V_{Q\bar{Q}}(r) \Rightarrow V_{Q\bar{Q}}(r,T)$

Wave function: $\Phi_{Q\bar{Q}}(r) \Rightarrow \Phi_{Q\bar{Q}}(r,T)$

J/ψ formation rate $\propto \left|\Phi_{Q\bar{Q}}(r,T)\right|^2$

J/ψ suppression ⇔ medium properties

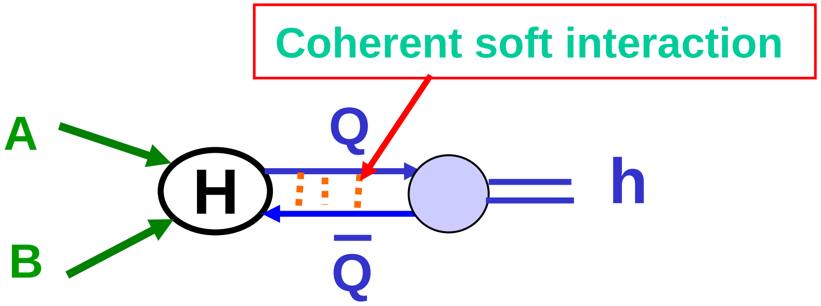
Matsui & Satz (1986)

- **Calibration:**

  Do we understand the production mechanism of J/ψ well enough to extract the information on QGP?

The basic production mechanism

❑ Quarkonium production:



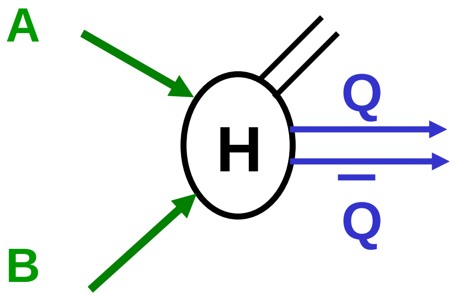
$$\sigma_{AB\to h} = \sum_{states} \int d\Gamma_{Q\bar{Q}} \frac{d\sigma_{AB\to states(Q\bar{Q})}}{d\Gamma_{Q\bar{Q}}} F_{states(Q\bar{Q})\to h}\left(p_Q, p_{\bar{Q}}, p_h\right)$$

Production of a heavy quark pair, and followed by the formation of a quarkonium via coherent soft interactions between the pair

❑ Production models:

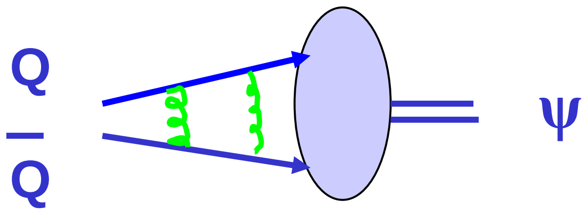
Different models ⬄ Different assumptions/treatments to the non-perturbative transition from the *QQ* pair to a quarkonium

Color singlet model



- color singlet pair
- right quantum numbers for the quarkonium
- same wave function for production and decay

$$\sigma_{AB\to\psi} \propto \sigma_{AB\to(Q\bar{Q})}\left|R_\psi\left(0\right)\right|^2$$

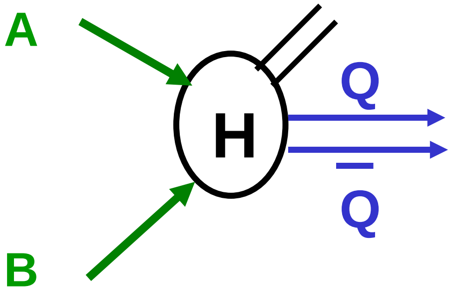


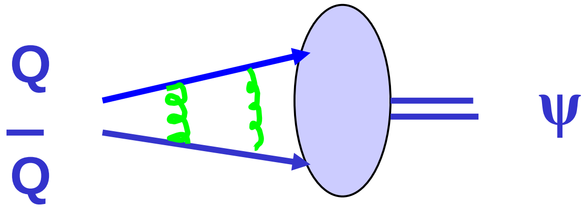
Einhorn and Ellis (1975), …

- absolutely normalized predictions
- predictions on polarization
- quantum interference between production and formation suppressed

Works well for J/ψ production in photo-production and others
But, one order of magnitude too small for CDF data, …

Color Evaporation Model





Fritsch (1978); Halzen; …

- all pairs with invariant mass less than open flavor threshold
- color and spin average

- a single constant for non-perturbative formation
- one constant for one quarkonium state

$$\sigma_{AB\to\psi} = f_{\psi} \int dm^2_{Q\bar{Q}} \frac{d\sigma_{AB\to(Q\bar{Q})}}{dm^2_{Q\bar{Q}}}$$

Works well for total cross sections, not perfect for distributions,
Predicts zero polarization for quarkonium production

Non-relativistic QCD (NRQCD) model

Bodwin, Braaten, Lapage (1985); …

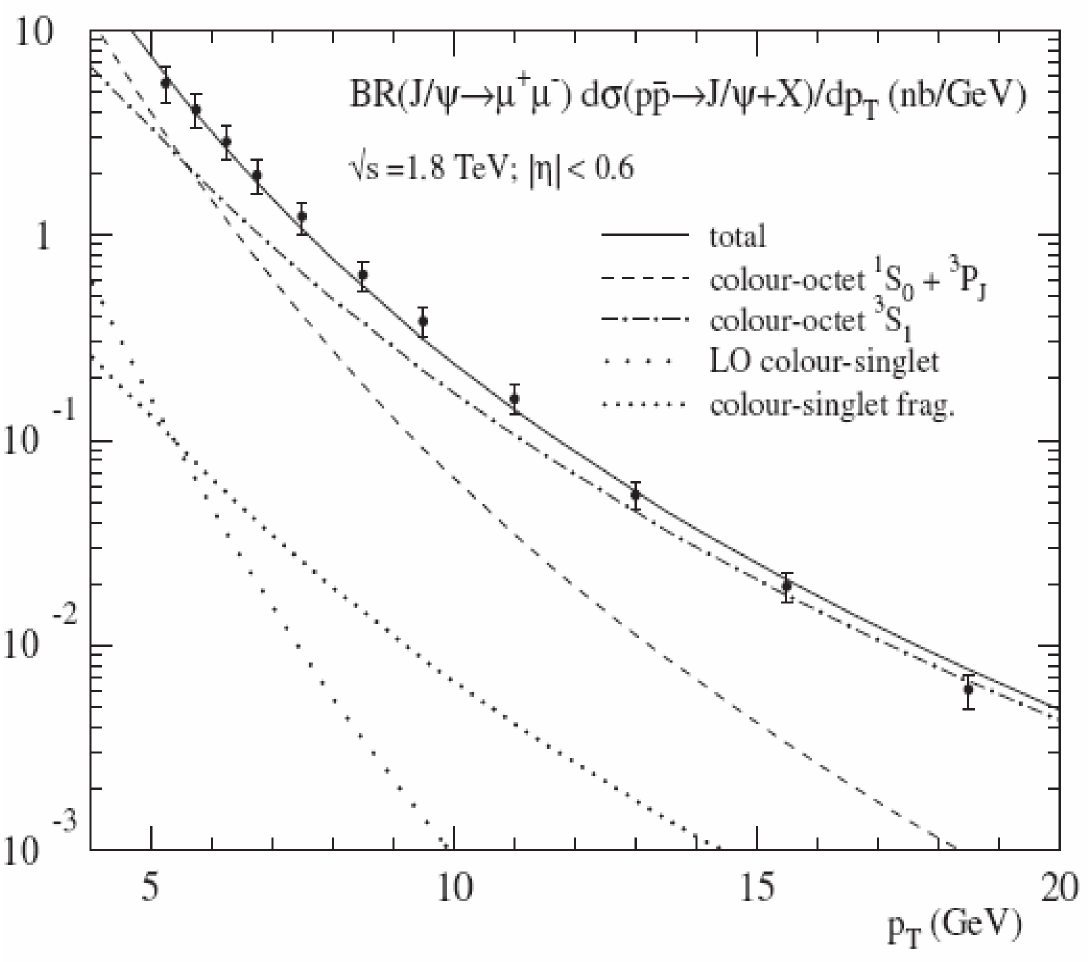
- ☐ All colored and uncolored pre-J/ψ heavy quark states can become color singlet J/ψ mesons
- ☐ Production rate of J/ψ is factorized

$$\sigma_{AB\to J/\psi}\left(M_{J/\psi}\right) \approx \sum_{[O]} \sigma_{AB\to[O]}\left(m_{c\bar{c}}^2 = M_{J/\psi}\right)\left\langle O_{J/\psi}(0)\right\rangle$$

- ☐ Quantum states [*O*] separated by spin and color
- ☐ Transition probability is proportional to non-perturbative, but, universal local matrix elements
- ☐ Approximations/assumptions:
  - ❖ velocity expansion – $\left\langle p_Q - p_{\bar{Q}} \right\rangle \ll 2m_Q$
  - ❖ factorization – not proved

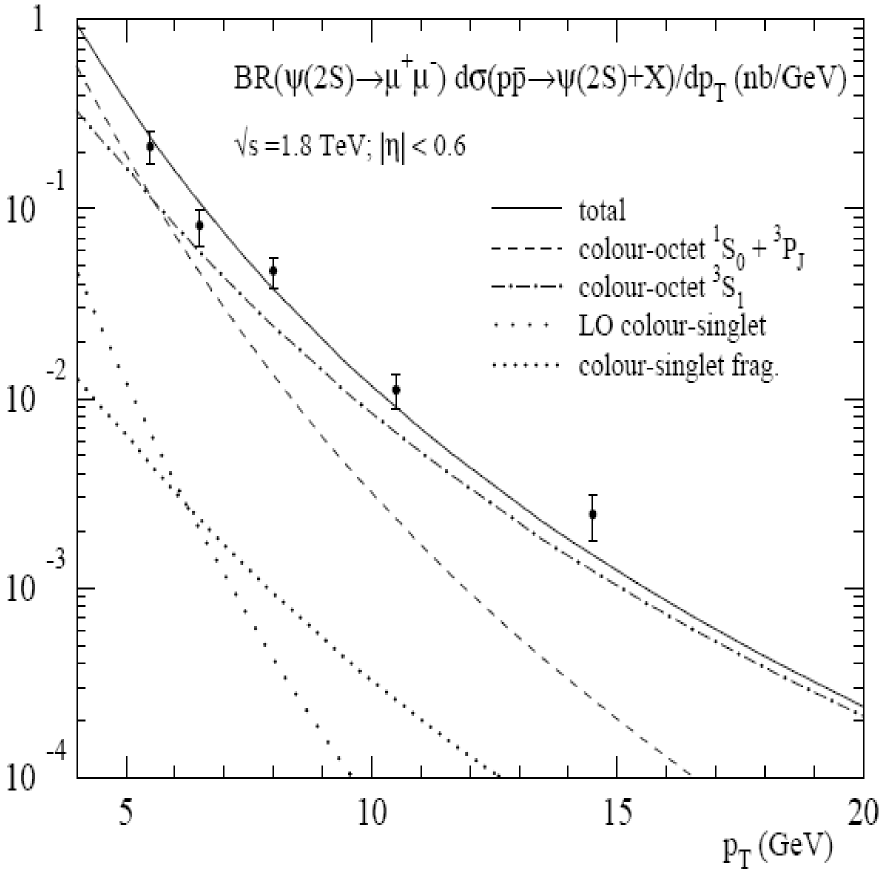
Successes of the production models

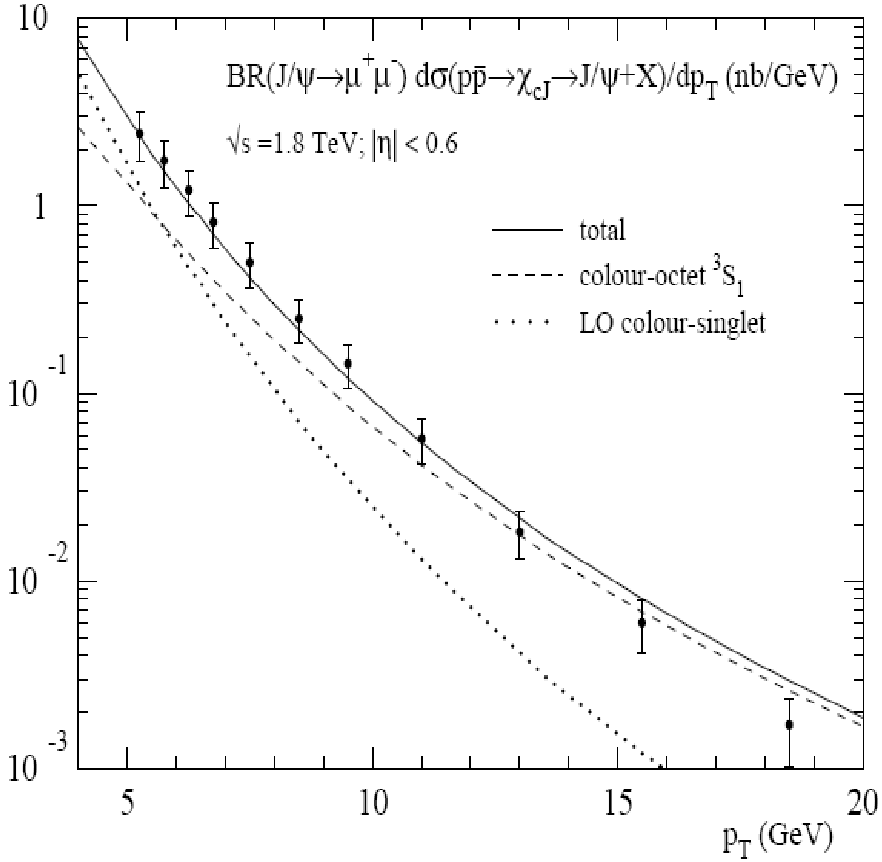
❑ Tevatron data for J/ψ production



- ❖ Data is not consistent with color singlet model
- ❖ Data is consistent with NRQCD, with matrix elements fixed by the data
- ❖ CEM predicts a similar p_T distribution

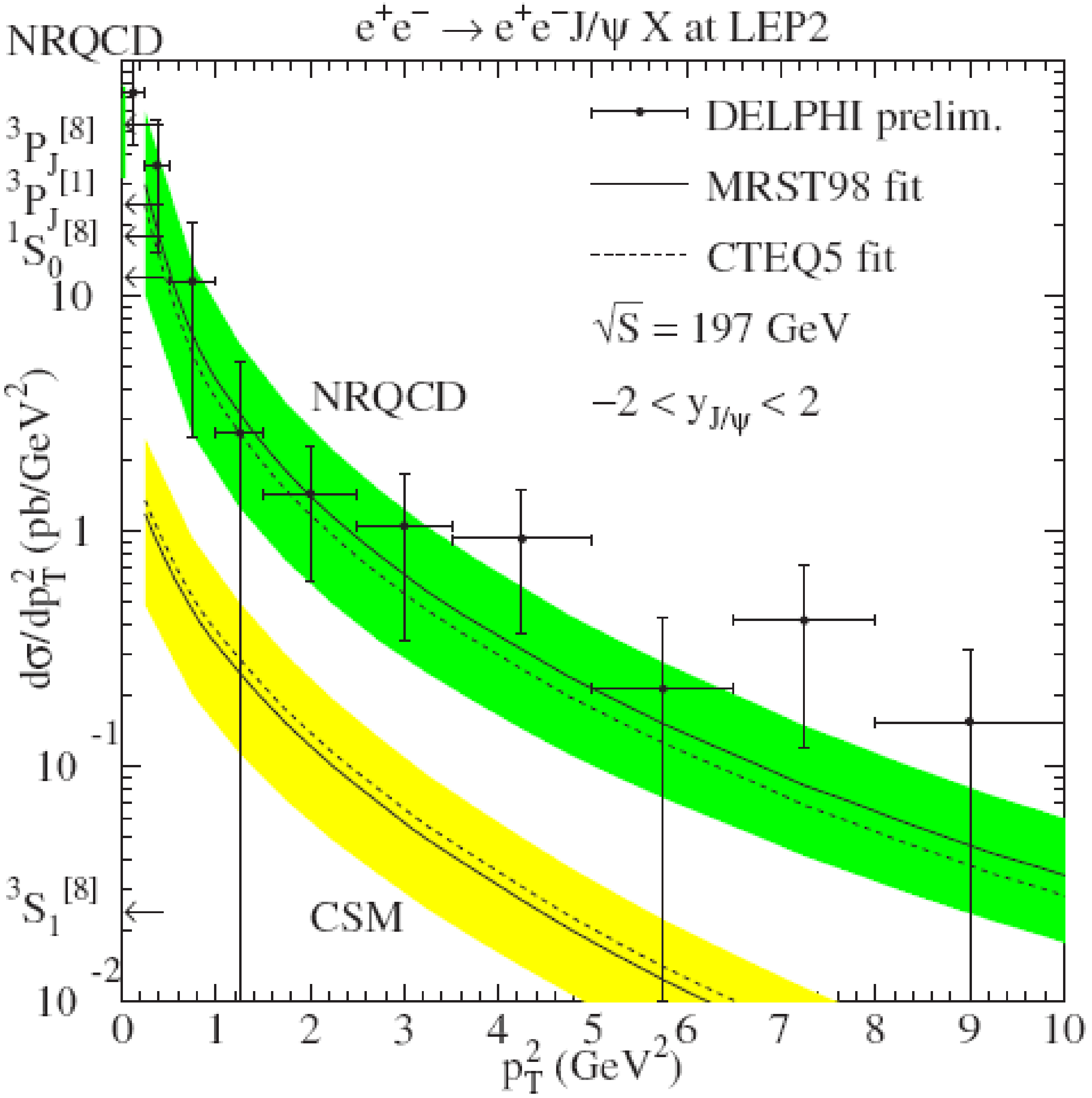
❑ Other states as well:





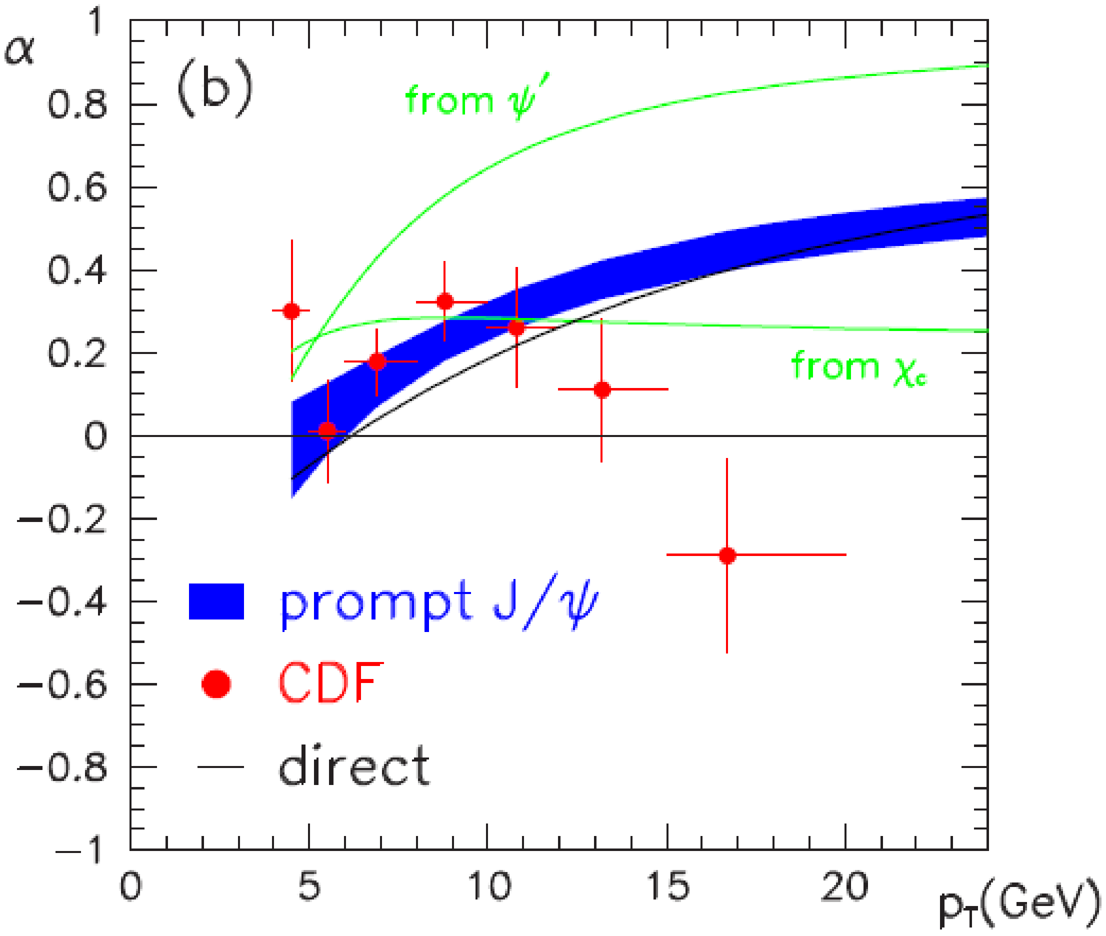
E. Braaten et al. Annu. Rev. Nucl. Part. Sci. 46, 197 (1996)

❑ LEP data on J/ψ photo-production: $\gamma\gamma \to J/\psi + X$

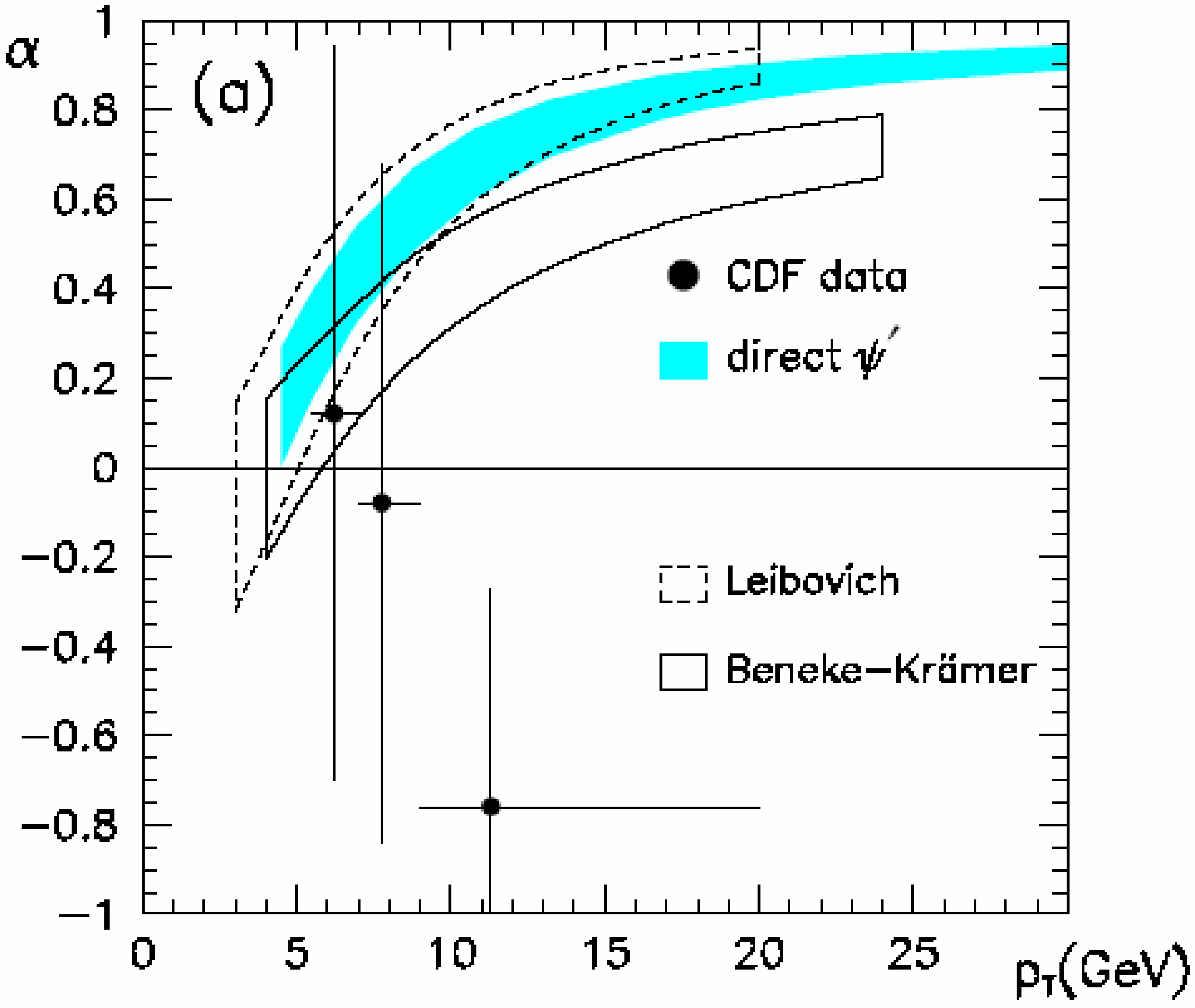


QWG's report

Surprises – experimentally

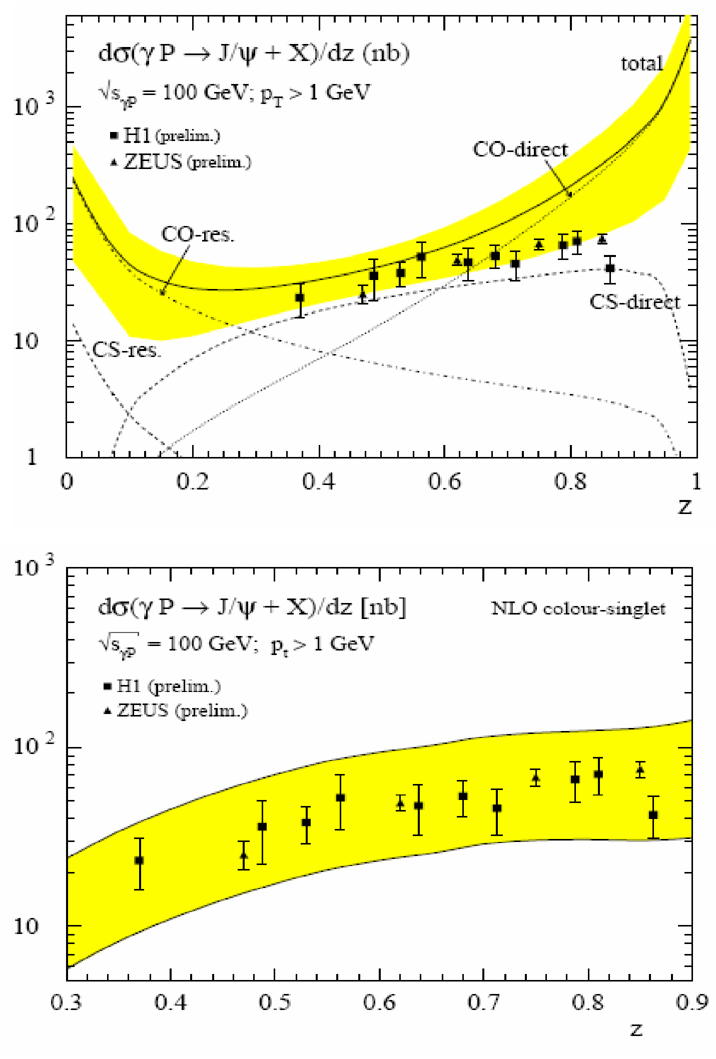


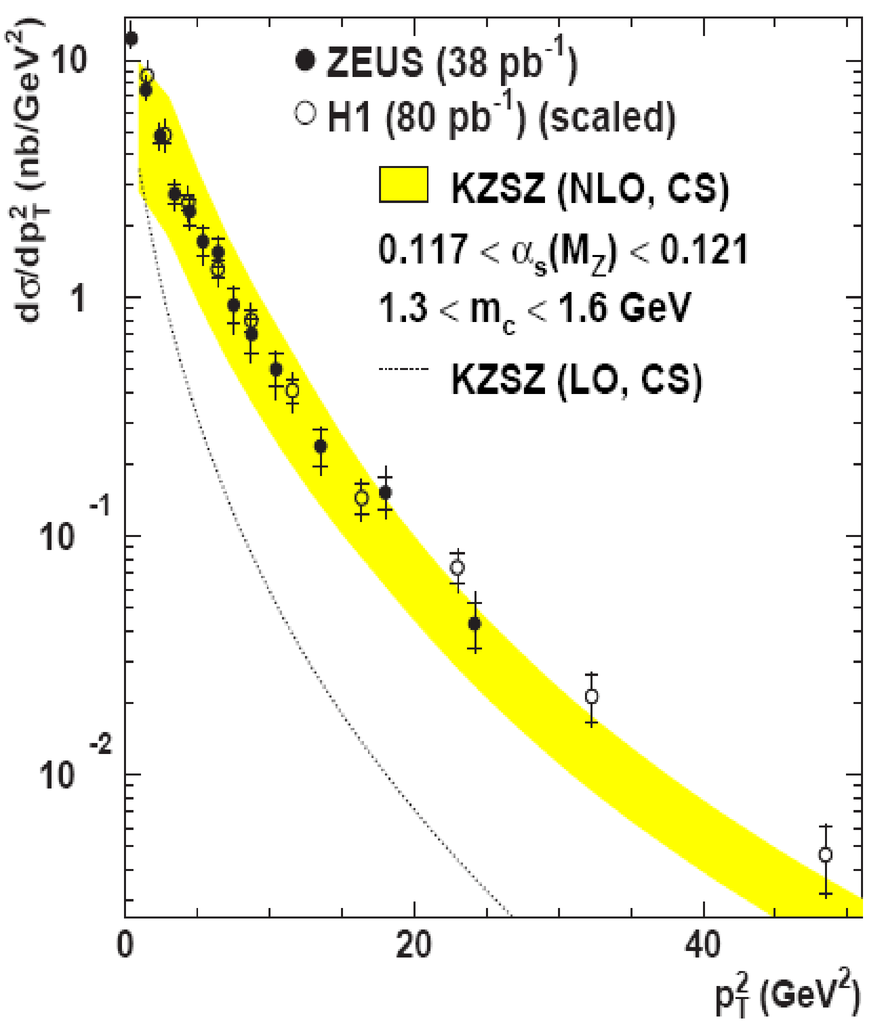
- ψ' polarization as a function of p_T:



E. Braaten et al. Phys. Rev. D62, 094005 (2000)

Inelastic Quarkonium Photoproduction at HERA





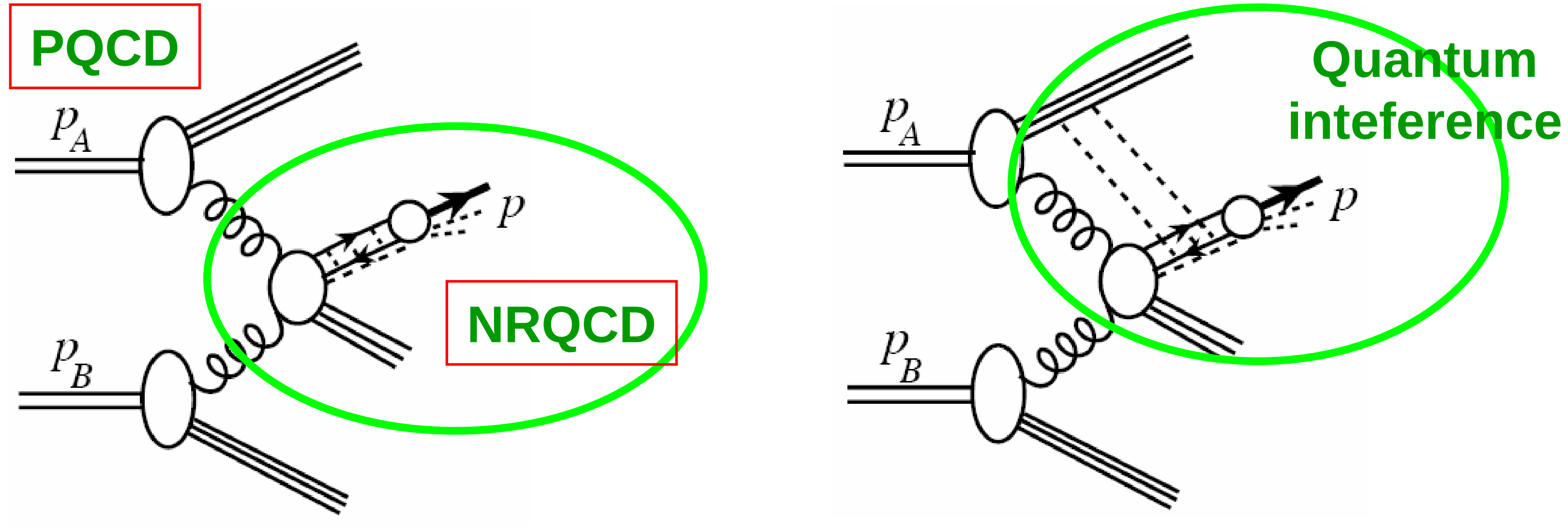
No room for color-octet contribution – universality?

QWG's report

Surprises – theoretically

- ❑ None of the factorized production models, including NRQCD model, were proved theoretically

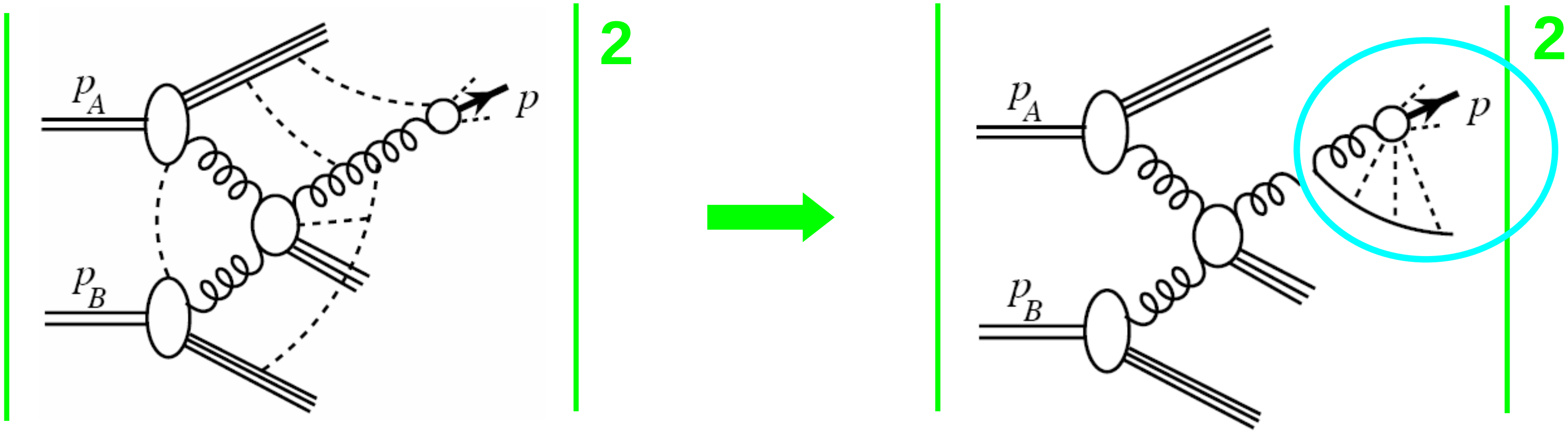
- ❑ Factorization of NRQCD model clearly fails for low p_T



- ❑ Factorization of NRQCD model might work for large p_T

Spectator interactions are suppressed by $(1/p_T)^n$

Factorization of single hadron inclusive at large p_T



$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z,\mu) \otimes D_{H/i}(z,m_c,\mu) + \mathcal{O}(m_H^2/p_T^2)$$

Prove NRQCD Factorization,

$$d\sigma_{A+B\to H+X}(p_T) = \sum_n d\hat{\sigma}_{A+B\to c\bar{c}[n]+X}(p_T)\,\langle \mathcal{O}_n^H \rangle$$

To prove:

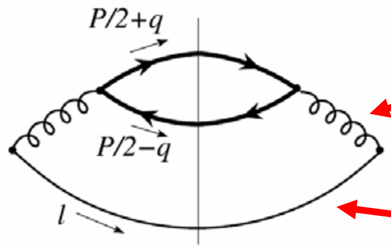
$$\boxed{D_{H/i}(z,m_c,\mu) = \sum_n d_{i\to c\bar{c}[n]}(z,\mu,m_c)\,\langle \mathcal{O}_n^H \rangle}$$

Divergences in $\langle \mathcal{O}_n^H \rangle$ cancels divergences in $D_{H/i}(z,m_c,\mu)$

Breakdown of NRQCD factorization at v^2 at NNLO:

Nayak, Qiu, Sterman, 2005

LO:

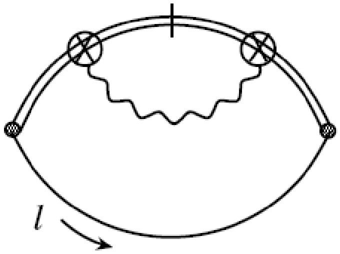


Short-distance

Eikonal line

$n=[Q\bar{Q}]_{\text{singlet}}$

NLO:

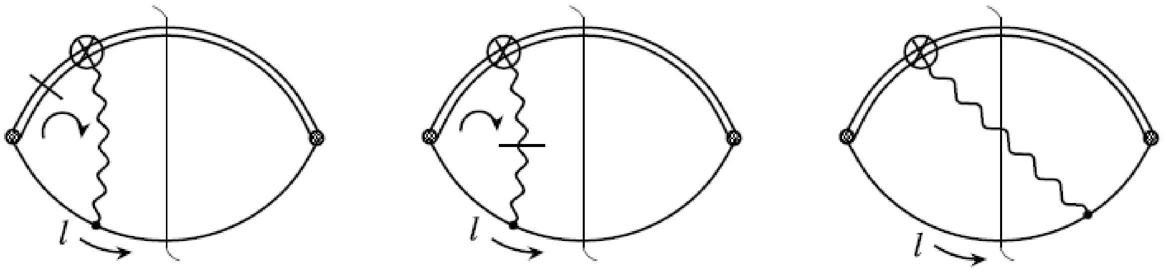


$$= \frac{16}{3}\,\frac{\alpha_s}{\pi}\,\frac{\vec{q}^{\,2}}{P^2}\,\frac{1}{-\varepsilon} + \dots$$

Topologially-factorized

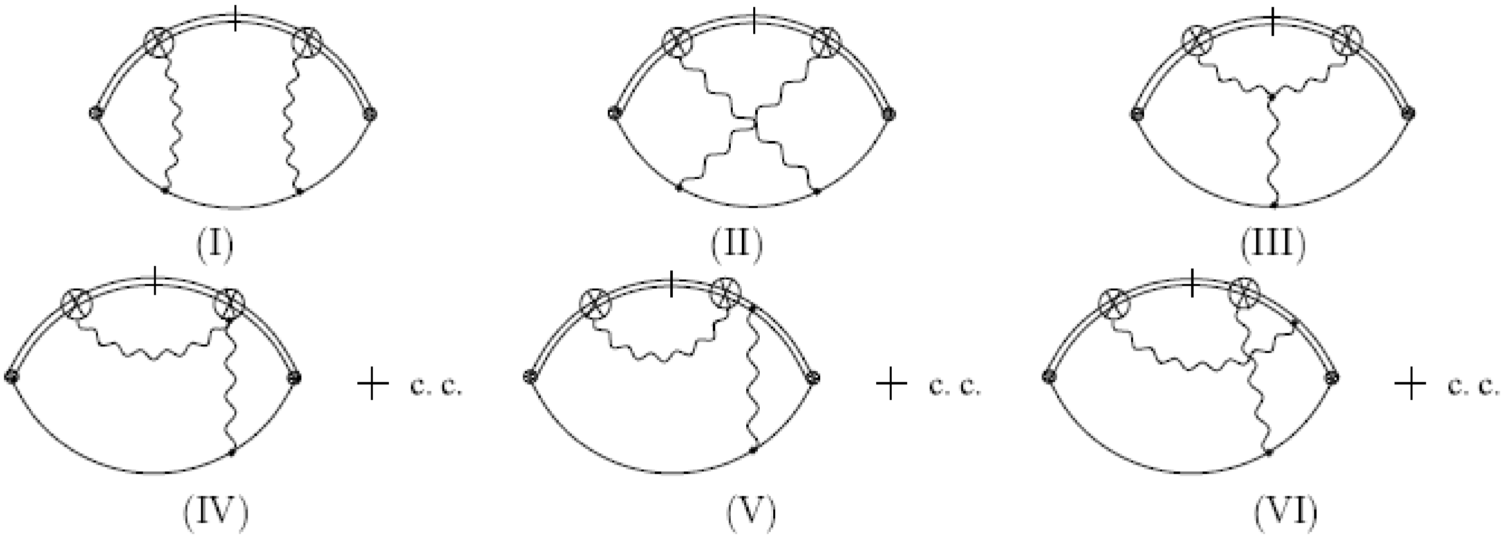
→ the matrix element $\langle \mathcal{O}_n^H \rangle$

Color neutralization is IR divergent – nonperturbative!

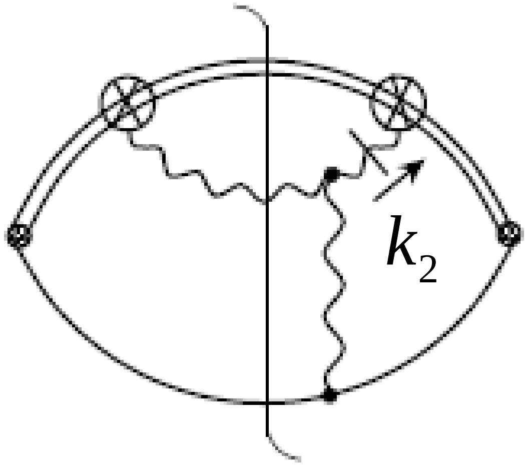


IR divergences cancel between real and virtual diagrams

NNLO:



All IR divergences cancel between real and virtual diagrams, except



$$2\,\mathrm{Re}\,III\,A^{(k_2^0\ \mathrm{pole})}(q) \;=\; -\,\alpha_s^2\,\frac{1}{3\varepsilon}\,\vec{q}^{\,2}$$

$$=\; -\,\alpha_s^2\,\frac{1}{3\varepsilon}\,\frac{\boxed{\vec{v}^2}}{4}$$

This non-topological IR divergence Cannot be absorbed into the matrix elements $\langle \mathcal{O}_n^H \rangle$

Breakdown of NRQCD factorization

$$D_{H/i}(z, m_c, \mu) = \sum_n d_{i\to c\bar{c}[n]}(z, \mu, m_c)\, \langle \mathcal{O}_n^H \rangle$$

- This non-topological IR divergence has to appear in the coefficient function, $d_{i\to c\bar{c}[n]}(z, \mu, m_c)$

→ **Breakdown of NRQCD factorization at v^2 and at NNLO in α_s**

- The way out?

Try to modify the matrix elements so that they can absorb all IR divergences of the fragmentation function

Redefinition or Extension for NRQCD?

- ☐ **NRQCD matrix elements of heavy quarkonium production are not gauge invariant**

$$\mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_n \psi(0) \left(a_H^\dagger a_H\right) \psi^\dagger \mathcal{K}_n' \chi(0)$$

$\mathcal{K}_n,\ \mathcal{K}_n'$: **Product of color, spin, and covariant derivatives**

Operator-valued gauge transformations do not always commute with $a_H^\dagger a_H$

- ☐ **Add gauge links to the operator:**

$$\mathcal{O}_n^H(0) \longrightarrow \chi^\dagger \mathcal{K}_{n,c} \psi(0)\, \Phi_l^\dagger[0,A]_{cb} \left(a_H^\dagger a_H\right) \Phi_l[0,A]_{ba}\, \chi^\dagger \mathcal{K}_{n,a}' \psi(0)$$

The IR poles from the gauge links cancel the same IR poles in the fragmentation functions leave the coefficient functions IR safe at NNLO

$$D_{H/i}(z, m_c, \mu) = \sum_n d_{i\to c\bar{c}[n]}(z, \mu, m_c)\, \langle \mathcal{O}_n^H \rangle$$

Factorization beyond v^2 and NNLO

- ❑ The IR pole $(1/\varepsilon)v^2$ is independent of the direction of the gauge link (or Wilson line)
  - ⇔ the modified, gauge invariant matrix elements are universal at least to NNLO in α_s and v^2
- ❑ Velocity expansion is not good for charmonium
  - ❖ Large phase space available for gluon radiation:

$$Q^2 - 4M_C^2 \Rightarrow 4M_D^2 - 4M_C^2 \approx 6\ \mathrm{GeV}^2$$

  - ❖ Large possible velocity in production:

$$v_{\mathrm{prod}} \sim \frac{|k_c|}{M_c} \sim \sqrt{\frac{4M_D^2 - 4M_c^2}{4M_c^2}} \sim 0.88$$

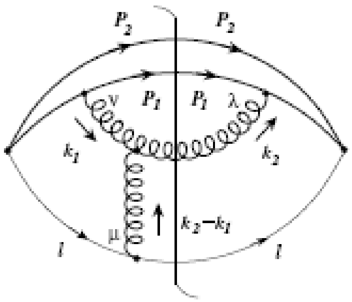
  - ❖ Very different from decay:

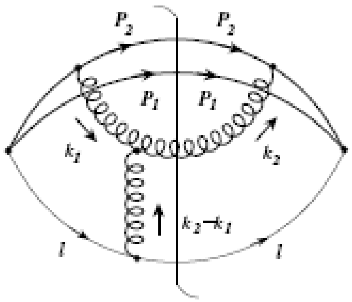
$$v_{\mathrm{decay}} \sim \sqrt{\frac{4M_{J/\psi}^2 - 4M_c^2}{4M_c^2}} \sim 0.48$$

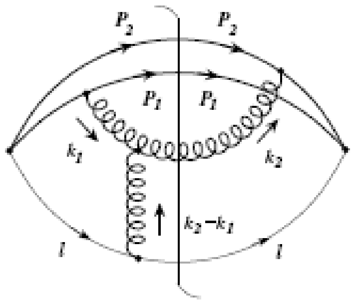
Factorization at NNLO and all order in v^2

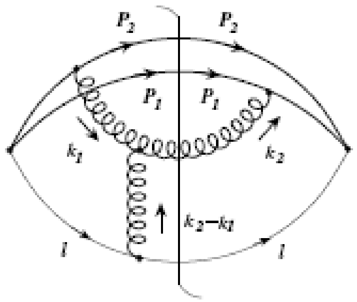
❑ New result:

The IR poles at all orders of v-expansion at NNLO are independent of the direction of the eikonal line









❑ Significance:

Factorization for producing a heavy quark pair with a finite invariant mass, at least at NNLO in α_s

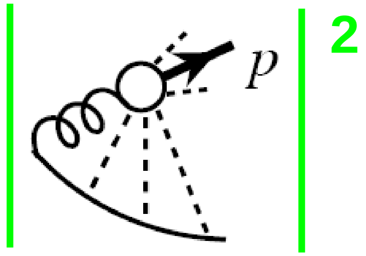
$$D_{H/i}(z,m_c)=\sum_n d_{n/i}(z,m_c,Q)\otimes\left\langle O_n^H(Q)\right\rangle$$

Recall: Collinear factorization vs. k_T factorization

Lessons from pQCD factorization

- ❑ **Predictive power of pQCD factorization:**
  - ❖ **Infrared Safety of the short-distance part**
  - ❖ **Universality of the parton distribution/fragmentation**
- ❑ **Fragmentation function represents a probability for a virtual parton to evolve into an observed hadron plus everything**

$$D_{g\rightarrow \mathrm{J}/\psi}(z,m_c) \propto$$



Not equal to a square of hadron wave function

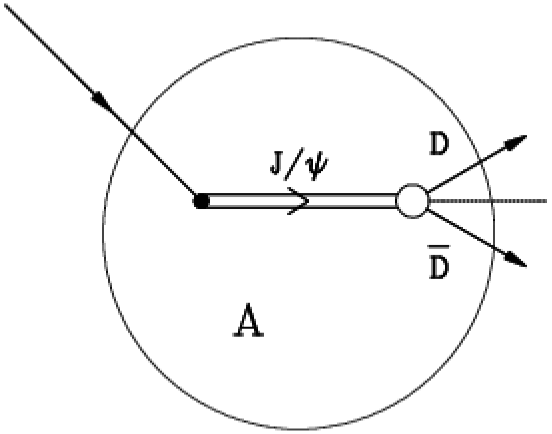
- ❑ **Formation of a heavy quarkonium from a pair of virtual heavy quarks – connection to the vacuum wave function? help of heavy quark mass?**

J/ψ suppression

- J/ψ is unlikely to be formed at the same time when the heavy quark pair was produced

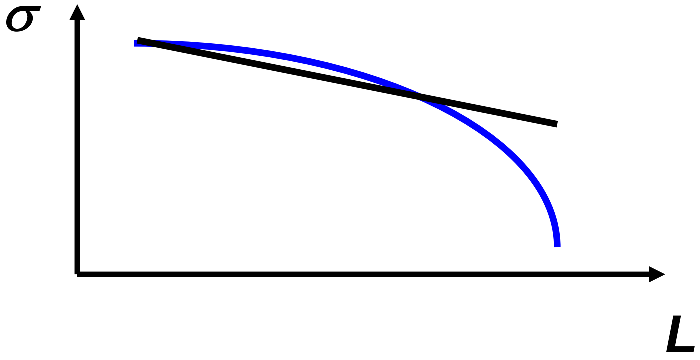
- If J/ψ were produced at the collision point:

  Expect Glauber model to work with a constant J/ψ – nucleon absorption



- If J/ψ were formed much later:

  Effective cross section for breaking the coherence of a heavy quark pair may not be a constant



Qiu,vary,zhang (2003), Accardi,Kang,Qiu

Summary and outlook

- ☐ Heavy quarkonium provides a "non-relativistic" system, and could offer some important perspectives to the formation of QCD bound states
- ☐ Heavy quarkonium has two intrinsic scales, and could be a good probe of QGP or other dense medium
- ☐ But, after 30 years, since the discovery of J/ψ, we still have not been able to fully understand the production mechanism of heavy quarkonia
- ☐ None of the factorized production models, including NRQCD model, were proved theoretically
- ☐ RHIC is offering an excellent opportunity to learn and exam the formation of QCD bound states – nuclear matter could be an effective filter to distinguish the production models.

Backup slices