Some comments on angular momentum sum rules and off-forward PDFs

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based on work done in collaboration with

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Introduction

- Draw attention to and comment on some issues

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- Raise some queries

Matrix elements of J

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The classic result of Jaffe and Manohar is incorrect.

The J-M result for the expectation value is:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} =$$
$$\frac{1}{4mp_0}\left[(3p_0^2 - m^2)\boldsymbol{s}_i - \frac{3p_0 + m}{p_0 + m}(\boldsymbol{p}.\boldsymbol{s})\boldsymbol{p}_i\right]$$

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We find, for any spin configuration (longitudinal, transverse etc) the remarkably simple result (suppressing a delta-function term):

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle = \frac{1}{2}\boldsymbol{s}_i$$

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This difference is critical for the purpose of deriving angular momentum sum rules, because these are derived for a fast moving nucleon i.e. for $p_0 \to \infty$.

For transverse spin i.e. for $\boldsymbol{s}$ perpendicular to $\boldsymbol{p}$ the J-M result gives:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{4mp_0} \left[(3p_0^2 - m^2) \boldsymbol{s}_i \right]$$

which $\to \infty$ as $p_0 \to \infty$, so no sum rule is possible.

Ji defines $J_{q,g}$ by

$$J_{q,g}\, 2\boldsymbol{S} = \langle p, S | \hat{\boldsymbol{J}}_{q,g} | p, S \rangle$$

which is incorrect, but I don't think it is ever used.

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But if one uses the approach via the energy momentum tensor $T^{\mu\nu}$ then to begin with the expectation value depends on some of the scalar functions appearing in the matrix element of $T^{\mu\nu}$. I'll use Ji's notation for these.

$$
\begin{aligned}
\langle \psi_{p,\boldsymbol{s}} | M^{ij} | \psi_{p,\boldsymbol{s}} \rangle &= \frac{A}{2M(p_0 + M)} [p^i (\boldsymbol{p} \times \boldsymbol{s})^j - p^j (\boldsymbol{p} \times \boldsymbol{s})^i] \\
&+ \frac{1/2(A+B)}{M} \epsilon^{ij\alpha\beta} S_\alpha p_\beta
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In terms of 2nd moment of off-diagonal PDFs:

$$A_q(\Delta^2 = 0) = H_q^{(2)}(\Delta^2 = 0) \qquad B_q(\Delta^2 = 0) = E_q^{(2)}(\Delta^2 = 0)$$

Something interesting!

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which agrees with Ji, but.....

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Of course J_x^{proton} is fine, since $\sum_q B_q = -B_g$.

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$$\therefore \qquad \left(\frac{\sum_q J_q}{J_g}\right) = \left(\frac{\sum_q p_z^q}{p_z^g}\right)$$

Consistent with known result that

A NEW APPROACH

- It is simple.

- It is short

- It works for any spin. Previous methods only work for spin 1/2.

We know how rotations affect states. If $|\boldsymbol{p}, m\rangle$ is a state with momentum $\boldsymbol{p}$ and spin projection m in the rest frame of the particle, and if $\hat{R}_i(\beta)$ is the operator for a rotation β about the axis i, then

$$\hat{R}_i(\beta)|\boldsymbol{p}, m\rangle = |\boldsymbol{R}_i(\beta)\boldsymbol{p}, m'\rangle \, D^s_{m'm}[R_i(\beta)]$$

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From the above we know what the matrix element of $\widehat{R}_i(\beta)$ looks like. So we simply differentiate, multiply by i, and put $\beta = 0$.

Thus we have

$$\langle \boldsymbol{p}', m' | \boldsymbol{J}_i | \boldsymbol{p}, m \rangle = i \frac{\partial}{\partial \beta} \langle \boldsymbol{p}', m' | R_i(\beta) | \boldsymbol{p}, m \rangle |_{\beta=0}$$

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$$= \quad i \frac{\partial}{\partial \beta} \left[\langle \boldsymbol{p}', m' | R_i(\beta) \boldsymbol{p}, n \rangle \, D^s_{nm}[R_i(\beta)] \right]_{\beta=0}$$

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One technical point: you have to know that the derivative of the rotation matrix for spin s at $\beta = 0$ is just the spin matrix for that spin. (more correctly: the matrix generator of rotations for that spin) e.g. for spin 1/2 just $\sigma_i/2$.

COMPARISON OF RESULTS

For the expectation values we find, for any spin configuration (longitudinal, transverse etc) the remarkably simple result (suppressing a delta-function term):

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle = \frac{1}{2} \boldsymbol{s}_i$$

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Recall that the JM result for longitudinal spin was precisely:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{2} \boldsymbol{s}_i$$

in complete agreement with our result.

But for transverse polarization JM had:

$$\langle\langle \boldsymbol{p}, \boldsymbol{s} | \boldsymbol{J}_i | \boldsymbol{p}, \boldsymbol{s} \rangle\rangle_{JM} = \frac{1}{4mp_0} \left[(3p_0^2 - m^2) \boldsymbol{s}_i \right]$$

which disagrees with our result and which, as we said, would imply no possibility of a transverse sum rule.

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which disagrees with our result and which, as we said, would imply no possibility of a transverse sum rule.

With our correct result there is no fundamental distinction between the transverse and longitudinal cases.

SUM RULES

Expand nucleon state as superposition of n-parton Fock states.

$$|\boldsymbol{p}, m\rangle \simeq \sum_n \sum_{\{\sigma\}} \int d^3\boldsymbol{k}_1 \ldots d^3\boldsymbol{k}_n$$
$$\psi_{\boldsymbol{p},m}(\boldsymbol{k}_1, \sigma_1, \ldots \boldsymbol{k}_n, \sigma_n)$$
$$\delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 \ldots - \boldsymbol{k}_n)|\boldsymbol{k}_1, \sigma_1, \ldots \boldsymbol{k}_n, \sigma_n\rangle.$$

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$$\delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 \ldots - \boldsymbol{k}_n)|\boldsymbol{k}_1, \sigma_1, \ldots \boldsymbol{k}_n, \sigma_n\rangle.$$

There are two independent cases:

(a)Longitudinal polarization i.e. $\boldsymbol{s}$ along OZ. The sum rule for $\boldsymbol{J}_z$ yields the well known result

$$1/2 = 1/2\,\Delta\Sigma + \Delta G + \langle L_z^q \rangle + \langle L_z^G \rangle$$

(b) Transverse polarization i.e. $\boldsymbol{s} \perp \boldsymbol{p}$. The sum rule for $\boldsymbol{J}_x$ or $\boldsymbol{J}_y$ yields a a new sum rule

$$1/2 = 1/2 \sum_{q,\bar{q}} \int dx\, \Delta_T q(x) + \sum_{q,\bar{q},G} \langle L_{\boldsymbol{s}_T} \rangle$$

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Here $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

The structure functions $\Delta_T q(x) \equiv h_1^q(x)$ are known as the quark transversity or transverse spin distributions in the nucleon.

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As mentioned no such parton model sum rule is possible with the J-M formula because, as $p \to \infty$, for $i = x, y$ the matrix elements diverge.

A comment about the calculation and meaning of the orbital angular momentum.

For each parton labelled r there is an orbital term involving

$$i(\boldsymbol{k_r} \times \boldsymbol{\nabla}_{\boldsymbol{k_r}})_i \delta^3(\boldsymbol{p} - \boldsymbol{k_1} - \boldsymbol{k_2} .... - \boldsymbol{k_r} .... - \boldsymbol{k_n})$$

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Beautifully consistently can show that this leads to two terms. The first

$$i(\boldsymbol{p} \times \boldsymbol{\nabla}_{\boldsymbol{p}})_i \delta^3(\boldsymbol{p'} - \boldsymbol{p})$$

i.e the angular momentum of the nucleon as a whole about the origin. This cancels the same term on the LHS of the sum rule.

The second is

$$\delta^{(3)}(\boldsymbol{p'} - \boldsymbol{p})\langle L_i \rangle_{m'm} \tag{1}$$

where $\langle L_i \rangle_{m'm}$ is the contribution from the internal angular momentum arising from the partons

$$\begin{aligned}\langle L_i \rangle_{m'm} \;=\; & \sum_n \sum_r \int d^3k_1 \ldots\ldots d^3k_n \psi^*_{\boldsymbol{p},m'}(\boldsymbol{k}_1, ...\boldsymbol{k}_n,) \\ & \{[-i(\boldsymbol{k}_r \times \boldsymbol{\nabla}_{k_r})_i]\psi_{\boldsymbol{p},m}(\boldsymbol{k}_1, ...\boldsymbol{k}_r, ...\boldsymbol{k}_n)\} \\ & \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 - ... - \boldsymbol{k}_n)\end{aligned}$$

The second is

$$\delta^{(3)}(\boldsymbol{p'} - \boldsymbol{p})\langle L_i \rangle_{m'm} \tag{2}$$

where $\langle L_i \rangle_{m'm}$ is the contribution from the internal angular momentum arising from the partons

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This is the usual QM form for angular momentum!

It is absolutely crucial to note that the sum rule involves a SUM of Quark and Antiquark densities.

Not realizing this has led to some misunderstandings.

What some people call the TENSOR CHARGE of the NUCLEON is the difference between quark and antiquark contributions.

Thus the transverse spin sum rule, although it involves the transverse spin or transversity quark and antiquark densities, does NOT involve the nucleon's transversity. The Tensor Charge operator is NOT related to the angular momentum.

Consistent with known result that

both $\left(\frac{\sum_q p_z^q}{p_z^g}\right)$ and $\left(\frac{\sum_q J_q}{J_g}\right)$ $\overset{Q^2\to\infty}{\Longrightarrow}$ $\left(\frac{3n_f}{16}\right)$

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Is the Teryaev argument convincing ?!?

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I have no idea!

Extra slides