

# New developments in the quantum statistical approach of the parton distributions

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## Outline

- **Basic procedure** to construct the statistical polarized parton distributions
- **Essential features** from unpolarized and polarized Deep Inelastic Scattering data
- **Predictions** tested against new data : DIS, Semi-inclusive DIS and several hadronic processes
- **Conclusions**

## Collaboration with [Claude Bourrely](#) and [Franco Buccella](#)

- A Statistical Approach for Polarized Parton Distributions  
Euro. Phys. J. [C23](#), 487 (2002)
- Recent Tests for the Statistical Parton Distributions  
Mod. Phys. Letters [A18](#), 771 (2003)
- The Statistical Parton Distributions: status and prospects  
Euro. Phys. J. [C41](#),327 (2005)
- The extension to the transverse momentum of the statistical parton distributions  
Mod. Phys. Letters [A21](#), 143 (2006)
- Strangeness asymmetry of the nucleon in the statistical parton model  
Phys. Lett. [B648](#), 39 (2007)
- New tests of the quantum statistical approach of the parton distributions  
(in preparation)

## Basic procedure

Use a simple description of the PDF, at input scale  $Q_0^2$ , proportional to  $[\exp[(x - X_{0p})/\bar{x}] \pm 1]^{-1}$ , *plus* sign for quarks and antiquarks, corresponds to a **Fermi-Dirac** distribution and *minus* sign for gluons, corresponds to a **Bose-Einstein** distribution.  $X_{0p}$  is a constant which plays the role of the *thermodynamical potential* of the parton  $p$  and  $\bar{x}$  is the *universal temperature*, which is the same for all partons.

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From the chiral structure of QCD, we have **two important properties**, allowing to relate quark and antiquark distributions and to restrict the gluon distribution:

- Potential of a quark  $q^h$  of helicity  $h$  is opposite to the potential of the corresponding antiquark  $\bar{q}^{-h}$  of helicity  $-h$ ,  $X_{0q}^h = -X_{0\bar{q}}^{-h}$ .
- Potential of the gluon  $G$  is zero,  $X_{0G} = 0$ .

## The polarized PDF at $Q_0^2 = 4\text{GeV}^2$

For light quarks  $q = u, d$  of helicity  $h = \pm$ , we take

$$xq^{(h)}(x, Q_0^2) = \frac{AX_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1},$$

consequently for antiquarks of helicity  $h = \mp$

$$x\bar{q}^{(-h)}(x, Q_0^2) = \frac{\bar{A}(X_{0q}^h)^{-1} x^{2b}}{\exp[(x + X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1}.$$

Note:  $q = q^+ + q^-$  and  $\Delta q = q^+ - q^-$  (idem for  $\bar{q}$ ).

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For strange quarks and antiquarks,  $s$  and  $\bar{s}$ , given our poor knowledge on both unpolarized and polarized distributions, we first took in 2002

$$xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) = \frac{1}{4}[x\bar{u}(x, Q_0^2) + x\bar{d}(x, Q_0^2)]$$

and

$$x\Delta s(x, Q_0^2) = x\Delta\bar{s}(x, Q_0^2) = \frac{1}{3}[x\Delta\bar{d}(x, Q_0^2) - x\Delta\bar{u}(x, Q_0^2)].$$

However given the **strange quark asymmetry**, this was improved in Phys. Lett. **B648**, 39 (2007).

For gluons we use a **Bose-Einstein** expression given by  $xG(x, Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1}$ , with a **vanishing potential** and the same temperature  $\bar{x}$ . We also need to specify the polarized gluon distribution and we take the particular choice  $x\Delta G(x, Q_0^2) = 0$ .

## Essential features from the DIS data

From well established features of  $u$  and  $d$  extracted from DIS data, we anticipate some simple relations between the potentials:

- $u(x)$  dominates over  $d(x)$ , therefore we should have
$$X_{0u}^+ + X_{0u}^- > X_{0d}^+ + X_{0d}^-$$
- $\Delta u(x) > 0$ , therefore  $X_{0u}^+ > X_{0u}^-$
- $\Delta d(x) < 0$ , therefore  $X_{0d}^- > X_{0d}^+$ .

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- $\Delta d(x) < 0$ , therefore  $X_{0d}^- > X_{0d}^+$ .

So we expect  $X_{0u}^+$  to be the largest potential and  $X_{0d}^+$  the smallest one. In fact, from our fit we have obtained the following ordering (see below)

$$X_{0u}^+ > X_{0d}^- \sim X_{0u}^- > X_{0d}^+ .$$

This ordering has important consequences for  $\bar{u}$  and  $\bar{d}$ , namely

- $\bar{d}(x) > \bar{u}(x)$ , flavor symmetry breaking expected from **Pauli exclusion principle**. This was already confirmed by the violation of the **Gottfried sum rule** (NMC).
- $\Delta\bar{u}(x) > 0$  and  $\Delta\bar{d}(x) < 0$ , a **PREDICTION** in agreement with polarized DIS (see below) and will be more precisely checked at RHIC-BNL from  $W^\pm$  production.

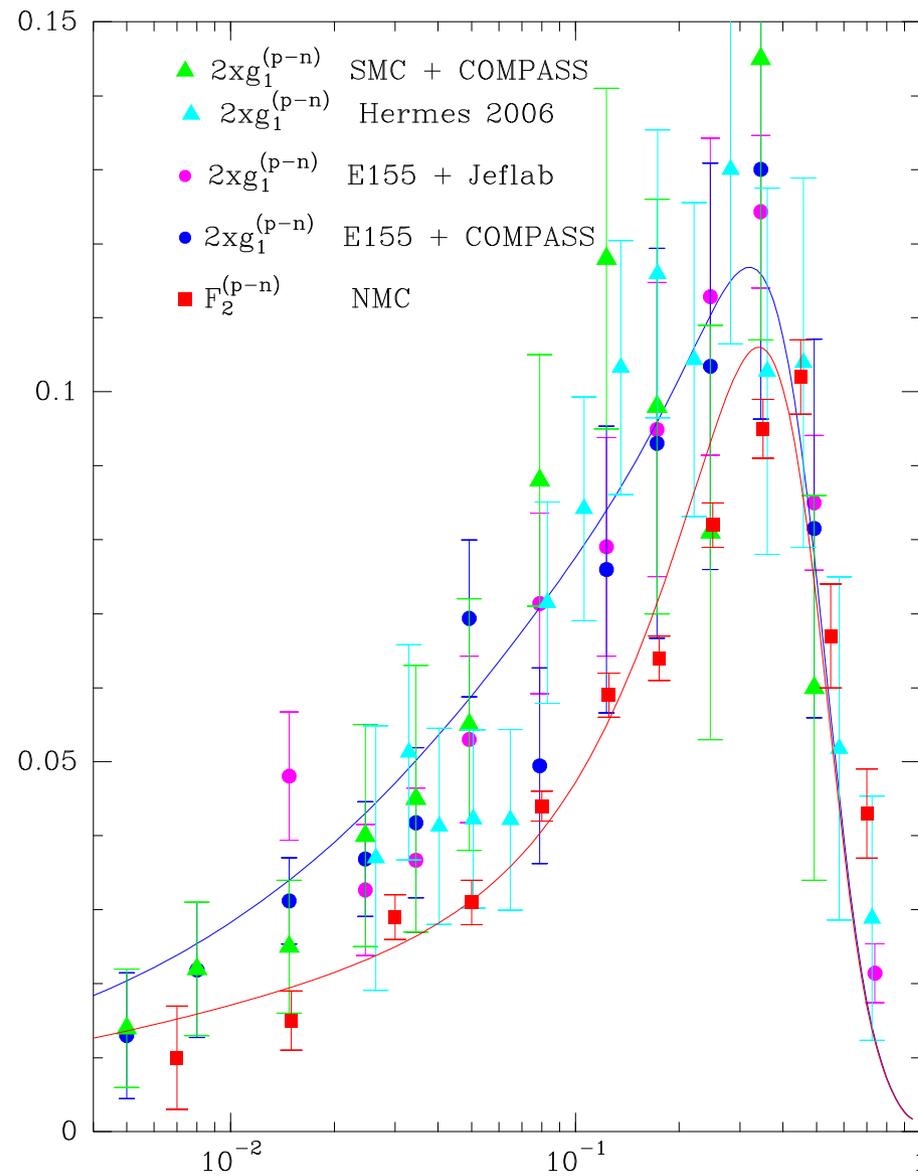
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- Note that since  $u^-(x) \sim d^-(x)$ , it follows that  $\bar{u}^+(x) \sim \bar{d}^+(x)$ , (**see next slide**) so we have

$$\Delta\bar{u}(x) - \Delta\bar{d}(x) \sim \bar{d}(x) - \bar{u}(x) ,$$

i.e. the flavor symmetry breaking is almost the **same** for unpolarized and polarized distributions ( $\bar{u}$  and  $\bar{d}$  polarizations contribute to about 10% to the **Bjorken sum**

# An interesting observation

$u^+$  dominates and  $u^- \simeq d^-$



## Nine free parameters

By performing a NLO QCD evolution of these PDF, we were able to obtain a good description of a large set of very precise data on  $F_2^p(x, Q^2)$ ,  $F_2^n(x, Q^2)$ ,  $xF_3^{\nu N}(x, Q^2)$  and  $g_1^{p,d,n}(x, Q^2)$ , in correspondance with **nine** free parameters with some physical significance:

- \* the four potentials  $X_{0u}^+$ ,  $X_{0u}^-$ ,  $X_{0d}^-$ ,  $X_{0d}^+$ ,
- \* the universal temperature  $\bar{x}$ ,
- \* **and**  $b$ ,  $\tilde{b}$ ,  $b_G$ ,  $\tilde{A}$ .

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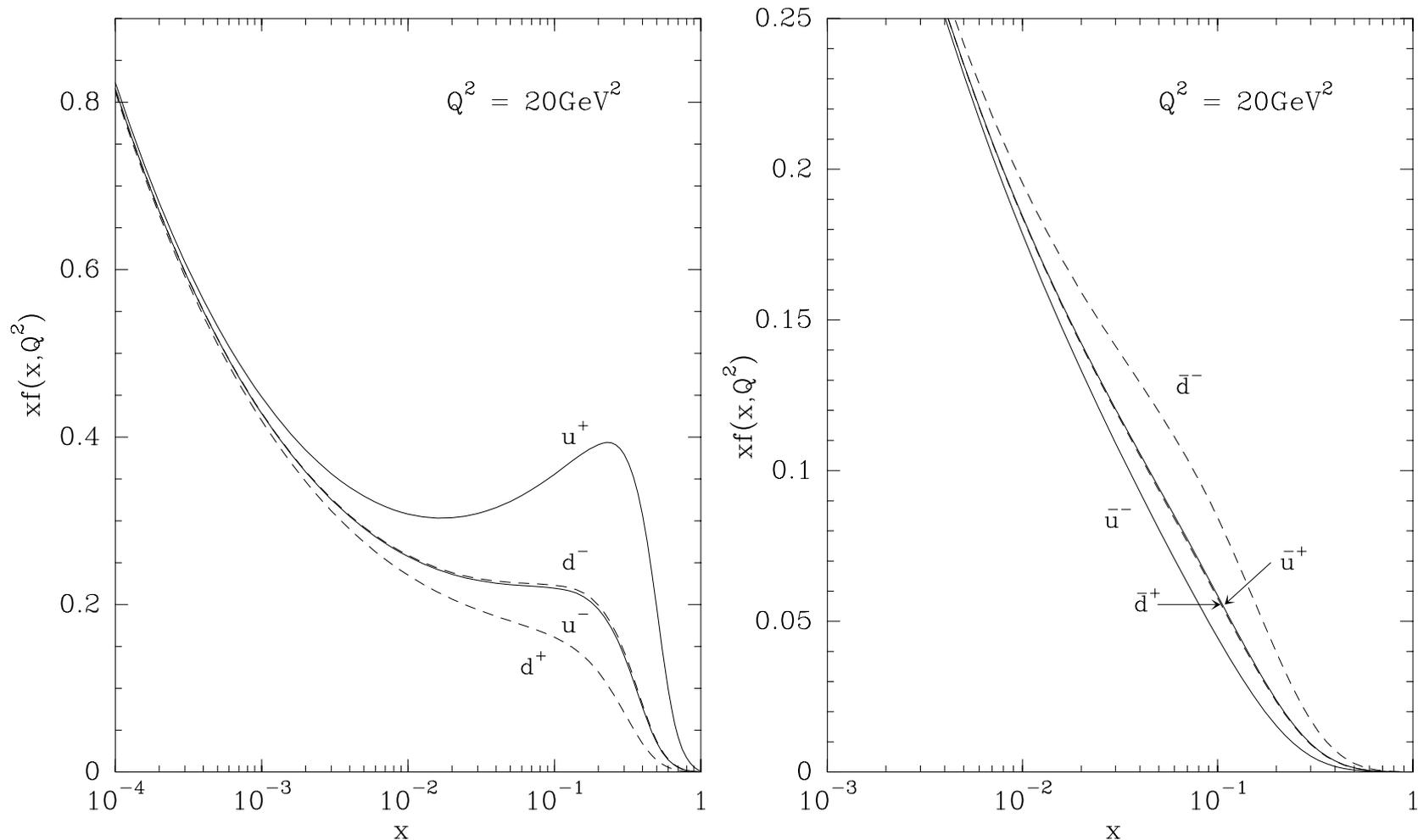
We also have three additional parameters,  $A$ ,  $\bar{A}$ ,  $A_G$ , which are fixed by 3 normalization conditions .

$$u - \bar{u} = 2, \quad d - \bar{d} = 1$$

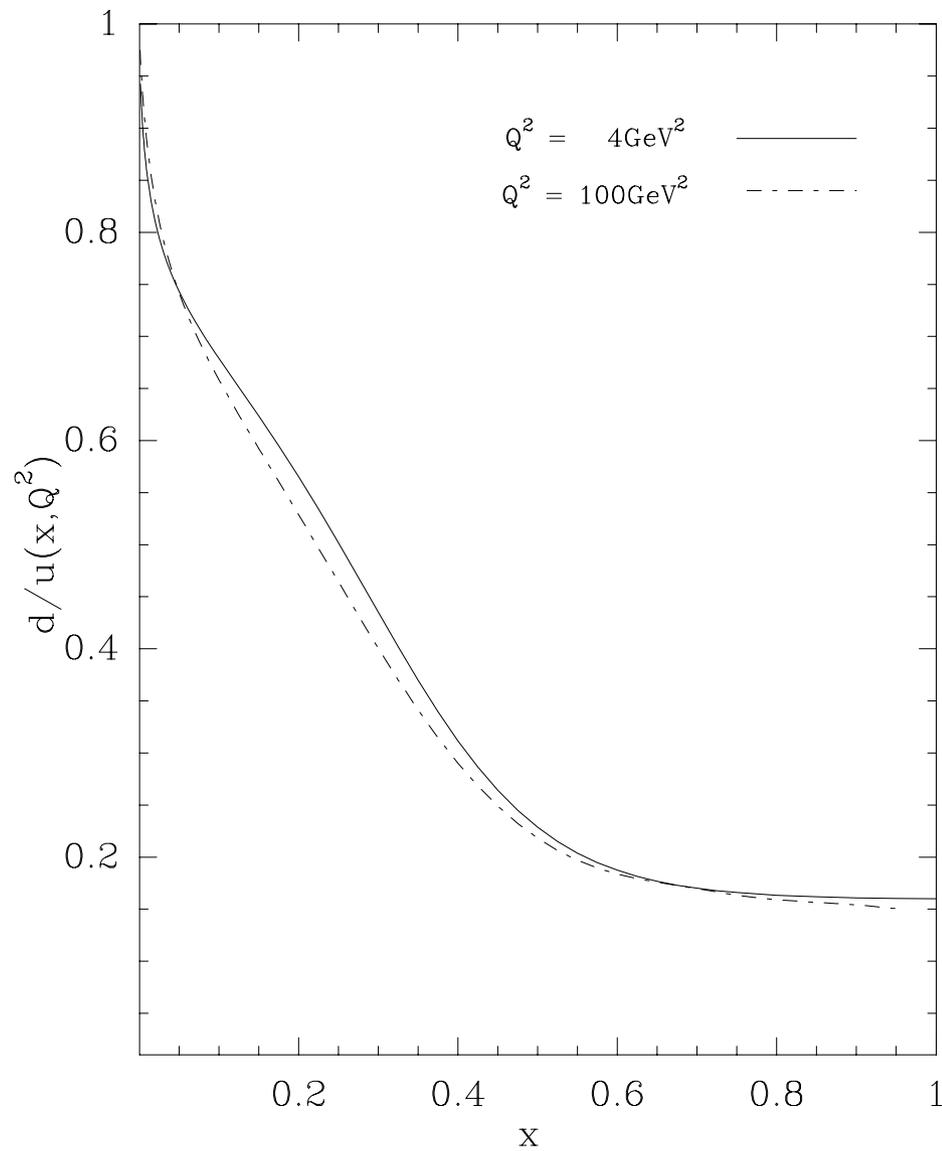
and the momentum sum rule.

# Polarized light quarks distributions versus x

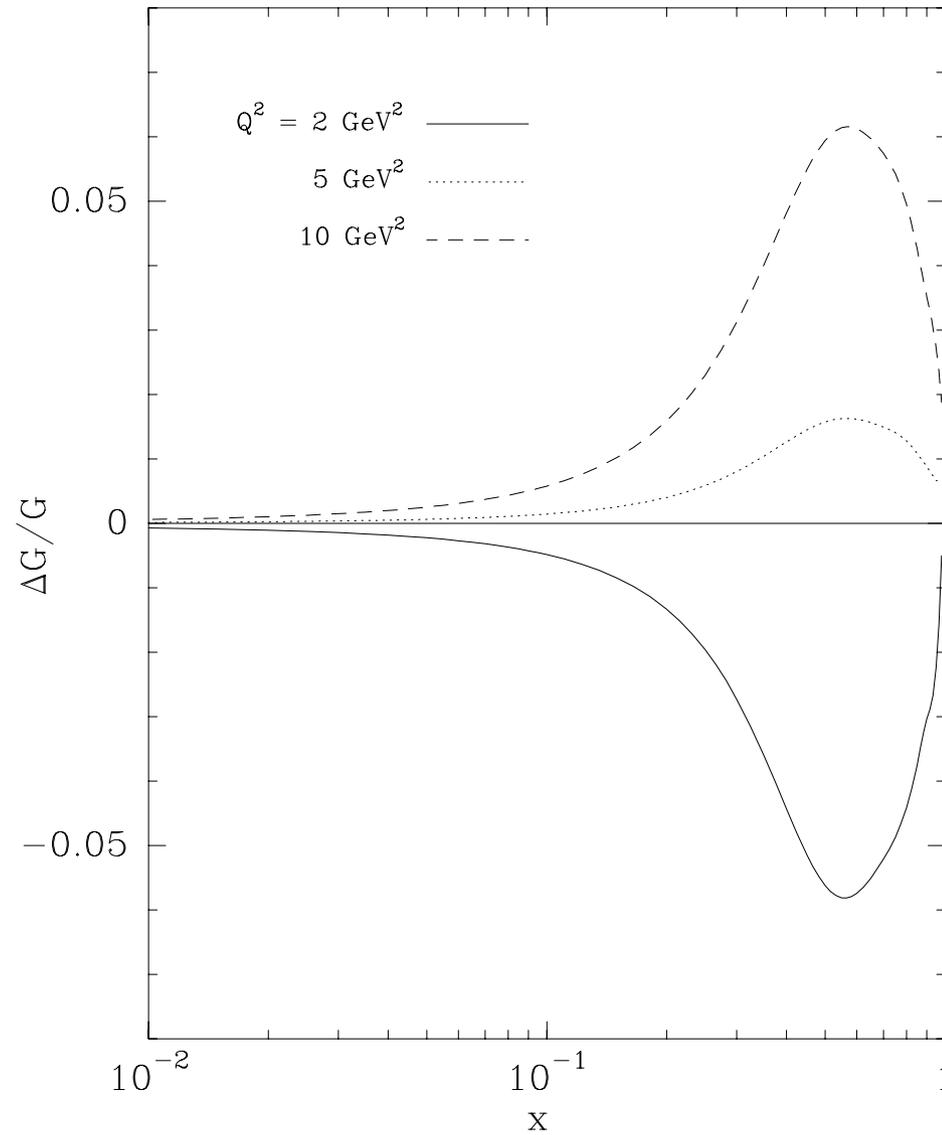
As we could anticipated  $u^+$ , is the largest one, is maximum near  $x = 0.3$  and  $u^- \simeq d^-$ .  
Therefore for the antiquarks  $\bar{d}^-$  is the largest one and  $\bar{u}^+ \simeq \bar{d}^+$   
Moreover we find  $\Delta\Sigma(Q_0^2) = 0.28$



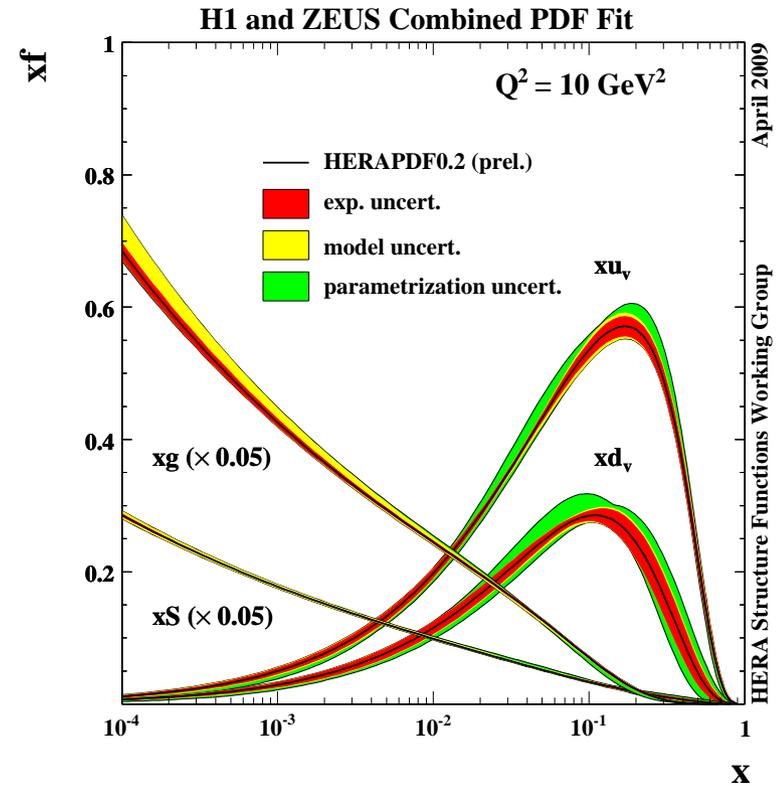
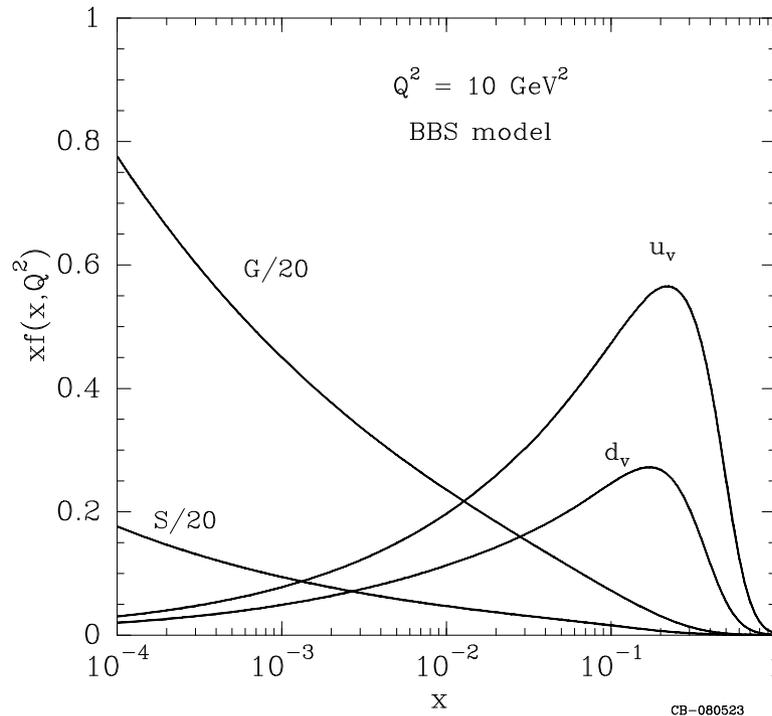
## The $d/u$ ratio versus $x$



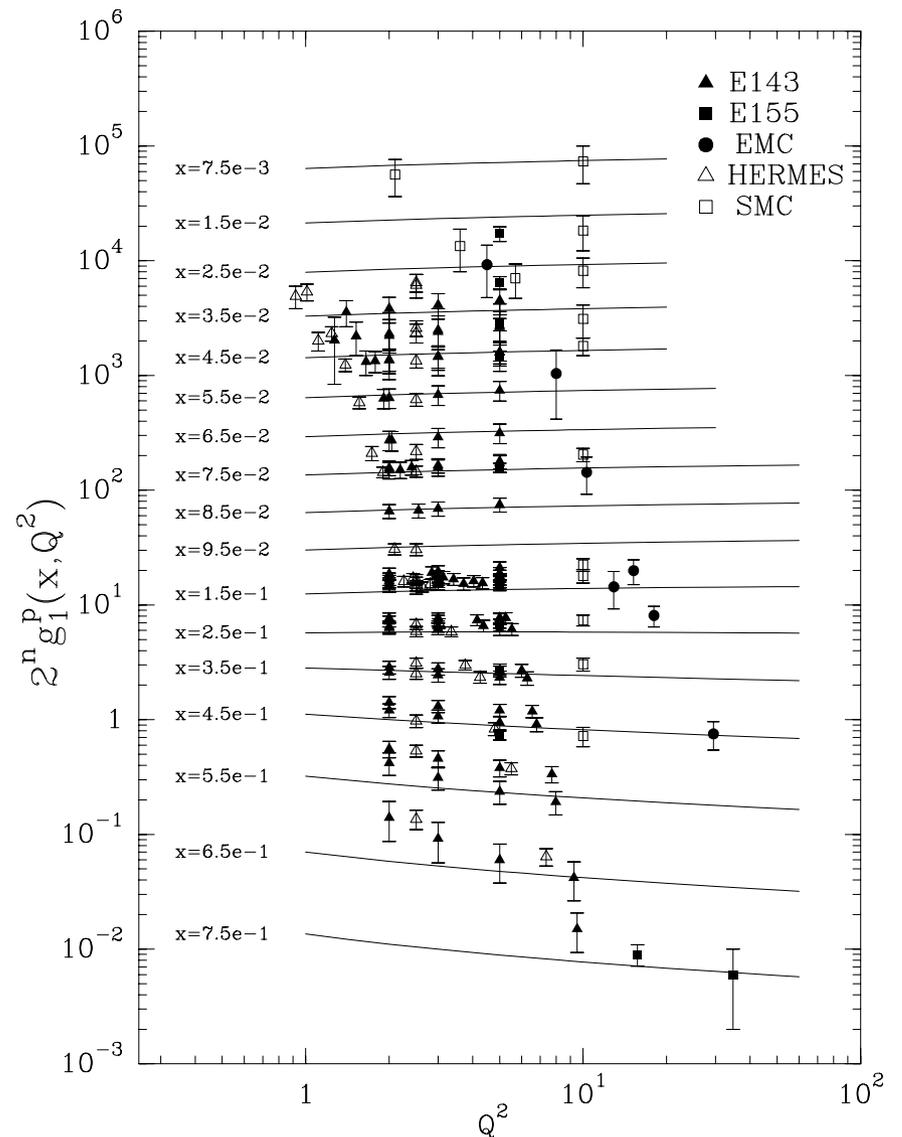
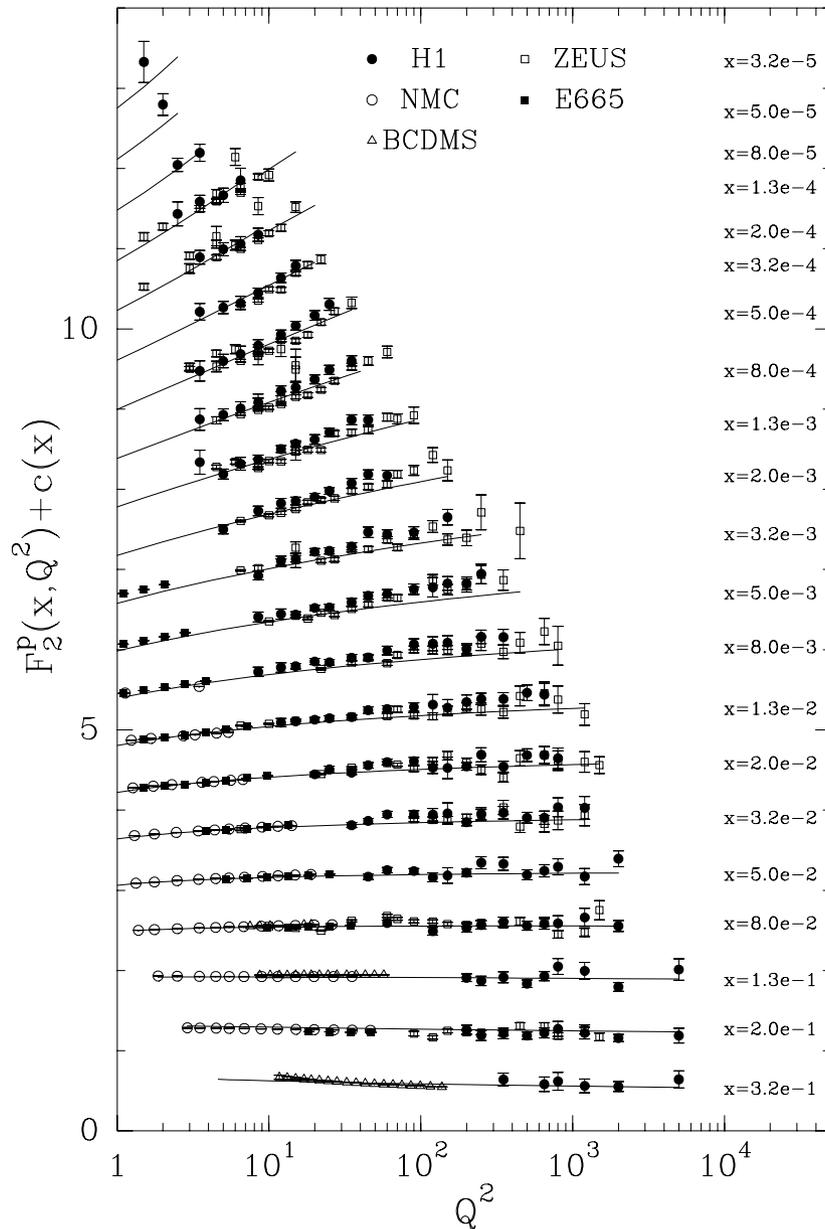
## The $\Delta G/G$ ratio versus $x$



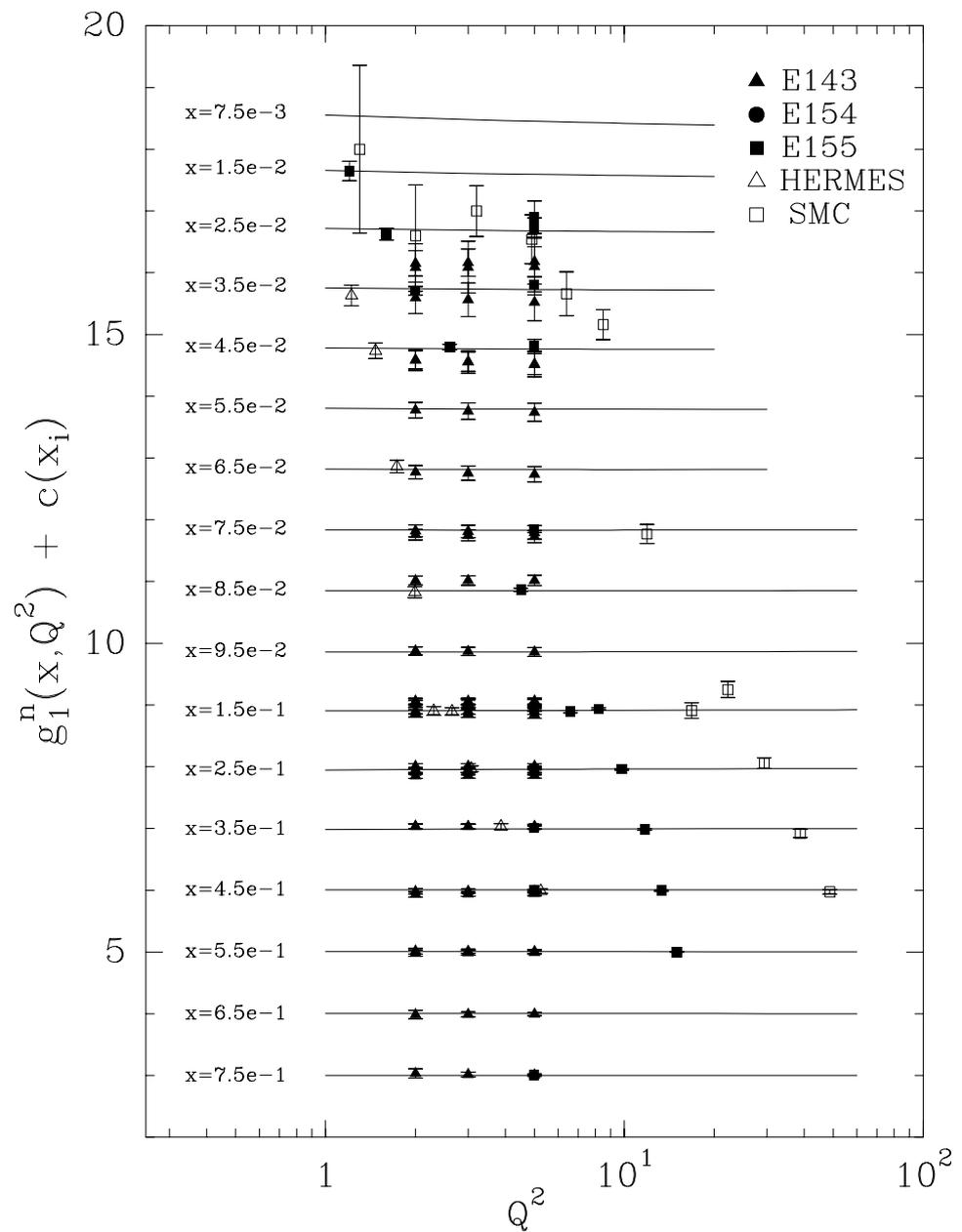
# A global view of the unpolarized parton distributions



# Earlier results on $F_2^p$ and $g_1^p$



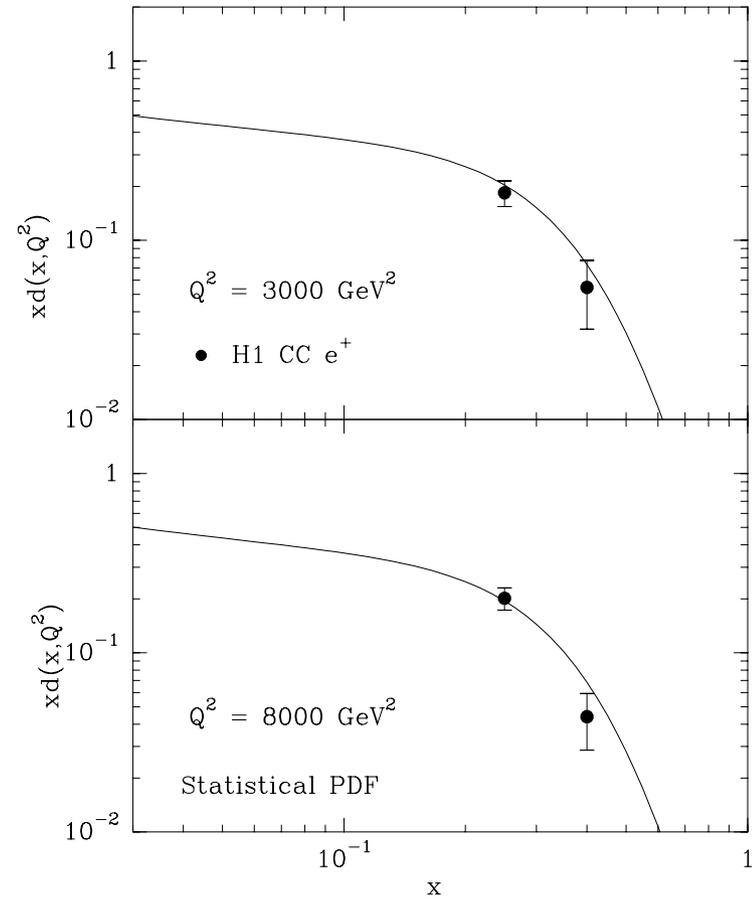
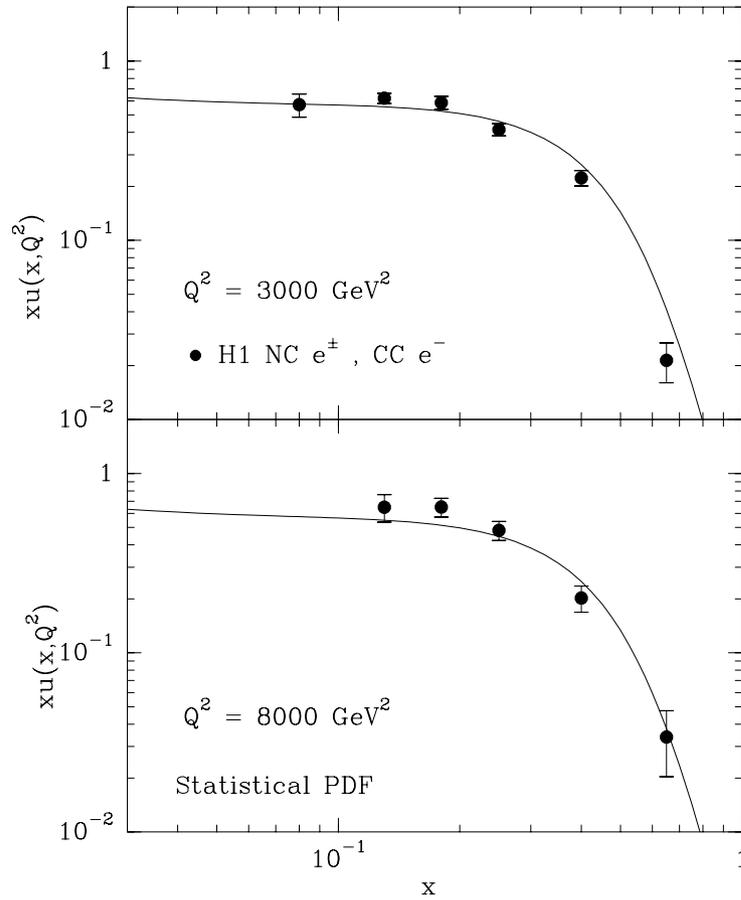
# Earlier results on $g_1^n$



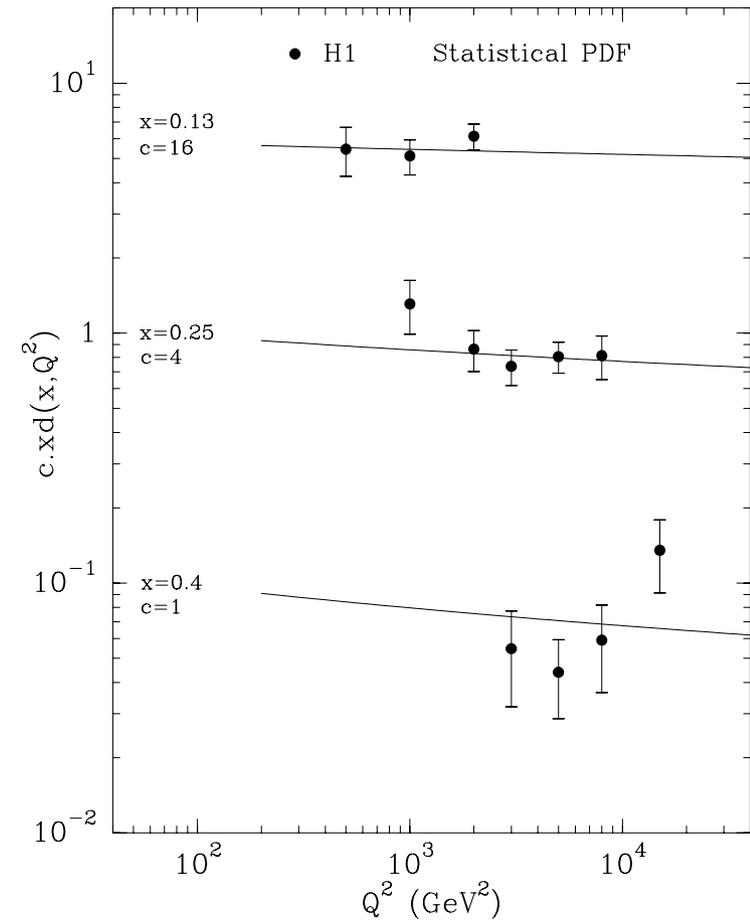
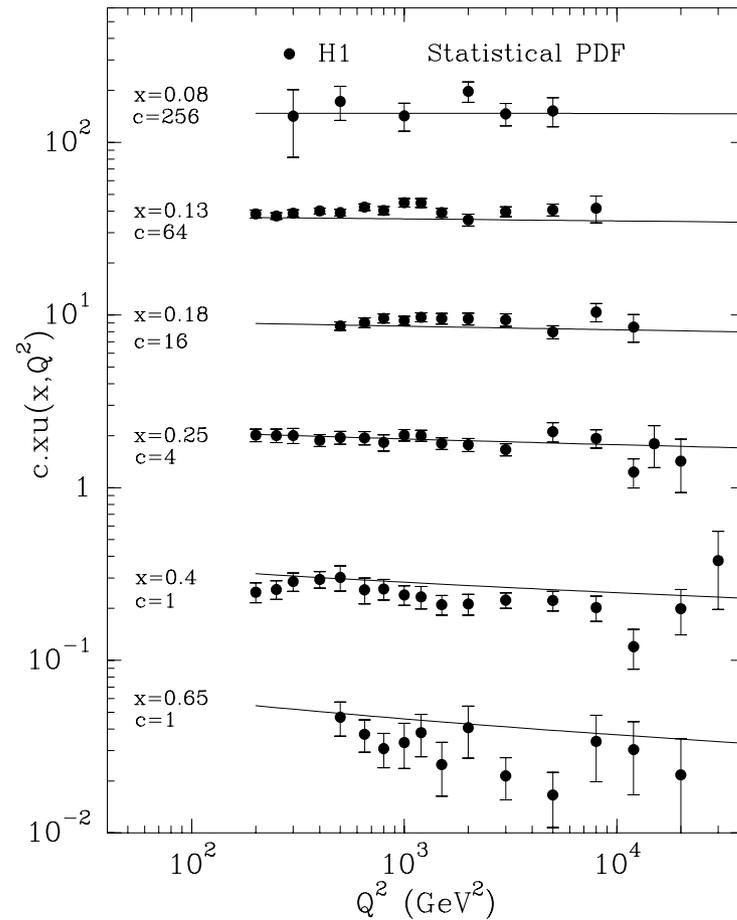
## Predictions tested against some data 2002 - 2005

- Deep Inelastic Scattering
- Hadronic Collisions

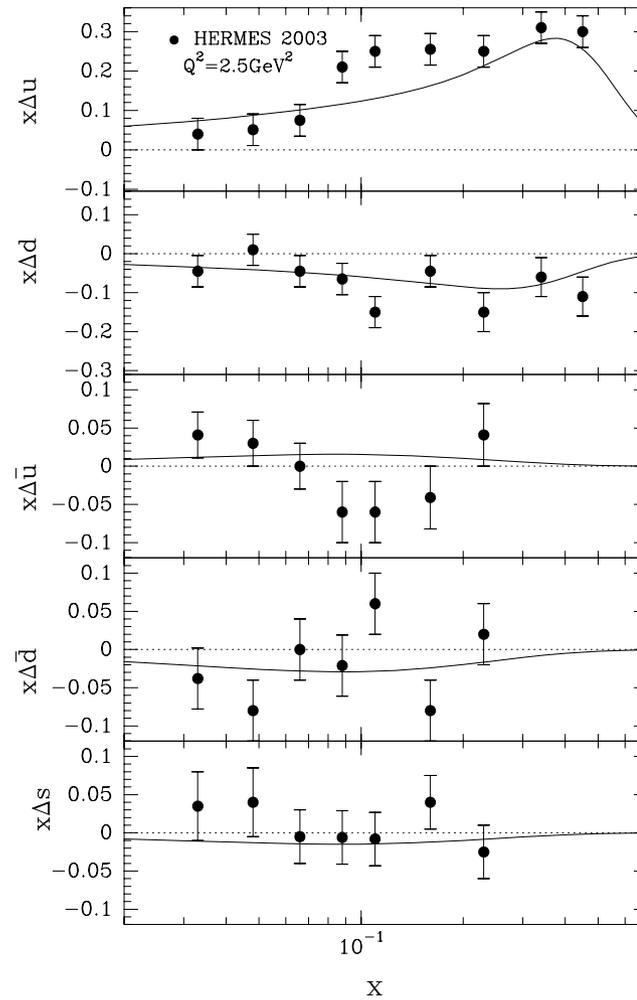
# Light quarks distributions versus x from HERA



# Light quarks distributions versus $Q^2$ from HERA



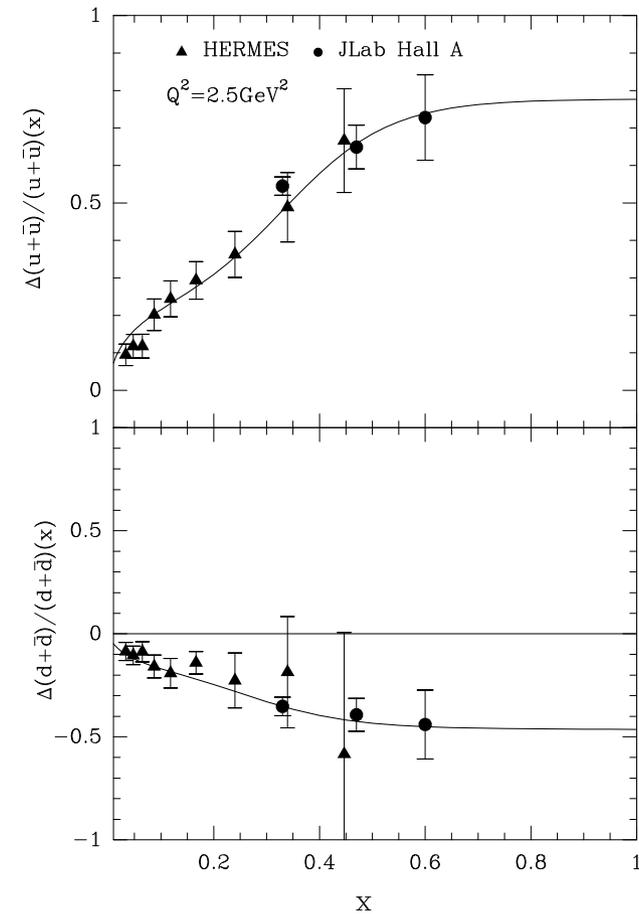
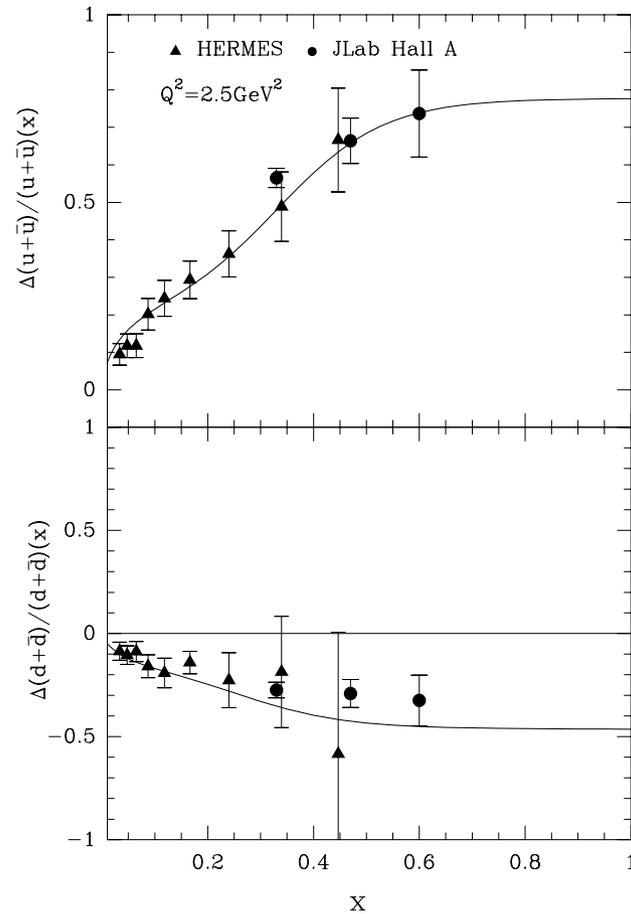
# Polarized quarks distributions vs x at DESY and JLab



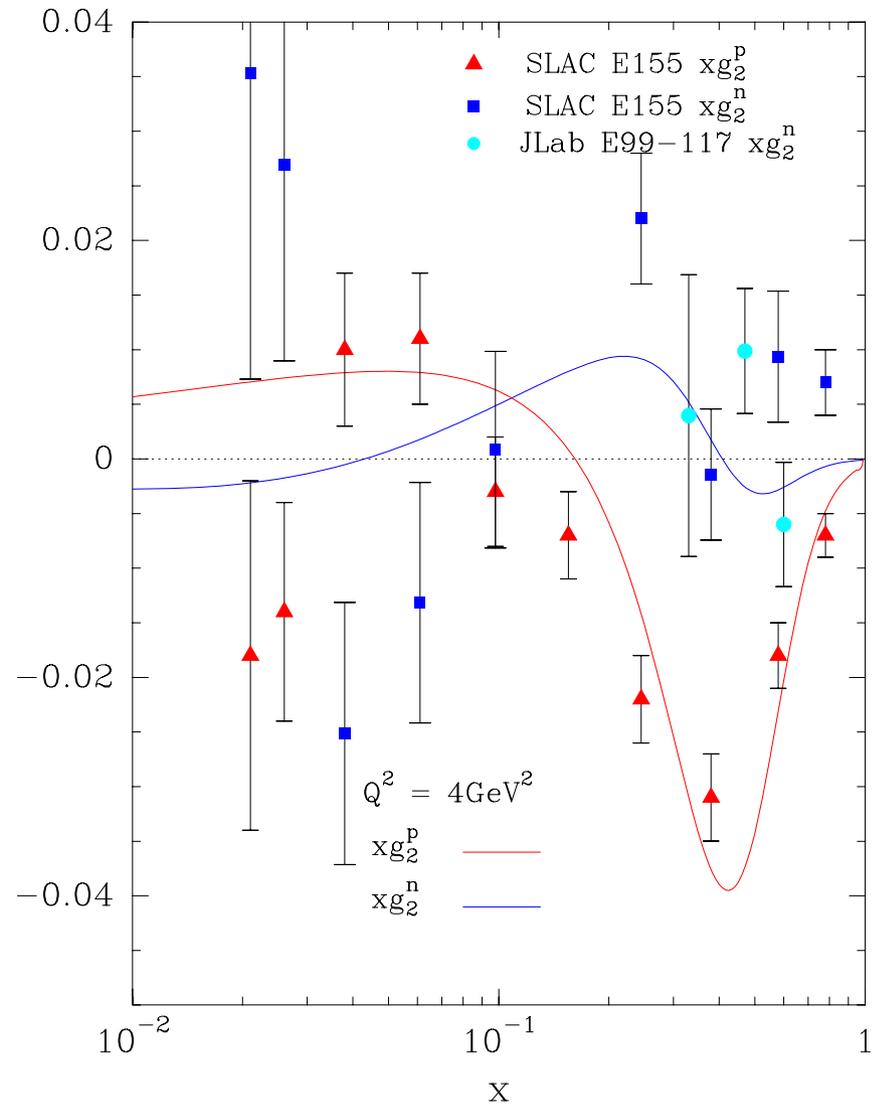
# Polarized quarks distributions vs x at DESY and JLab

Data before correction

Data after correction

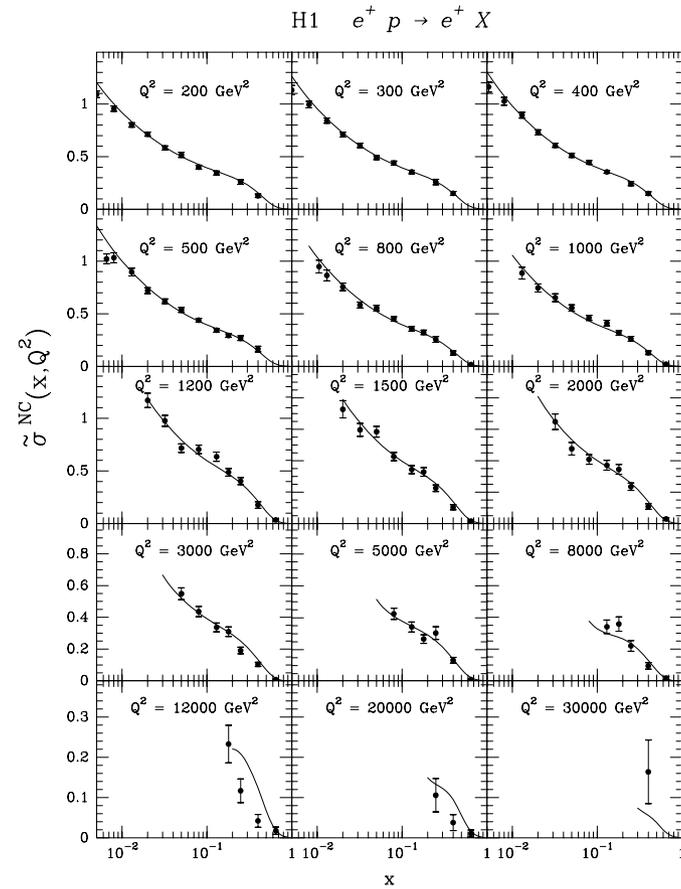
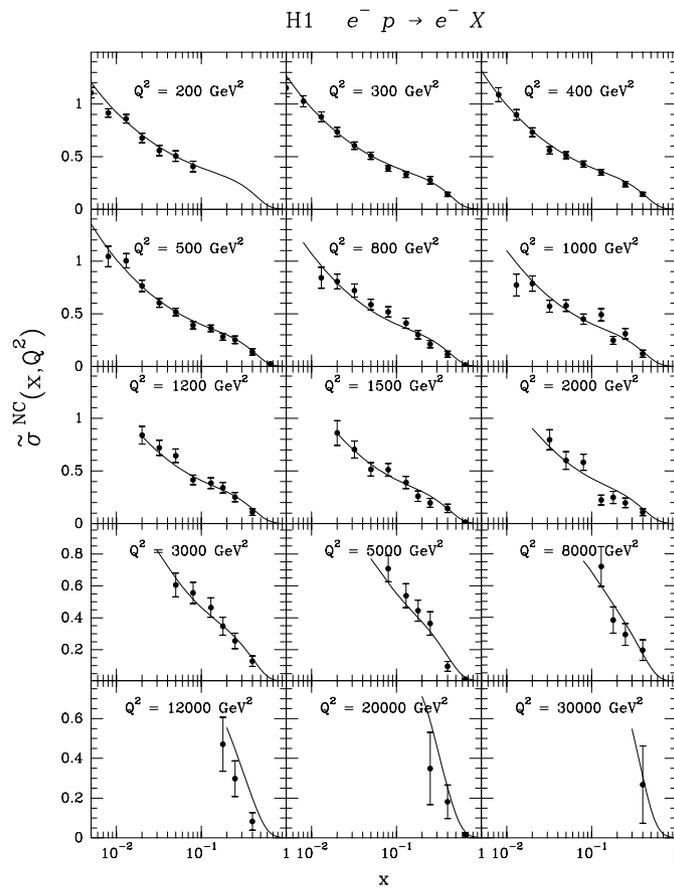


# The $g_2^{p,n}$ structure functions versus $x$

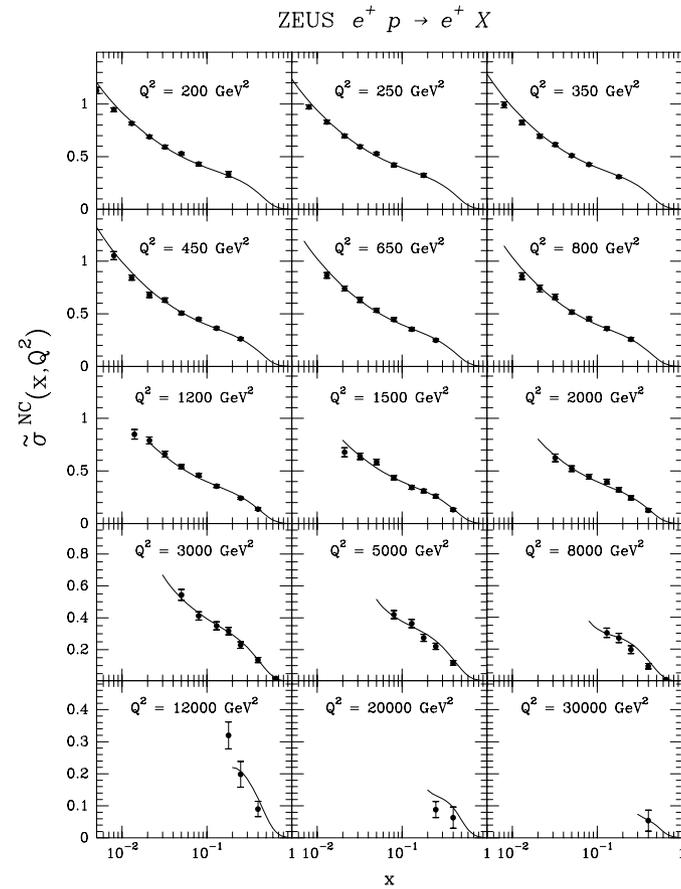
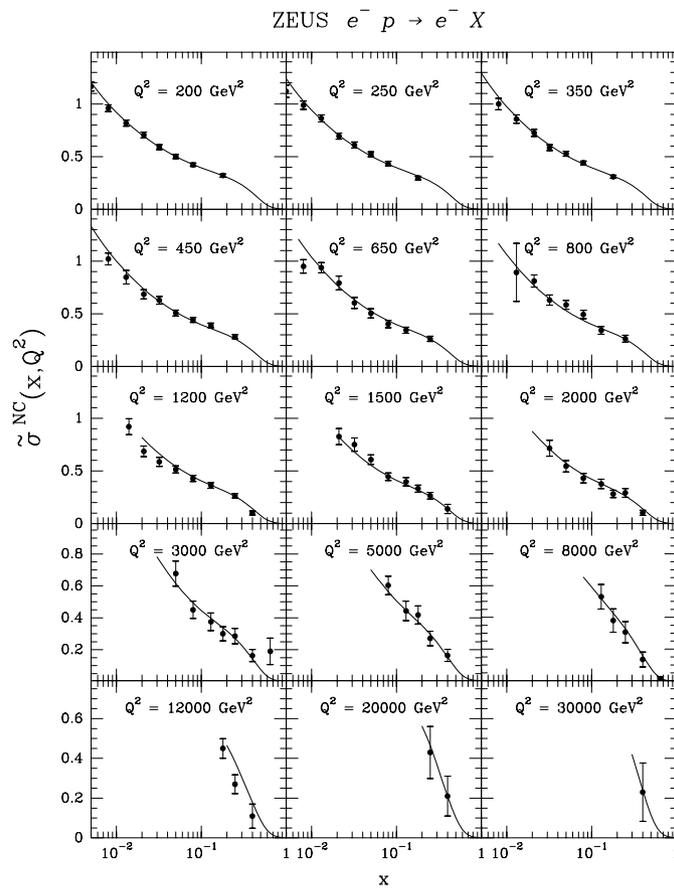


Predictions at leading twist assuming Wandzura-Wilczek sum rule

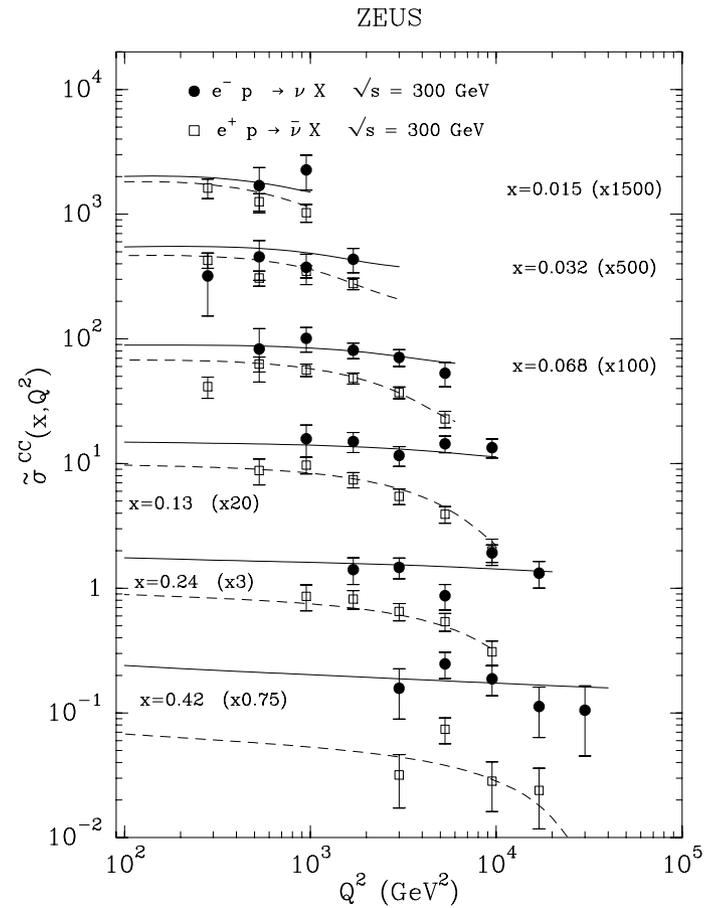
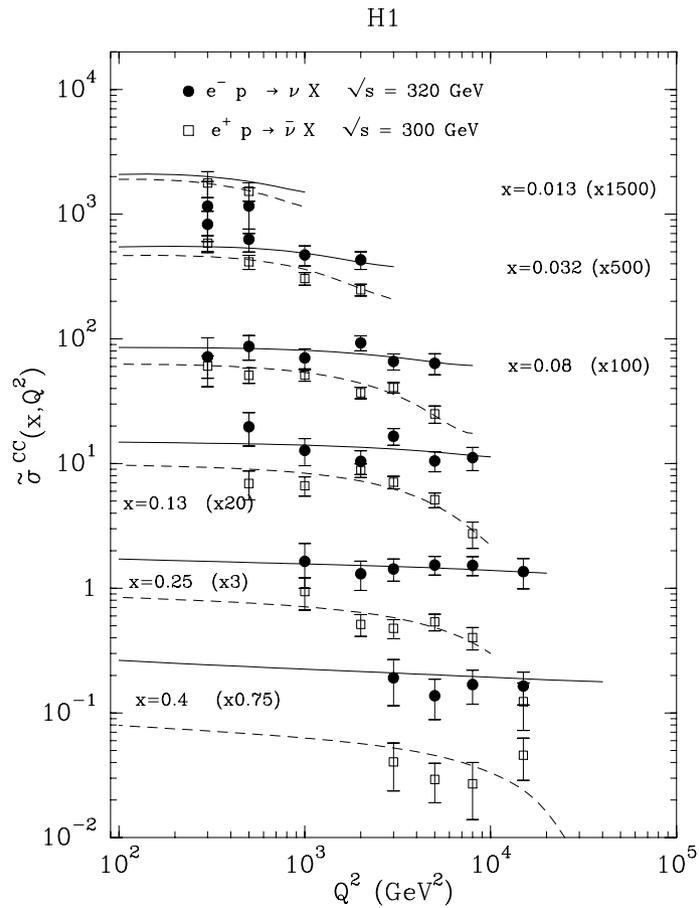
# Neutral current in unpolarized $e^\pm p$ collisions (H1)



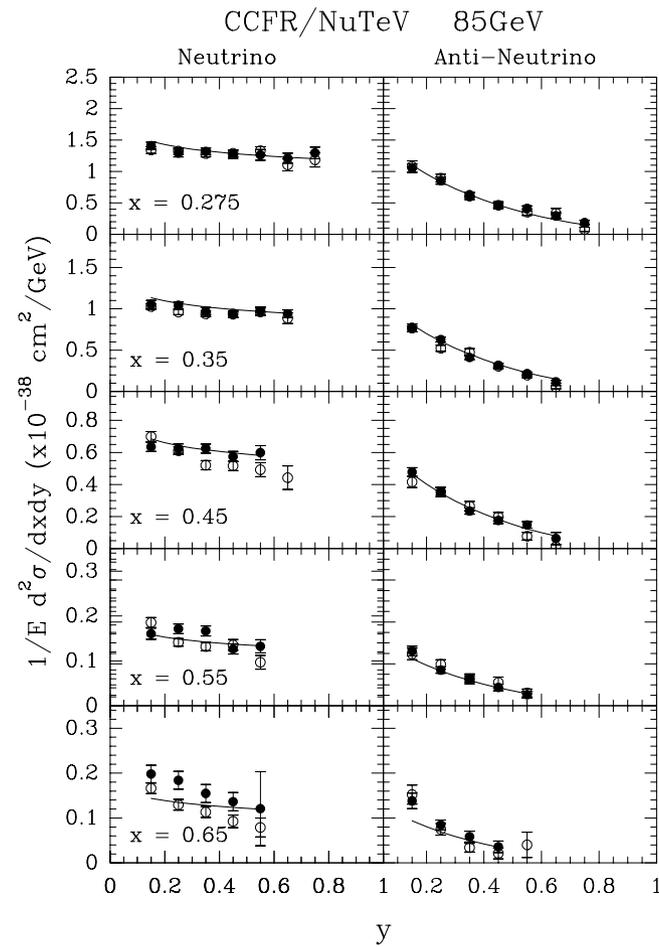
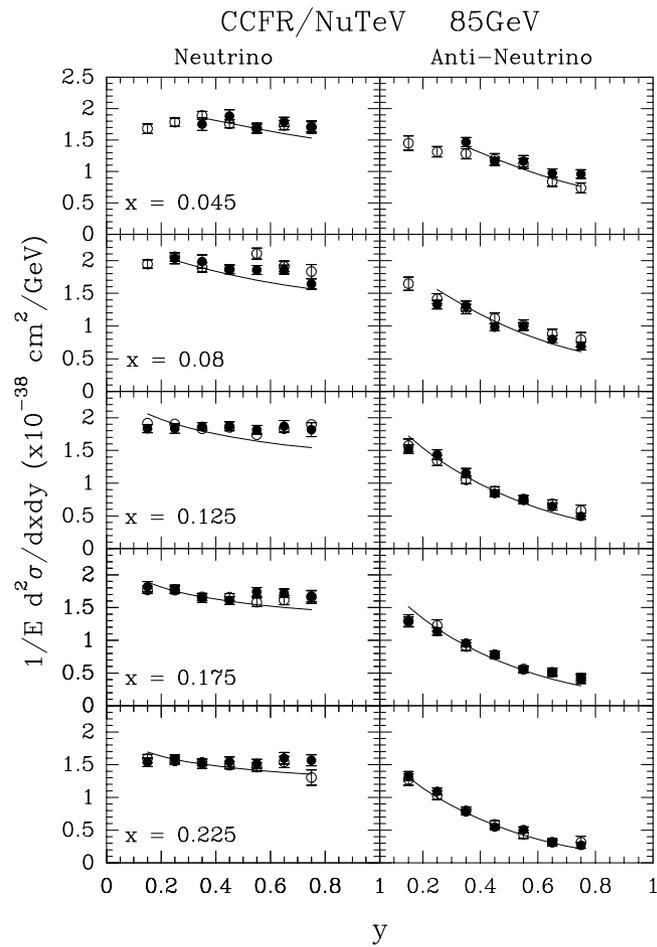
# Neutral current in unpolarized $e^\pm p$ collisions (ZEUS)



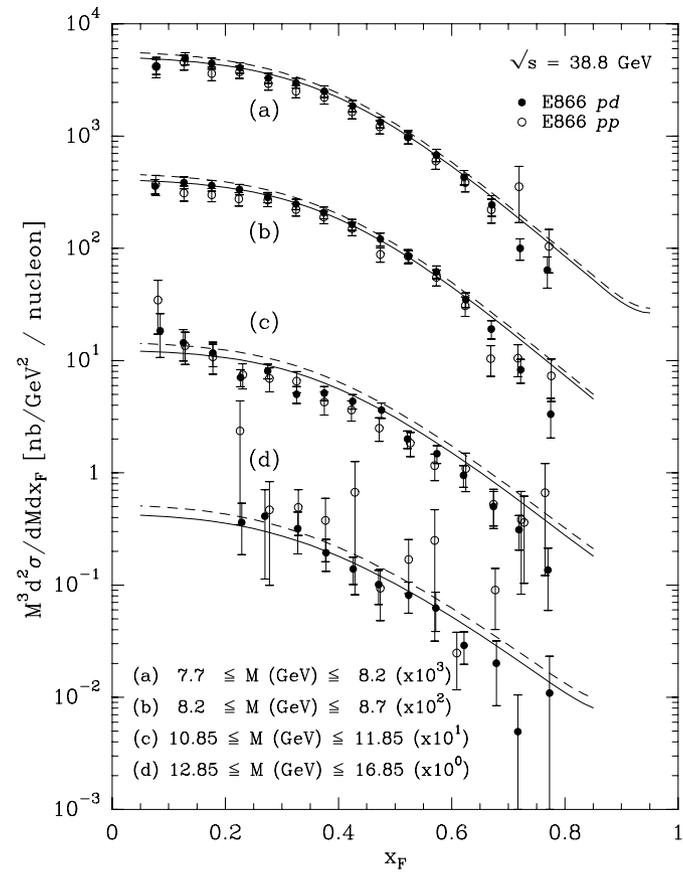
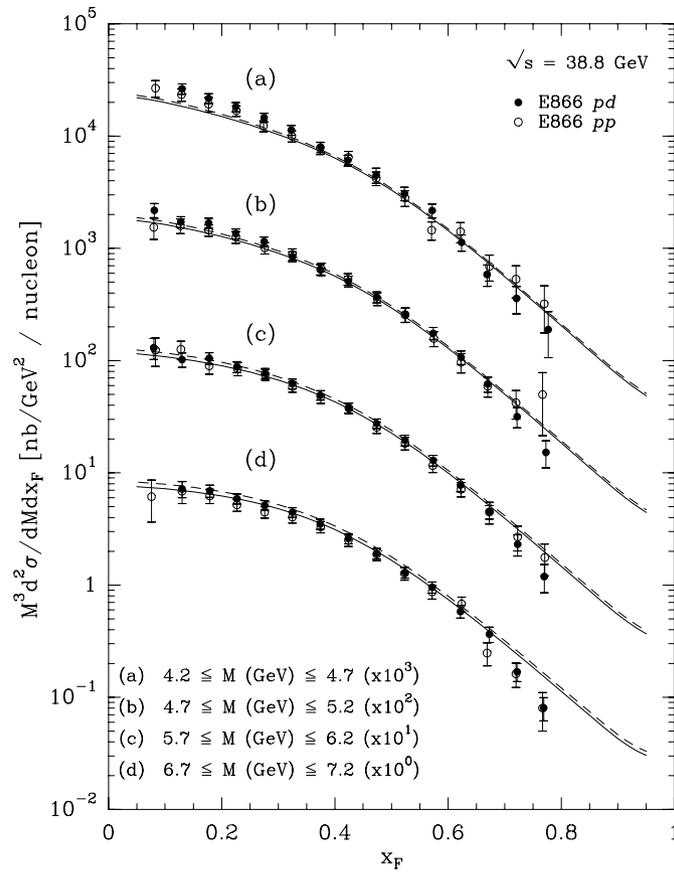
# Charged current in $e^\pm p$ collisions at HERA



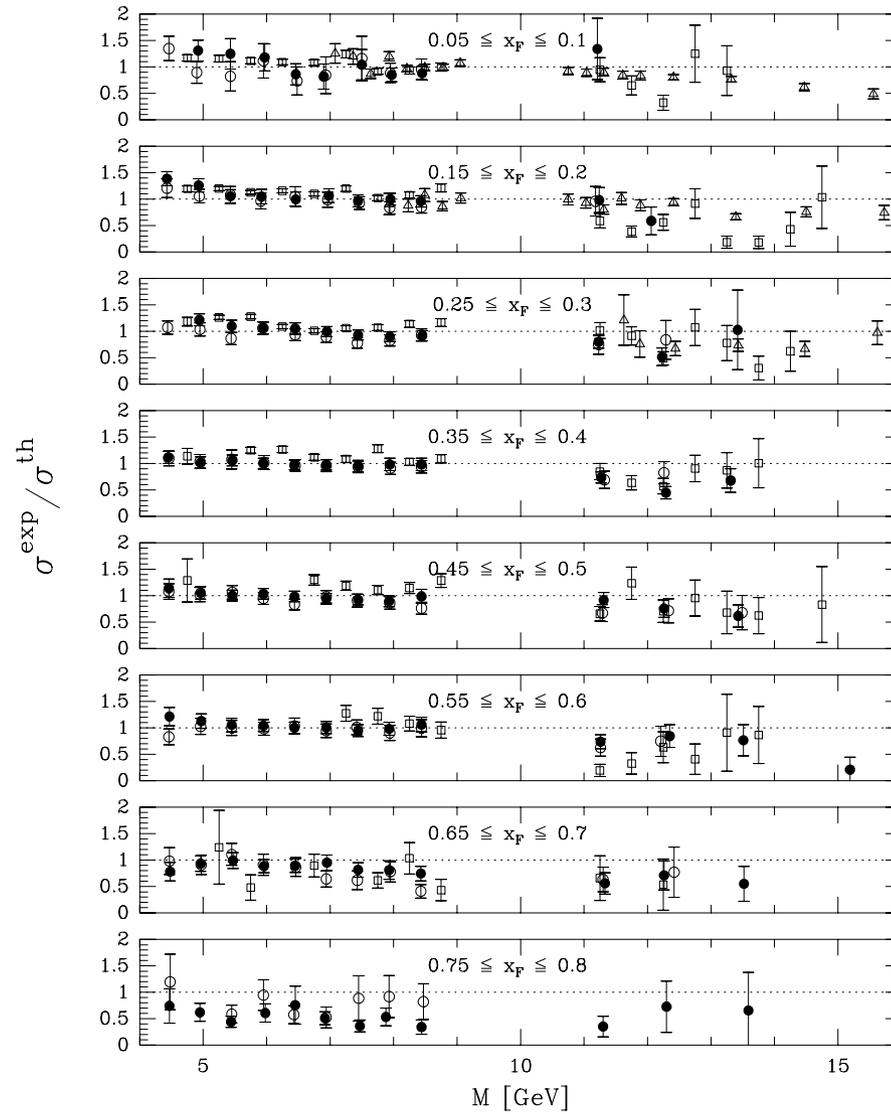
# Charged current neutrino cross sections at FNAL



# Drell-Yan processes at FNAL



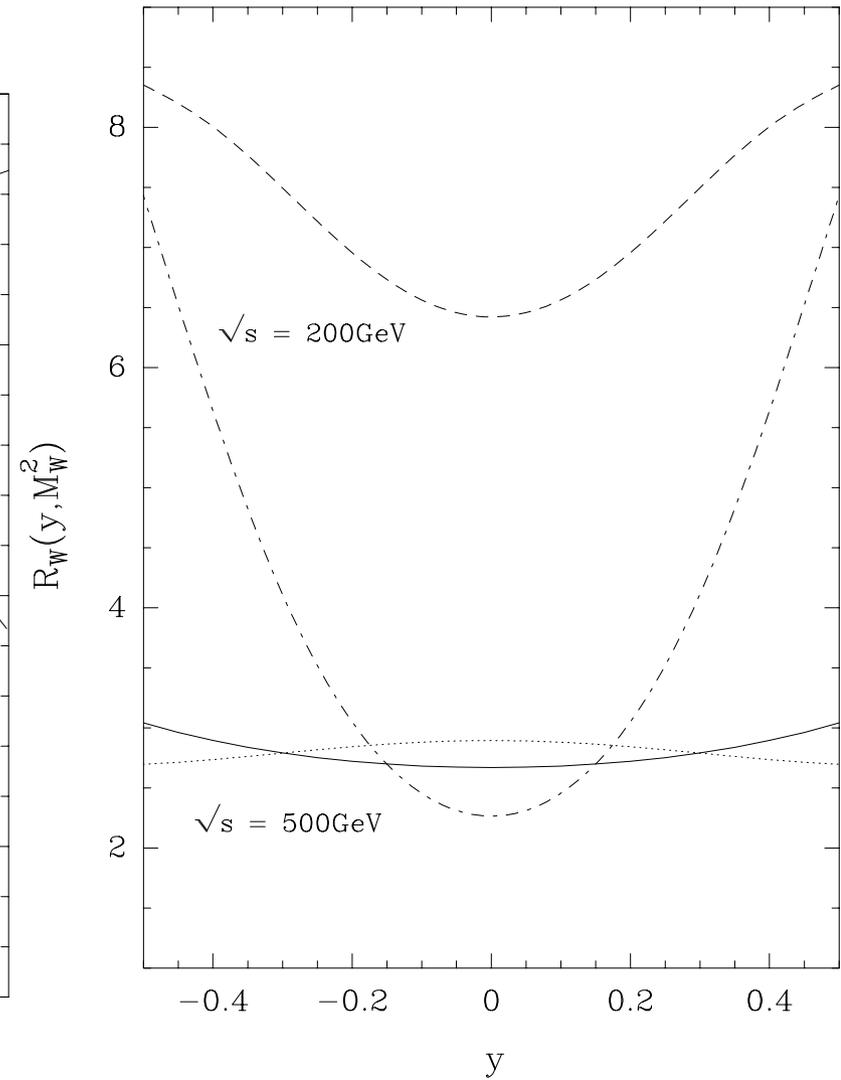
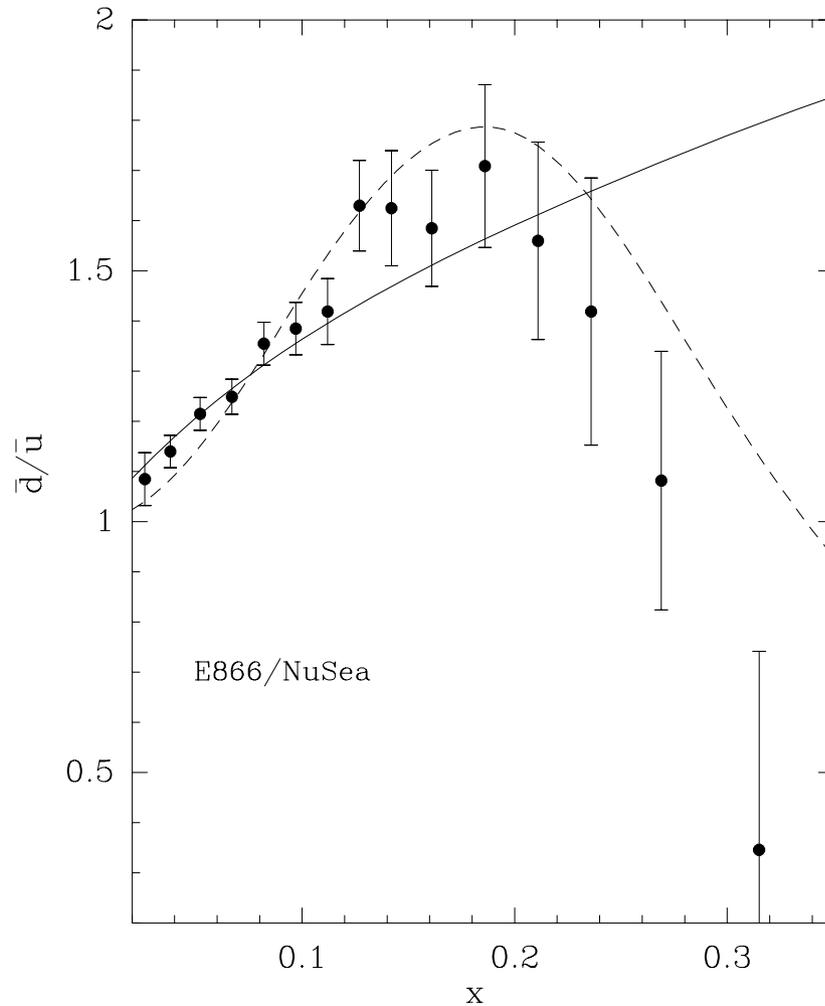
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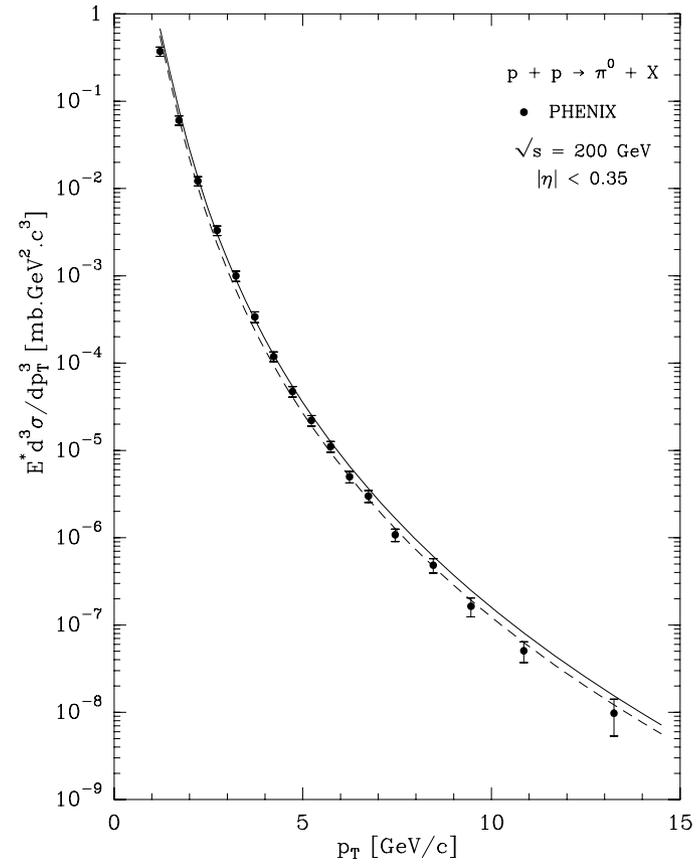
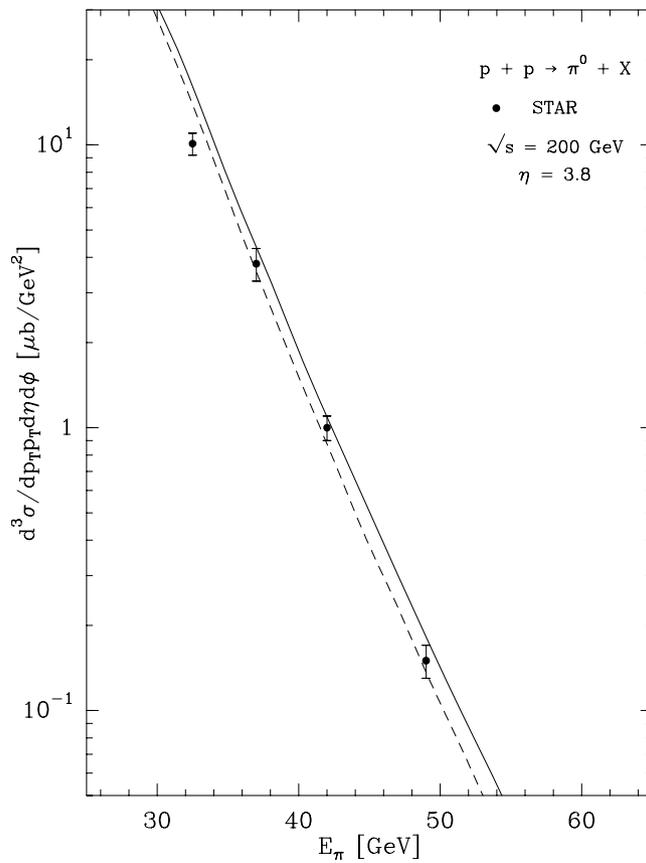
# The important issue of $\bar{d}/\bar{u}$ at large $x$

$$Q^2 = 54\text{GeV}^2$$

$$R_W = (d\sigma_W^+/dy)/(d\sigma_W^-/dy) \quad Q^2 = M_W^2$$

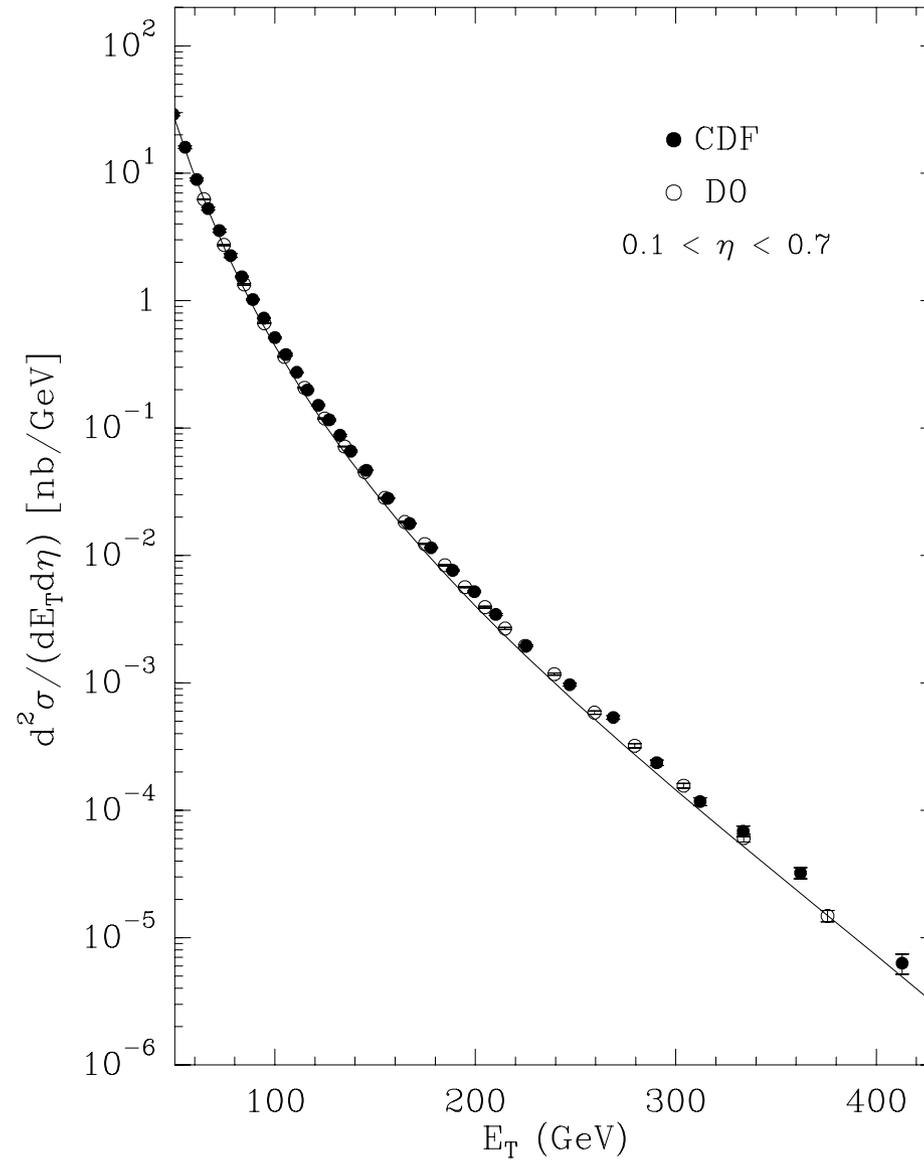


# Inclusive $\pi^0$ production in $pp$ collisions at RHIC

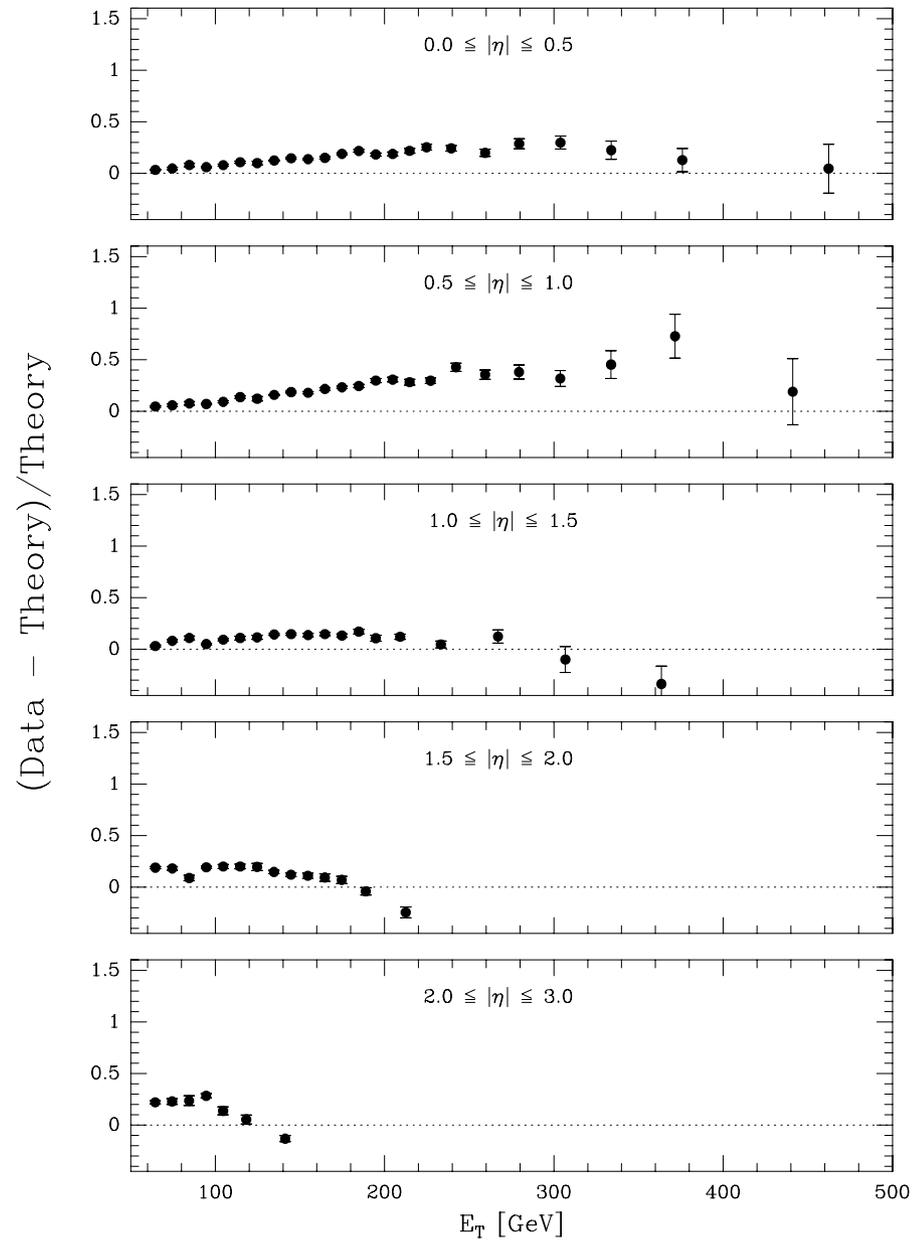


Mid-rapidity and central region

# Single-jet production in $\bar{p}p$ collisions at FNAL



# Single-jet production in $\bar{p}p$ collisions at FNAL



## Predictions tested against some very recent data

- Unpolarized Deep Inelastic Scattering

- Gluon

- \* Can be extracted from scaling violations of  $F_2$ , i.e. derivatives w.r.t  $Q^2$  and  $x$

- \* The structure function  $F_L$  is a direct sensitivity to the gluon:  $F_L = 0$  in quark-parton model, but  $F_L \neq 0$  in NLO pQCD

- Strange quark and antiquark

- First determined from NuTeV and tested against Semi-inclusive DIS from Hermes

- Valence light quarks

- From  $\gamma - Z$  interference in neutral current  $e^\pm p$  collisions

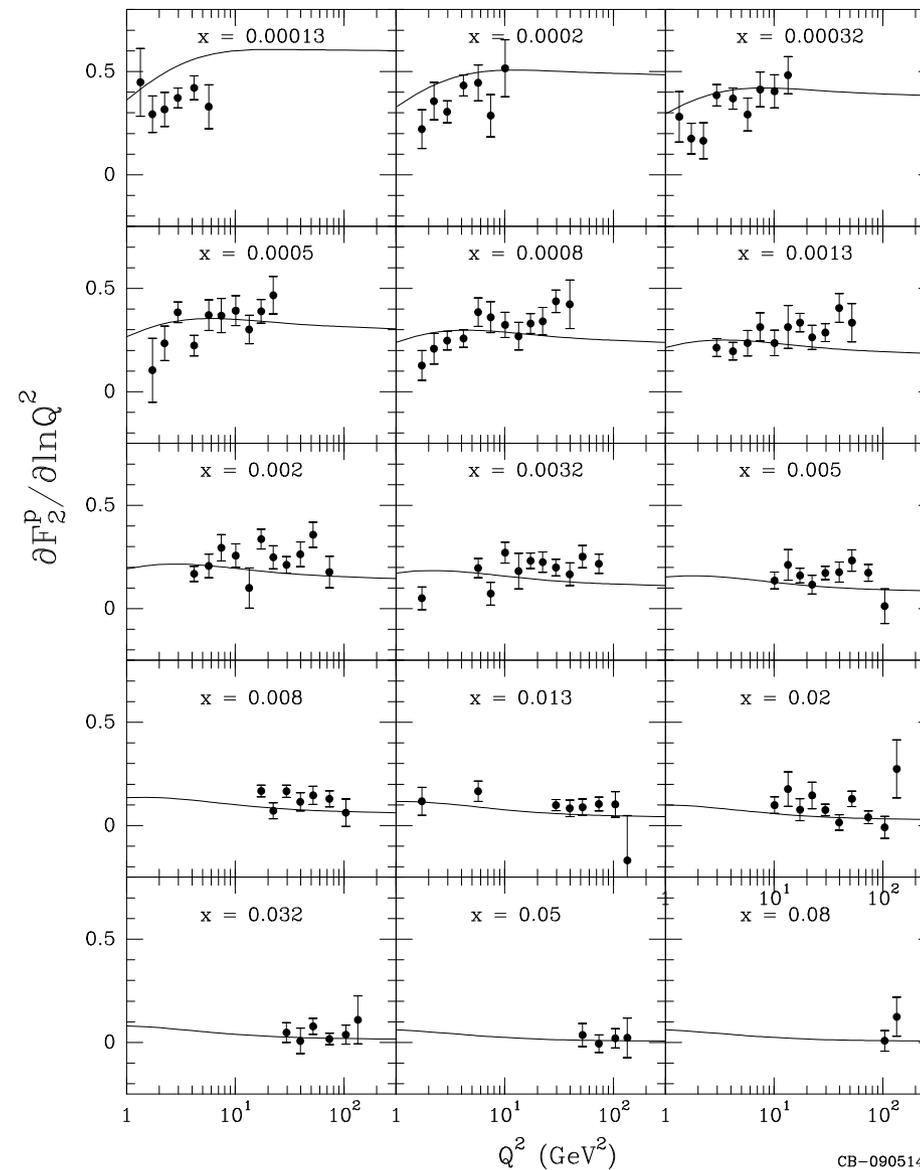
- Polarized Deep Inelastic Scattering

- \* Polarized valence light quarks from Semi-inclusive DIS on Deuterium

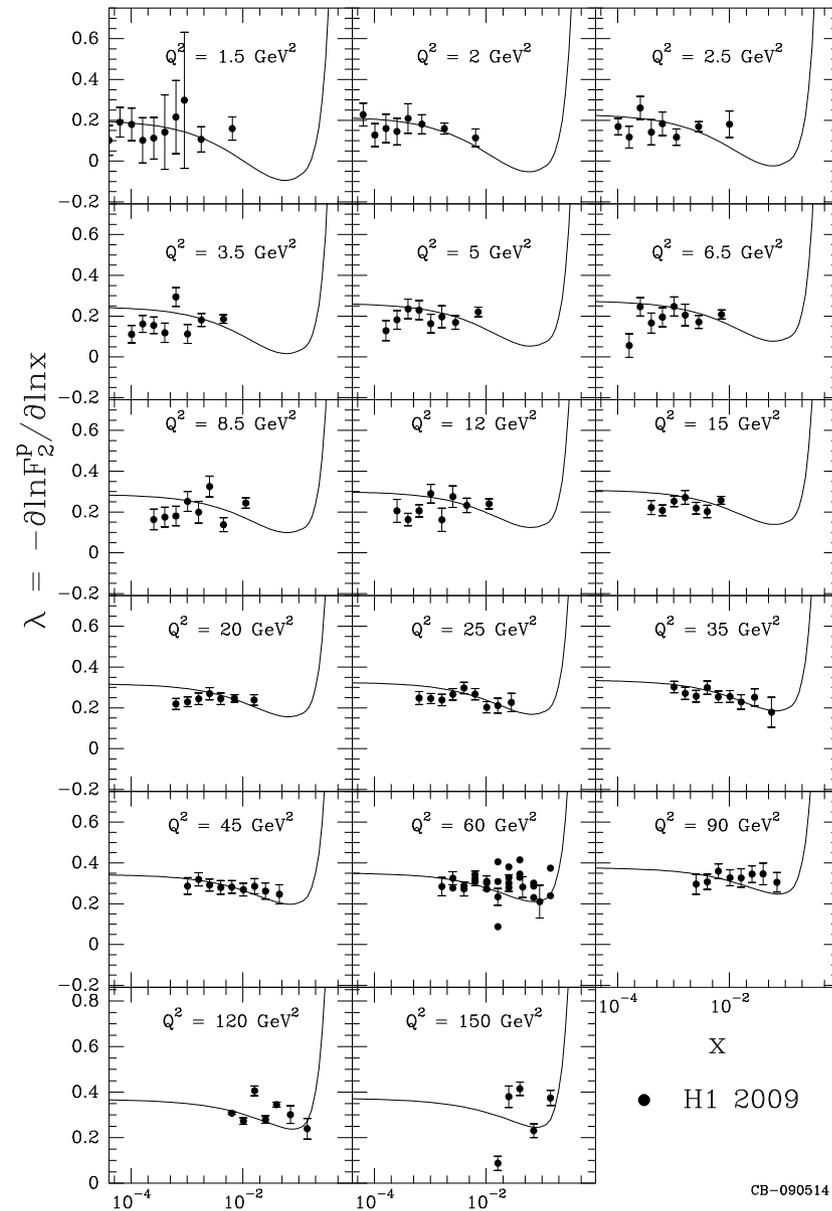
- \* Non-symmetric polarized sea quarks

# The derivative of $F_2^p$ with respect to $\ln Q^2$

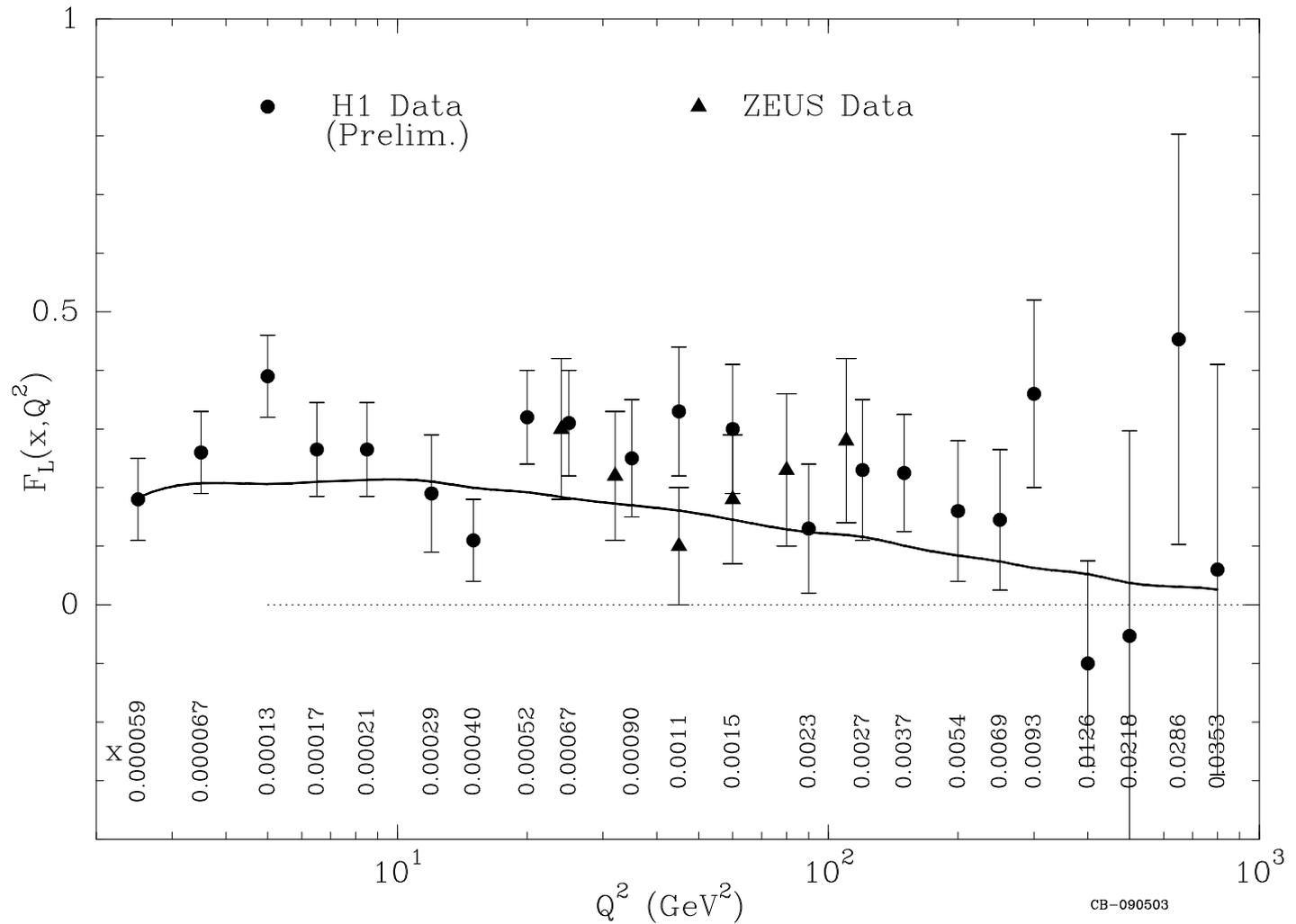
• H1 2009



# The derivative of $\ln F_2^p$ with respect to $\ln x$

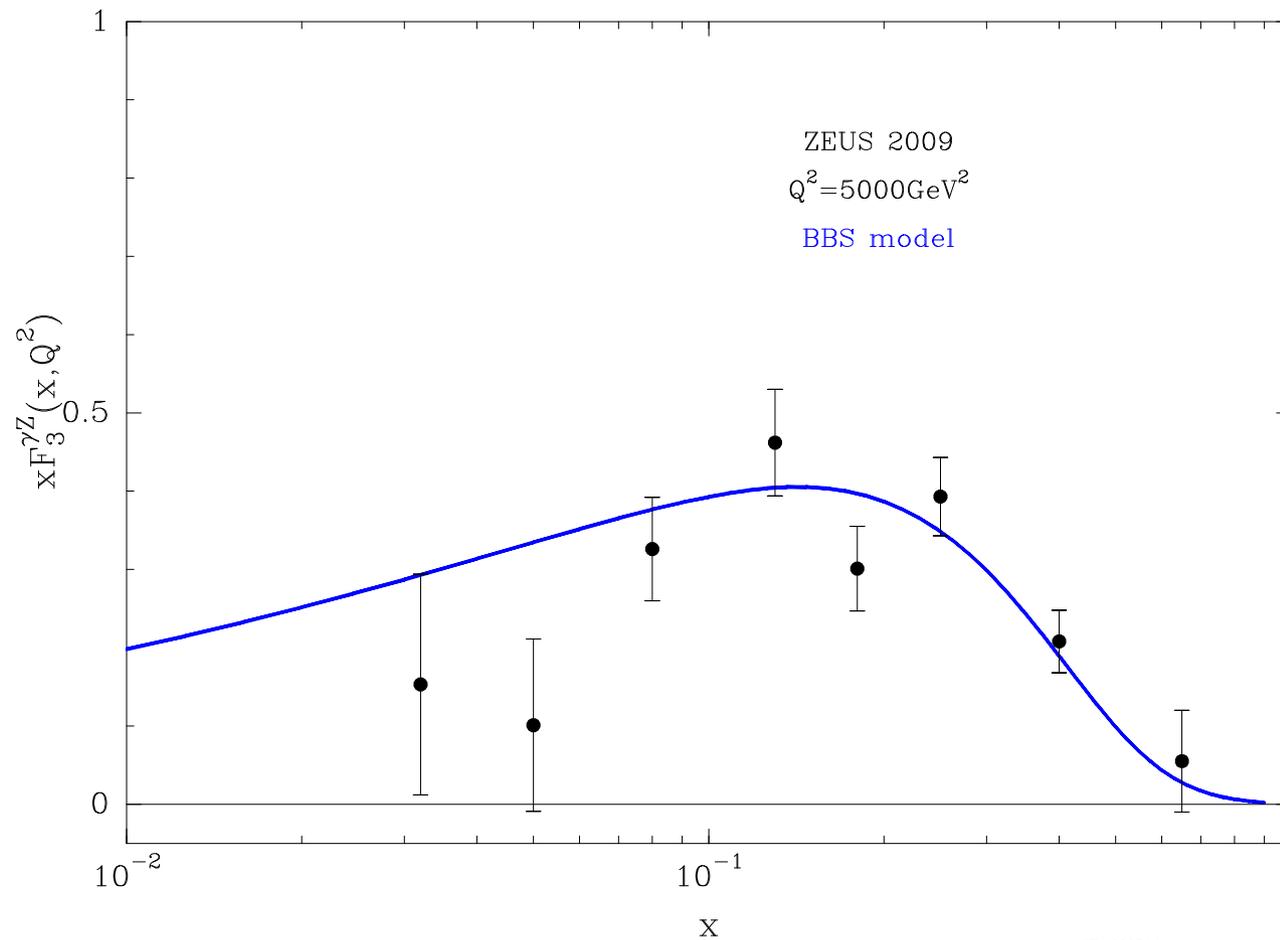


# The longitudinal structure function $F_L$



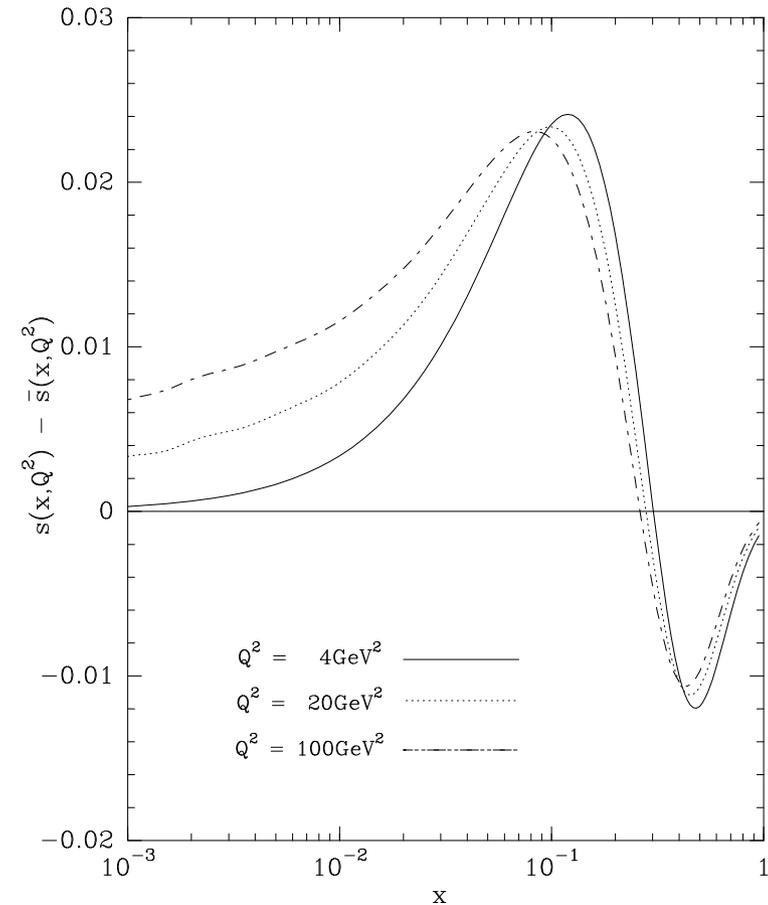
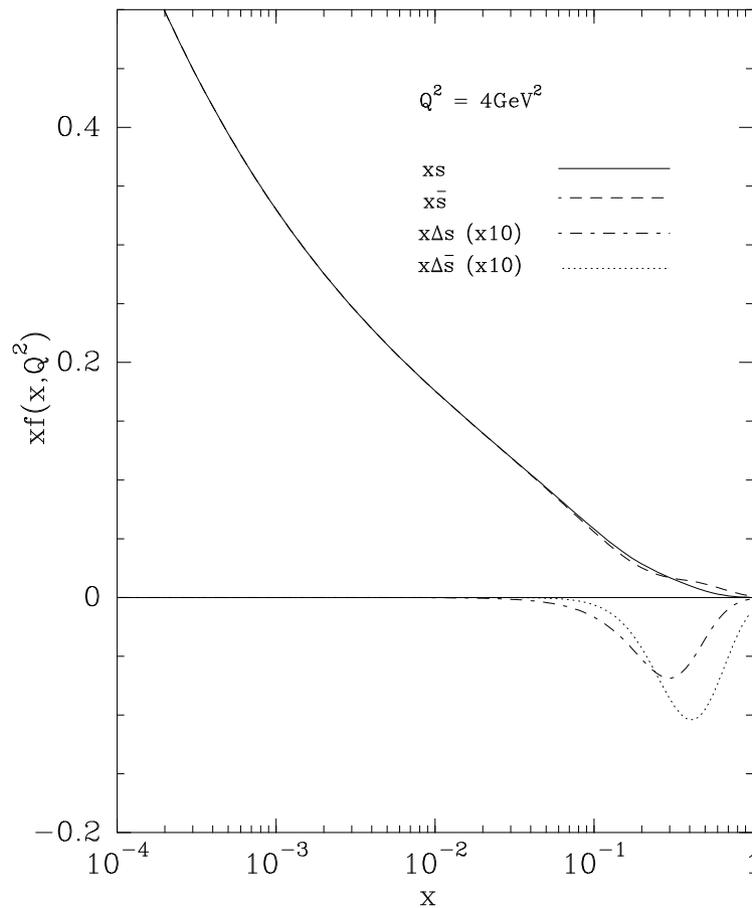
# The structure function $F_3^{\gamma Z}$

Interference term which can be isolated in neutral current  $e^\pm p$  collisions at high  $Q^2$   
We have to a good approximation  $x F_3^{\gamma Z} = \frac{x}{3}(2u_v + d_v)$



CB-090514

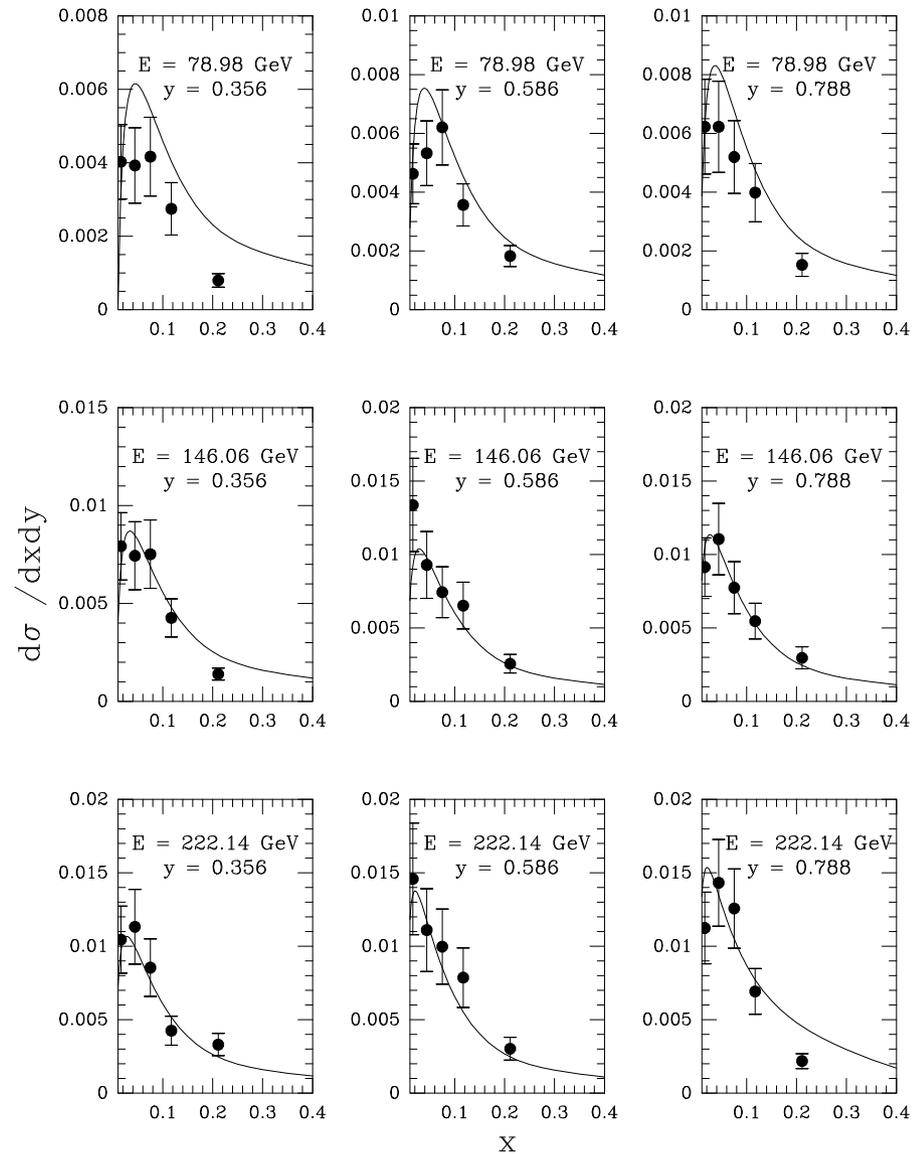
# The strange quark and antiquark distributions



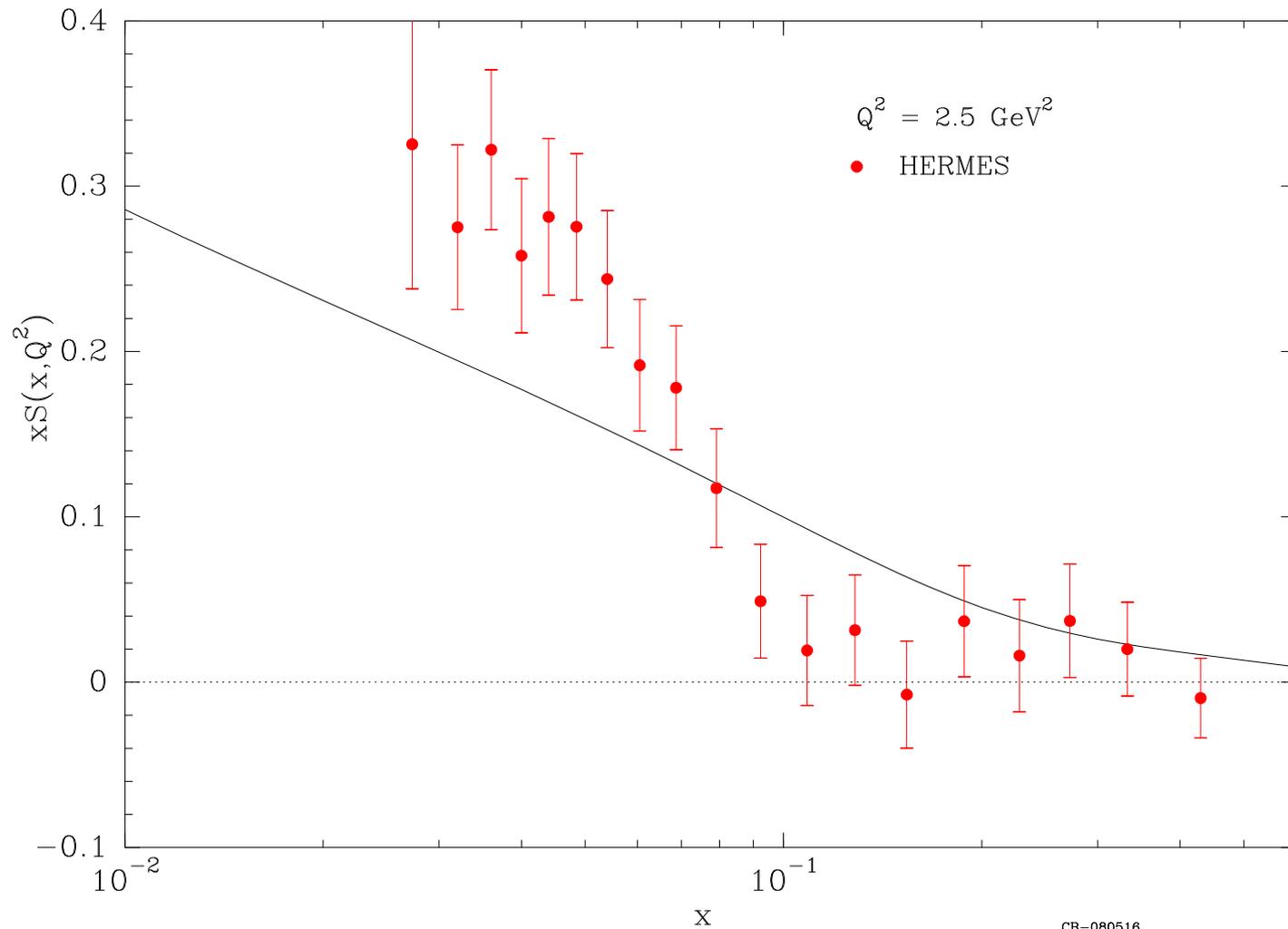
This requires four new parameters  $X_{0s}^{\pm}$ ,  $b_s$ ,  $\tilde{A}_s$  to fit the CCFR and NuTeV neutrino data for dimuon production

# The antineutrino NuTeV data

NUTEV  $\bar{\nu} Fe \rightarrow \mu \mu X$



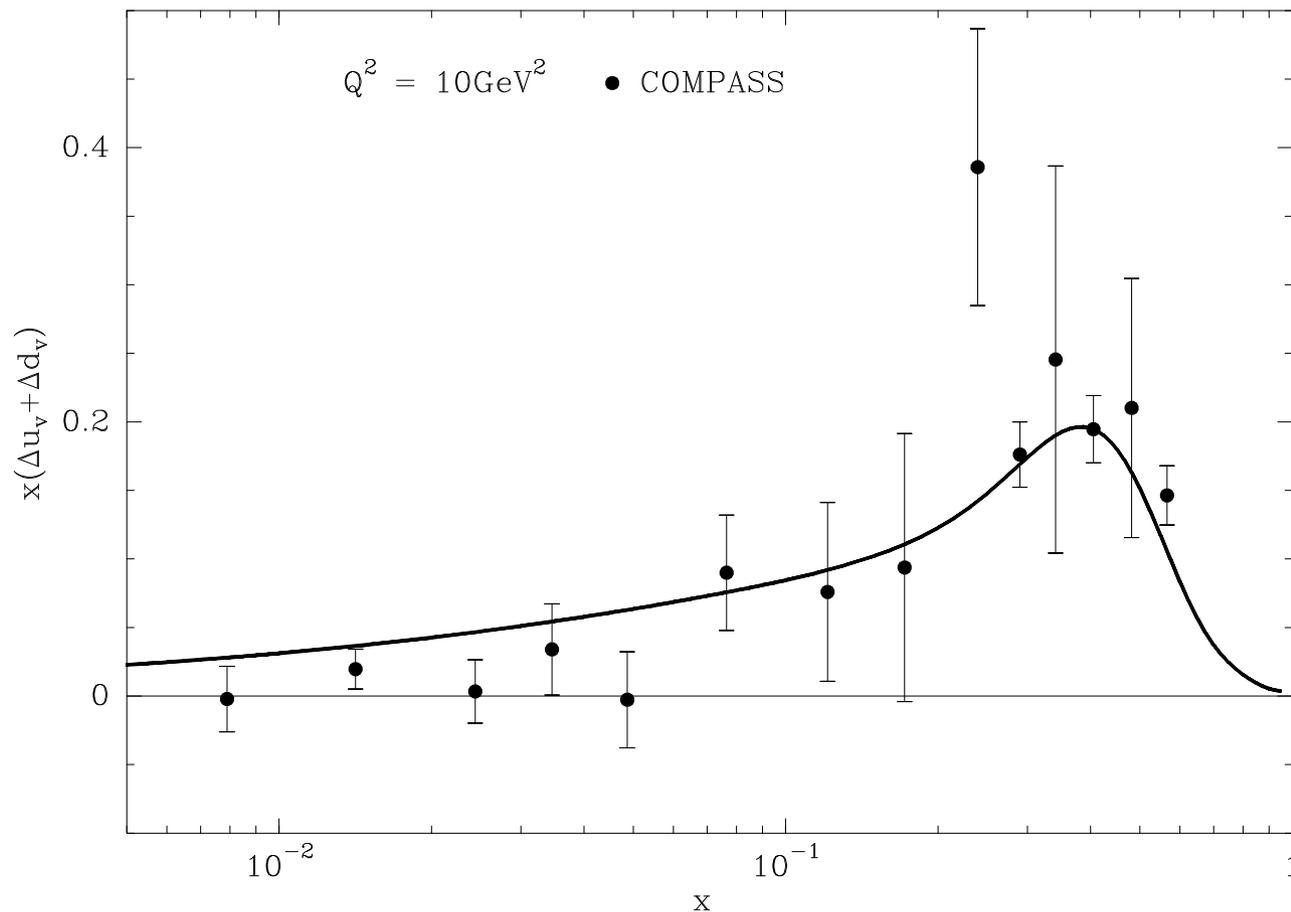
# The $xS(x) = xs(x) + x\bar{s}(x)$ distribution from Hermes



CB-080516

# The valence quark helicity distributions versus $x$

From semi-inclusive DIS  $\mu p \rightarrow \mu h^\pm X$  can determine the valence quark helicity distributions  
Combined with  $g_1^d$  it leads to  $\Delta\bar{u} + \Delta\bar{d} = 0.0 \pm 0.04.03$ , *i.e.* non-symmetric polarized sea



## Conclusions

- A new set of PDF is constructed in the framework of a statistical approach of the nucleon
- All **unpolarized and polarized** distributions depend upon **nine** free parameters, with some physical meaning.
- New tests against experimental (unpolarized and polarized) data on DIS, Semi-inclusive DIS and hadronic processes are very satisfactory
- Special features and good predictive power

**Not discussed here:**

\* The extension to the  $k_T$  dependence of the parton distributions

For practical use of our PDF see [www.cpt.univ-mrs.fr/bourelly/research/bbs-dir/bbs.html](http://www.cpt.univ-mrs.fr/bourelly/research/bbs-dir/bbs.html)