The Story Behind a Half

from the DSSV perspective

Marco Stratmann

U Regensburg/U Wurzburg

fundamental questions driving spin physics

- *explore and understand QCD:* dynamics, models, lattice, …
- *test our ability to use QCD:* asymptotic freedom, pert. methods, factorization, …

this talk

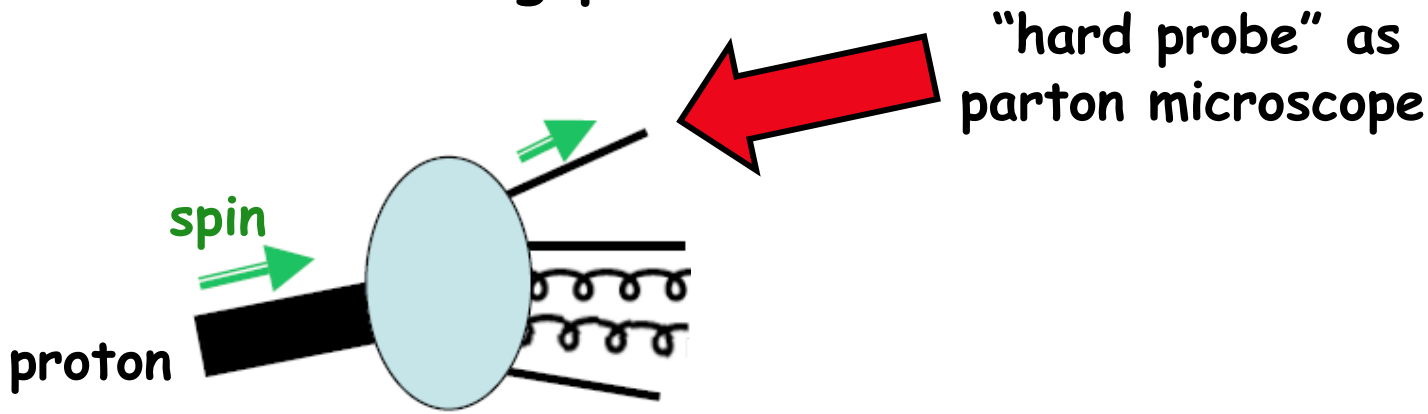
- *how do quarks and gluons carry the proton spin:* helicity PDFs, spin sum rule, …
- *understand transverse spin phenomena:* Sivers & Collins effect; TMDs, gauge links, …
- *what is the 3-D structure of the nucleon:* GPDs, …

map out the nucleon

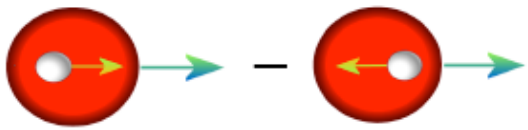
its complete spin, flavor, and gluon "landscape"

line of attack

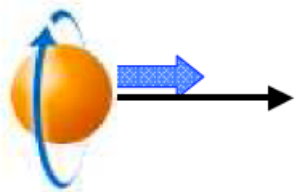
(1) study polarized scattering processes



(2) extract nucleon helicity structure

$\Delta f \equiv$  $\langle P, \frac{1}{2} | \mathcal{O}_{q,\bar{q},g} | P, \frac{1}{2} \rangle$

(3) learn about proton spin in terms of quarks and gluons



$$\frac{1}{2} = \langle P, \frac{1}{2} | \hat{J}_z | P, \frac{1}{2} \rangle$$

hard scattering goes roughly like this …

not possible without tremendous experimental efforts in the past 20+ years … matched by theoretical progress

SLAC

E142, E143, E154, E155

CERN

EMC, SMC, COMPASS

DESY

HERMES

JLab

Hall A, CLAS

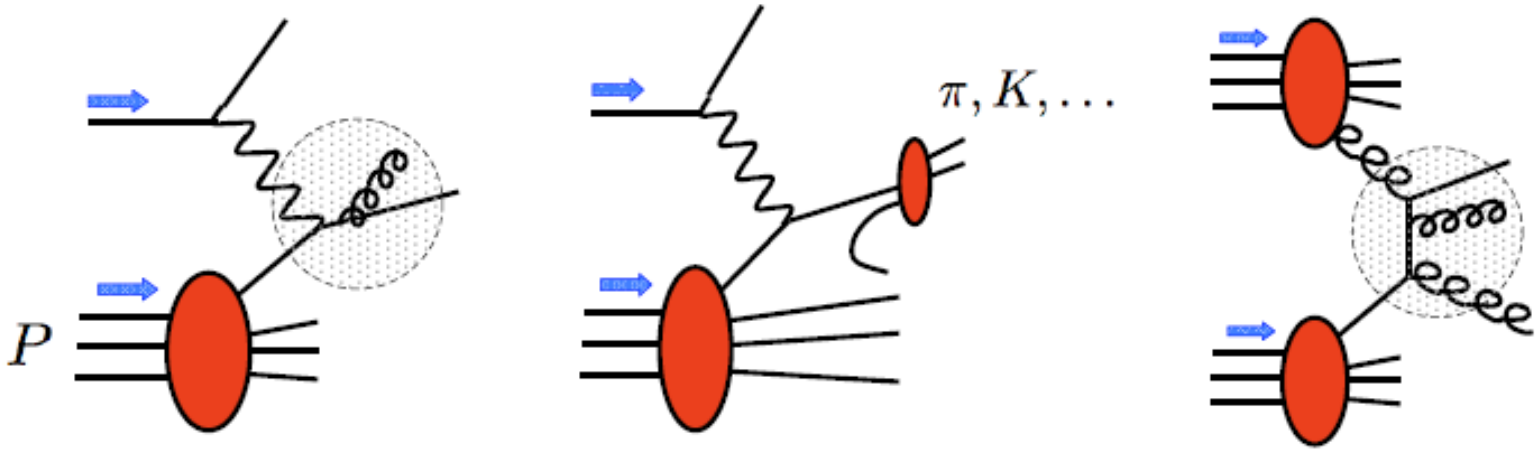
BNL

PHENIX, STAR

1st pp collisions at 500 GeV recently

How to determine PDFs from data?

information on nucleon (spin) structure available from



DIS **SIDIS** **hadron-hadron**

task: extract reliable pdfs not just compare some curves to data

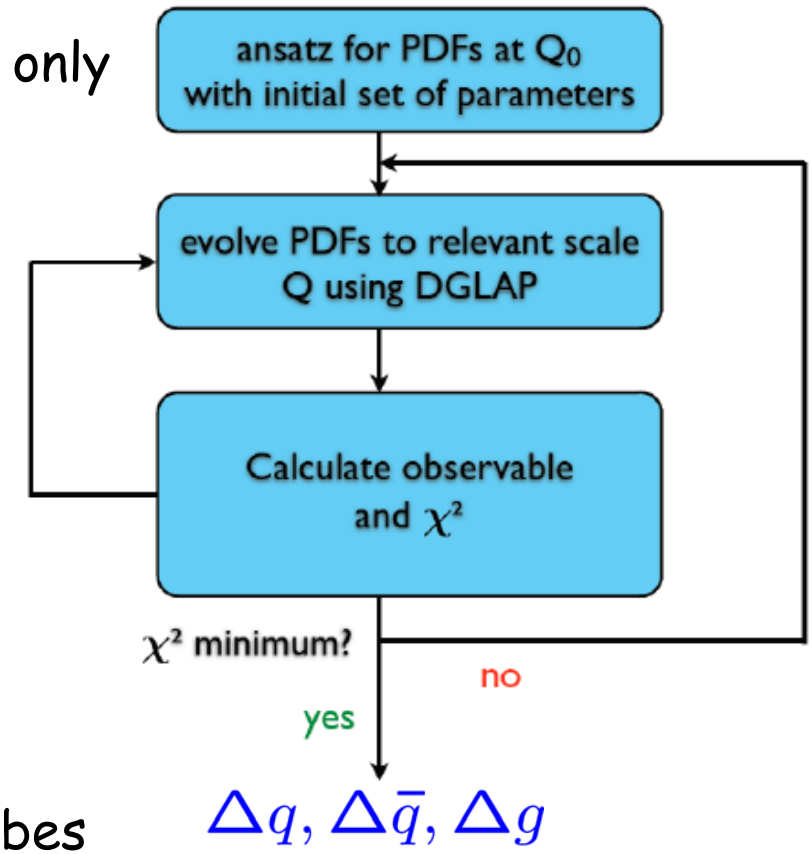
- all processes tied together: universality of pdfs & Q^2 - evolution
- each reaction provides insights into *different* aspects and kinematics
- need at least NLO for quantitative analyses; PDFs are not observables!
- information on PDFs "hidden" inside complicated (multi-)convolutions

→ a **"global QCD analysis"** is required

QCD analyses of helicity parton densities

long history of NLO χ^2 fits to DIS data only

GRSV Gluck, Reya, MS, Vogelsang
GS Gehrmann, Stirling
ABFR Altarelli, Ball, Forte, Ridolfi
BB Blumlein, Bottcher
BBS Bourrely, Buccella, Soffer
LSS Leader, Sidorov, Stamenov
AAC Hirai, Kumano, Saito
DNS de Florian, Navarro, Sassot
...

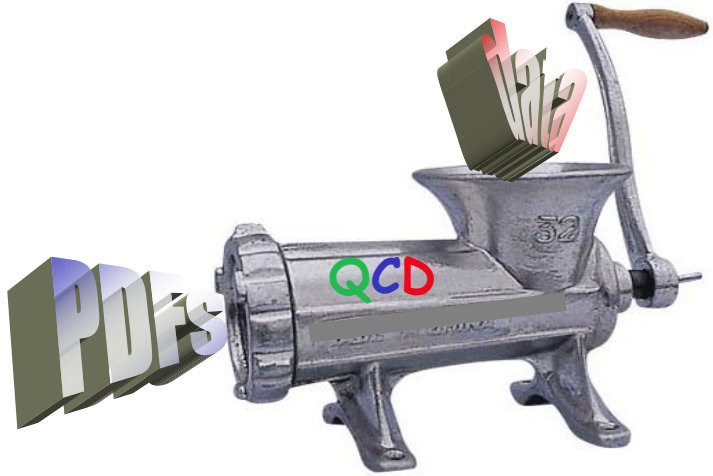


not really sufficient in view of other probes

recently: 1st global NLO analysis based on all probes: DIS, SIDIS, RHIC pp

DSSV de Florian, Sassot, MS, Vogelsang

PRL 101 (2008) 072001; arXiv:0904.3821 [hep-ph]



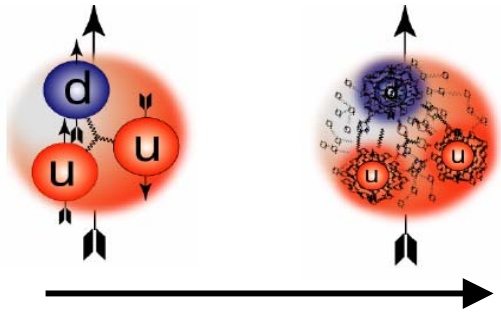
the charge:

analyze a large body of data
from many experiments on different processes
with diverse characteristics and errors
within a theoretical model with many parameters
and hard to quantify uncertainties
without knowing the optimum "ansatz" a priori

QCD toolbox

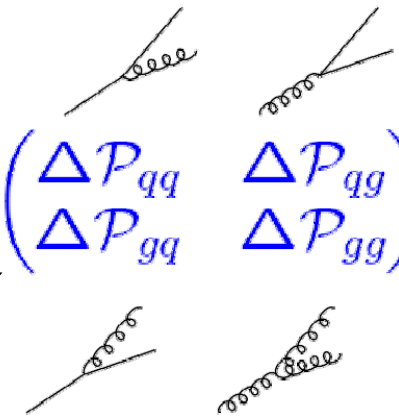
- **scale (=DGLAP) evolution**

more and more parton-parton splittings
resolved as the "resolution" scale μ increases



"resolution scale" μ
$r \sim 1/\mu$

key prediction of pQCD



$$\mu\frac{d}{d\mu}\begin{pmatrix}\Delta q(x,\mu)\\ \Delta g(x,\mu)\end{pmatrix}=\int_x^1\frac{dz}{z}\begin{pmatrix}\Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg}\\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg}\end{pmatrix}_{(z,\alpha_s(\mu))}\cdot\begin{pmatrix}\Delta q\\ \Delta g\end{pmatrix}\left(\frac{x}{z},\mu\right)$$

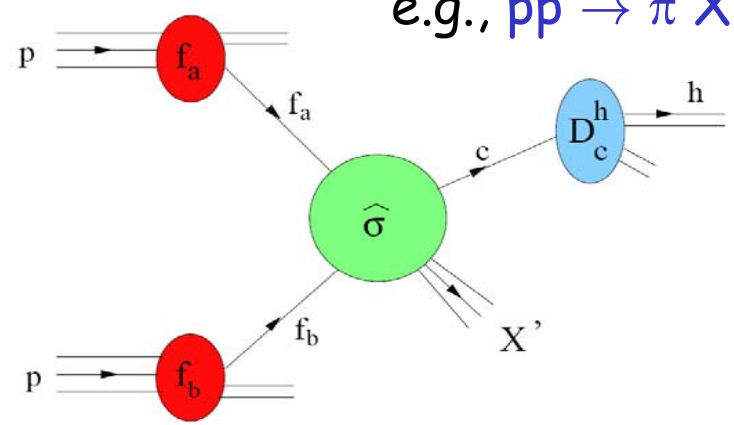
"splitting kernels" known to next-to-leading order (NLO) Mertig, van Neerven; Vogelsang

NNLO results already on the horizon Moch, Vermaseren, Vogt
(crucial for future precision studies)

- **factorization**

allows to separate universal PDFs from calculable but process-dependent hard scatterring cross sections

e.g., pp → π X



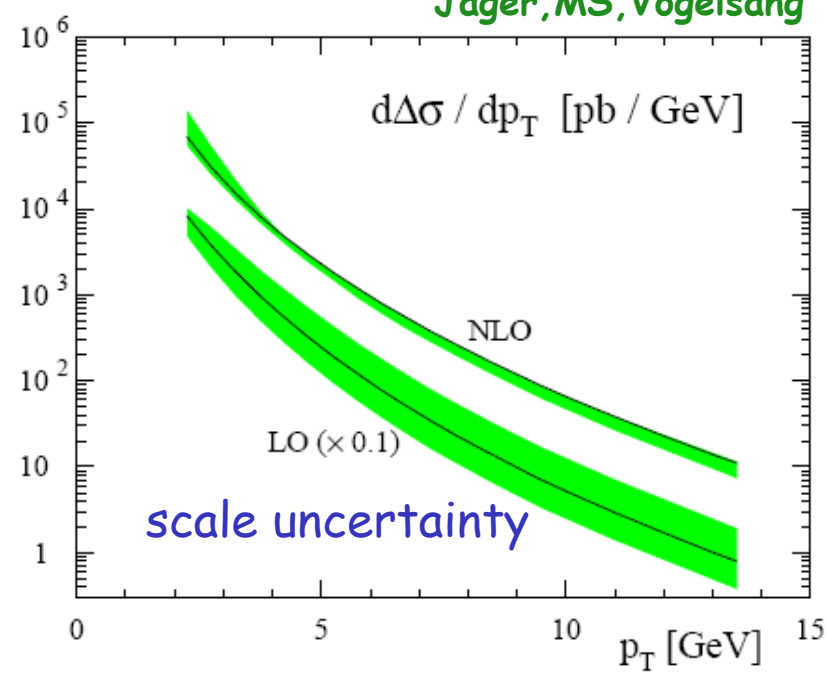
- **higher order QCD corrections**

essential to estimate/control theoretical uncertainties

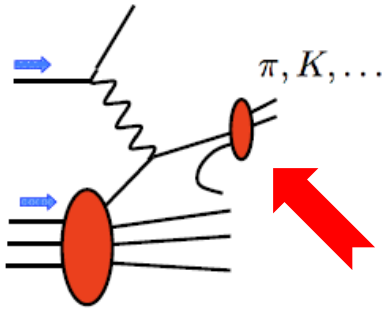
much closer to experiment (jets,...)

most relevant observables available at NLO accuracy

Jäger, MS, Vogelsang



interlude: fragmentation functions

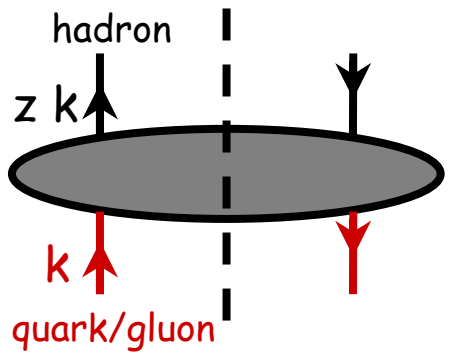


crucial for pQCD interpretation (factorization!) of data with detected hadrons, e.g., SIDIS (HERMES, COMPASS), $pp \to \pi X$ (PHENIX, ...)

observation: FFs based only on e^+e^- (LEP) data do a bad job here

some properties of $D_i^h(z,\mu)$ [very similar to PDFs]:

- non-perturbative but universal; pQCD predicts μ-dep.
- describe the *collinear* transition of a parton "i" into a massless hadron "h" carrying fractional momentum z



recent progress: global QCD analysis of $\pi^\pm$, $K^\pm$ yields in e^+e^- and ep, pp

DSS fit (de Florian, Sassot, MS) **Phys. Rev. D75 (2007) 114010; D76 (2007) 074033**

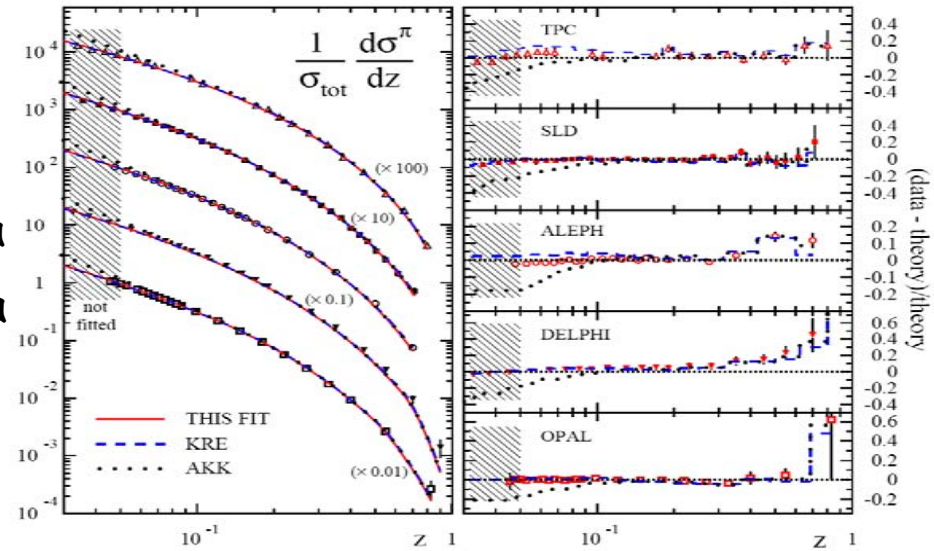
fits all relevant data; crucial impact on extraction of helicity PDFs

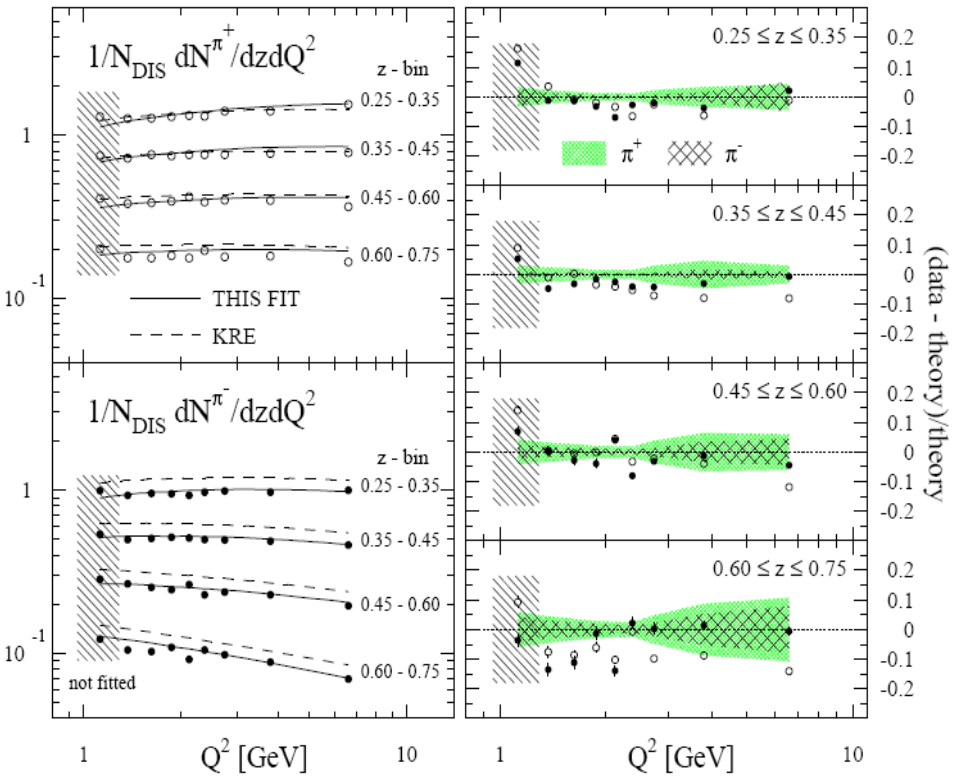
DSS: good global fit of all e^+e^-, ep, and pp hadron data

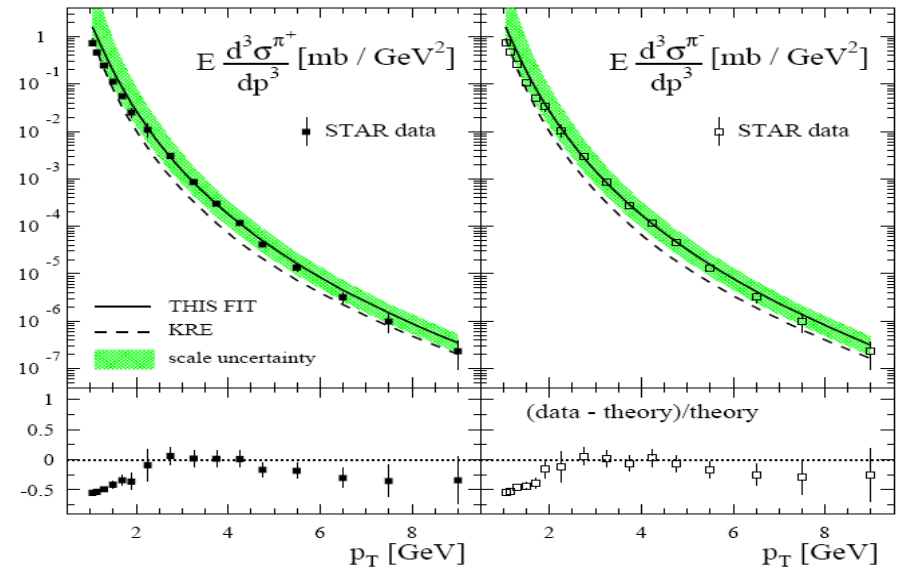
main results:

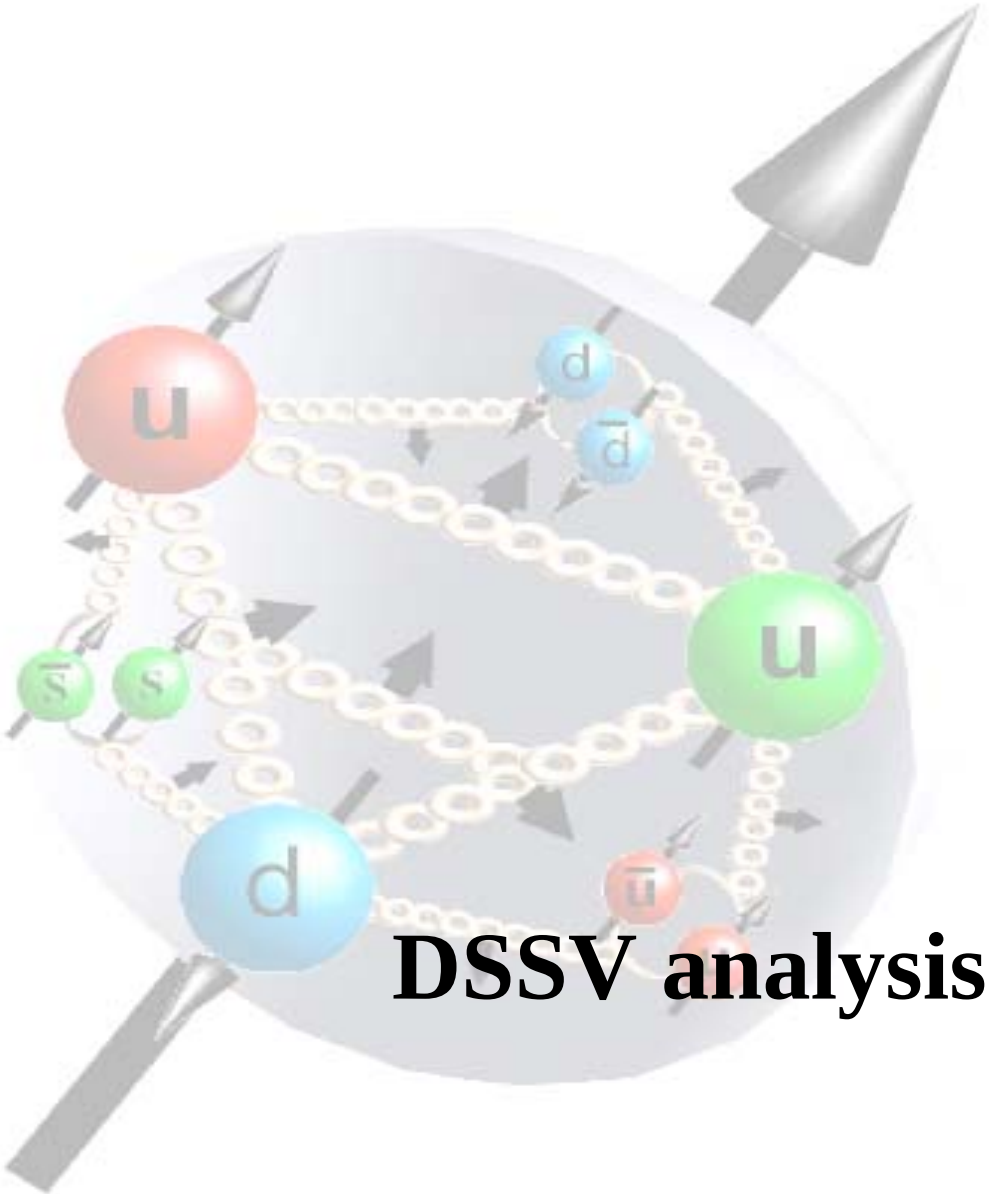
de Florian, Sassot, MS

- results for $\pi^\pm$, $K^\pm$, chg. hadrons
- full flavor separation for $D_i^H(z)$ and D_g^H
- uncertainties (L.M.) well under control
- fits all LEP, HERMES, SMC, RHIC, … data
- supersede old fits based only on e^+e^- data









DSSV analysis in detail

setup of DSSV analysis

- flexible, MRST-like input form

possible nodes

$$x\Delta f_j(x, 1\,\text{GeV}) = N_j\, x^{\alpha_j}\,(1-x)^{\beta_j}\left[1 + \kappa_j\sqrt{x} + \gamma_j x\right]$$

input scale

simplified form for sea quarks and Δg: $\kappa_j = 0$

- take α_s from MRST; also use MRST for positivity bounds
- NLO fit, $\overline{\text{MS}}$ scheme
- avoid assumptions on parameters $\{a_j\}$ unless data cannot discriminate

need to impose: $\alpha_{\bar{u}} = \alpha_{u+\bar{u}}$ $\qquad \alpha_{\bar{d}} = \alpha_{\bar{s}} = \alpha_{d+\bar{d}}$

let the fit decide about F,D value constraint on 1st moments:

$$\Delta u_{\text{tot}} - \Delta d_{\text{tot}} = (F+D)[1+\varepsilon_{SU(2)}]$$

1.269±0.003

fitted (end up close to zero)

$$\Delta u_{\text{tot}} + \Delta d_{\text{tot}} - 2\Delta s_{\text{tot}} = (3F-D)[1+\varepsilon_{SU(3)}]$$

0.586±0.031

data "selection"

initial step: verify that the theoretical framework is adequate !

→ use only data where unpolarized results agree with NLO pQCD

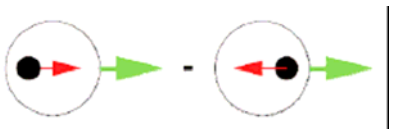
DSSV global analysis uses all three sources of data:

	experiment	data type	data point fitted
"classic" inclusive DIS data routinely used in PDF fits → $\Delta q + \Delta\bar{q}$	EMC, SMC	DIS	34
	COMPASS	DIS	15
	E142, E143, E154, E155	DIS	123
	HERMES	DIS	39
	HALL-A	DIS	3
	CLAS	DIS	20
semi-inclusive DIS data so far only used in DNS fit → flavor separation	SMC	SIDIS, $h^\pm$	48
	HERMES	SIDIS, $h^\pm$	54
		SIDIS, $\pi^\pm$	36
		SIDIS, $K^\pm$	27
	COMPASS	SIDIS, $h^\pm$	24
first RHIC pp data (never used before) → Δg	PHENIX (in part prel.)	200 GeV pp, π^0	20
	PHENIX (prel.)	62 GeV pp, π^0	5
	STAR (in part prel.)	200 GeV pp, jet	19
	TOTAL:		467

467 data pts in total (≈10% from RHIC)

How can we use all this in a global PDF fit?

several crucial differences w.r.t. an unpolarized fit:

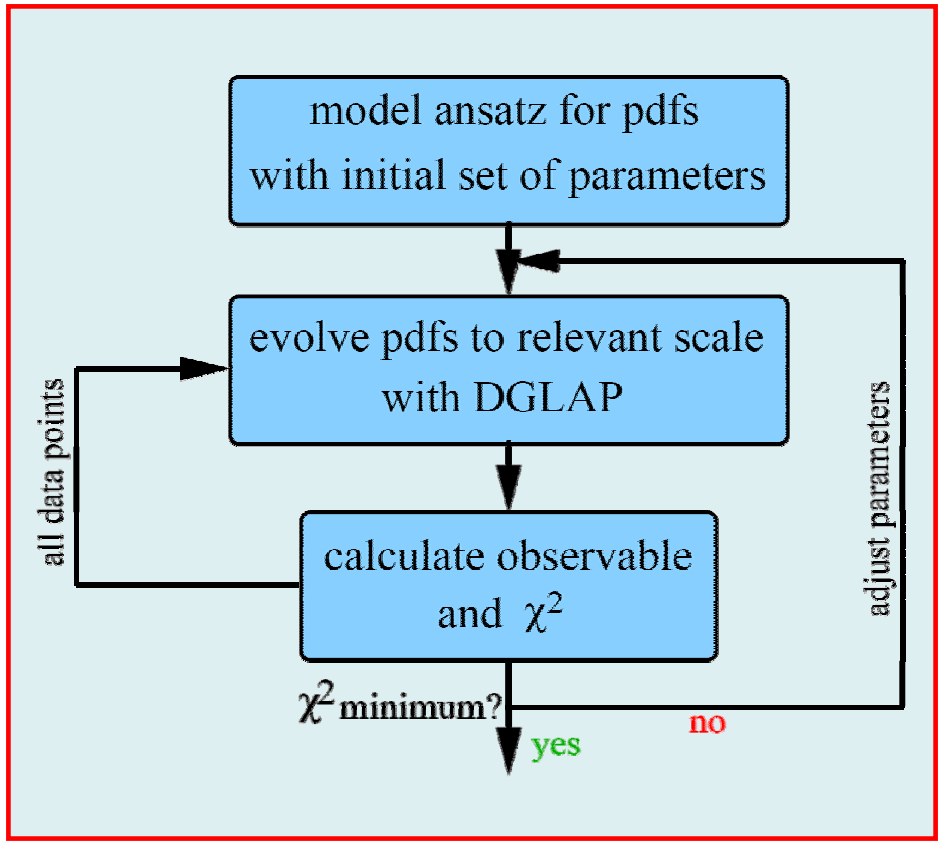
- **no sum rule** which relates quarks and gluons (unpolarized: momentum sum)
-  $\Delta f(x,\mu)$ not restricted to be positive; **nodes** possible

  "positivity bound" $|\Delta f(x,\mu)| \leq f(x,\mu)$ of limited use (valid only at LO !)
- **much less data**:
  - DIS in limited x, Q^2 range $\rightarrow$ much less constrained gluon
  - no νN-DIS data $\rightarrow$ flavor separation relies on SIDIS data
    possible uncertainties from fragmentation
  - pp data have to constrain Δg
    (more complicated to analyze than DIS scaling violations)

fit procedure

$$\chi^2 = \sum_i \frac{(T_i - E_i)^2}{\delta E_i^2}$$

467 data pts



change O(20) parameters $\{a_j\}$ about 5000 times

another 50000+ calls for studies of uncertainties

bottleneck !

computing time for a global analysis at NLO becomes excessive

problem: NLO expression for pp observables are very complicated

→ problem can be solved with the help of 19th century math

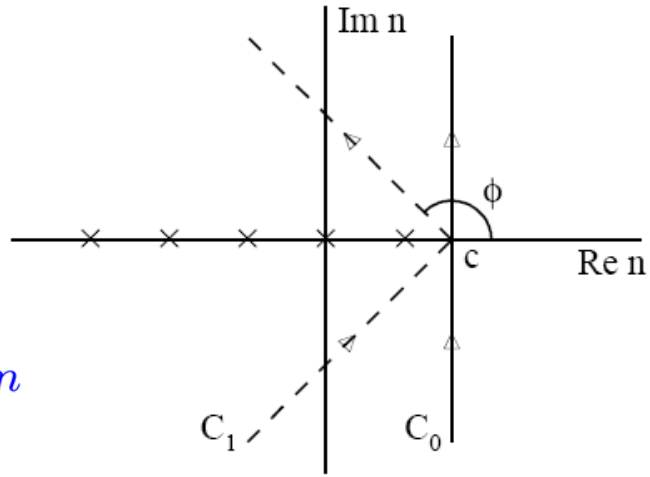
R.H. Mellin
Finnish mathematician

idea: take **Mellin n-moments**

$$h^n \equiv \int_0^1 dx\, x^{n-1}\, h(x)$$

inverse

$$h(x) \equiv \frac{1}{2\pi i} \int_{C_n} dn\, x^{-n}\, h^n$$



well-known property: convolutions factorize into simple products

- ✓ analytic solution of DGLAP evolution equations for moments
- ✓ analytic expressions for DIS and SIDIS coefficient functions

… however, NLO expression for pp processes too complicated

MS, Vogelsang

earlier ideas: Berger, Graudenz, Hampel, Vogt; Kosower

here is how it works:

example: pp→ π X

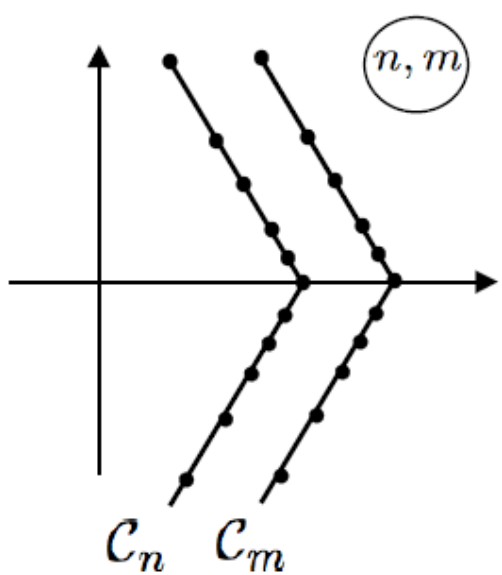
$$d\Delta\sigma = \sum_{abc} \int \Delta f_a \, \Delta f_b \, d\Delta\hat{\sigma}_{ab\to cX} \, D_c \, dx_a dx_b dz_c$$

express pdfs by their Mellin inverses

$$\frac{1}{2\pi i}\int_{\mathcal{C}_n} dn \, x_a^{-n} \, \Delta f_a^n \qquad \frac{1}{2\pi i}\int_{\mathcal{C}_m} dm \, x_b^{-m} \, \Delta f_b^m$$

$$= \frac{1}{(2\pi i)^2} \sum_{abc} \int_{\mathcal{C}_n} dn \int_{\mathcal{C}_m} dm \, \Delta f_a^n \, \Delta f_b^m \int x_a^{-n} x_b^{-m} \, d\Delta\hat{\sigma}_{ab\to cX} \, D_c \, dx_a dx_b dz_c$$

standard Mellin inverse	fit	completely indep. of pdfs pre-calculate prior to fit



discretize on **64 × 64 grid**

$$d\Delta\tilde{\sigma}_{ab\to cX}(n,m)$$

for fast Gaussian integration

applicability & performance

- **computing load**

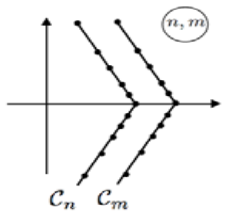
  O(10 sec)/data pt. $\rightarrow$ O(1 msec)/data pt.

  recall: need thousands of calls to perform a single fit !

- **obtaining the grids** $d\Delta\tilde{\sigma}_{ab\to cX}(n,m)$ **once prior to the fit**

  $$64 \times 64 \times 4 \times 10 \simeq O(10^5) \text{ calls per pp data pt.}$$

  n m n,m complex # subproc's

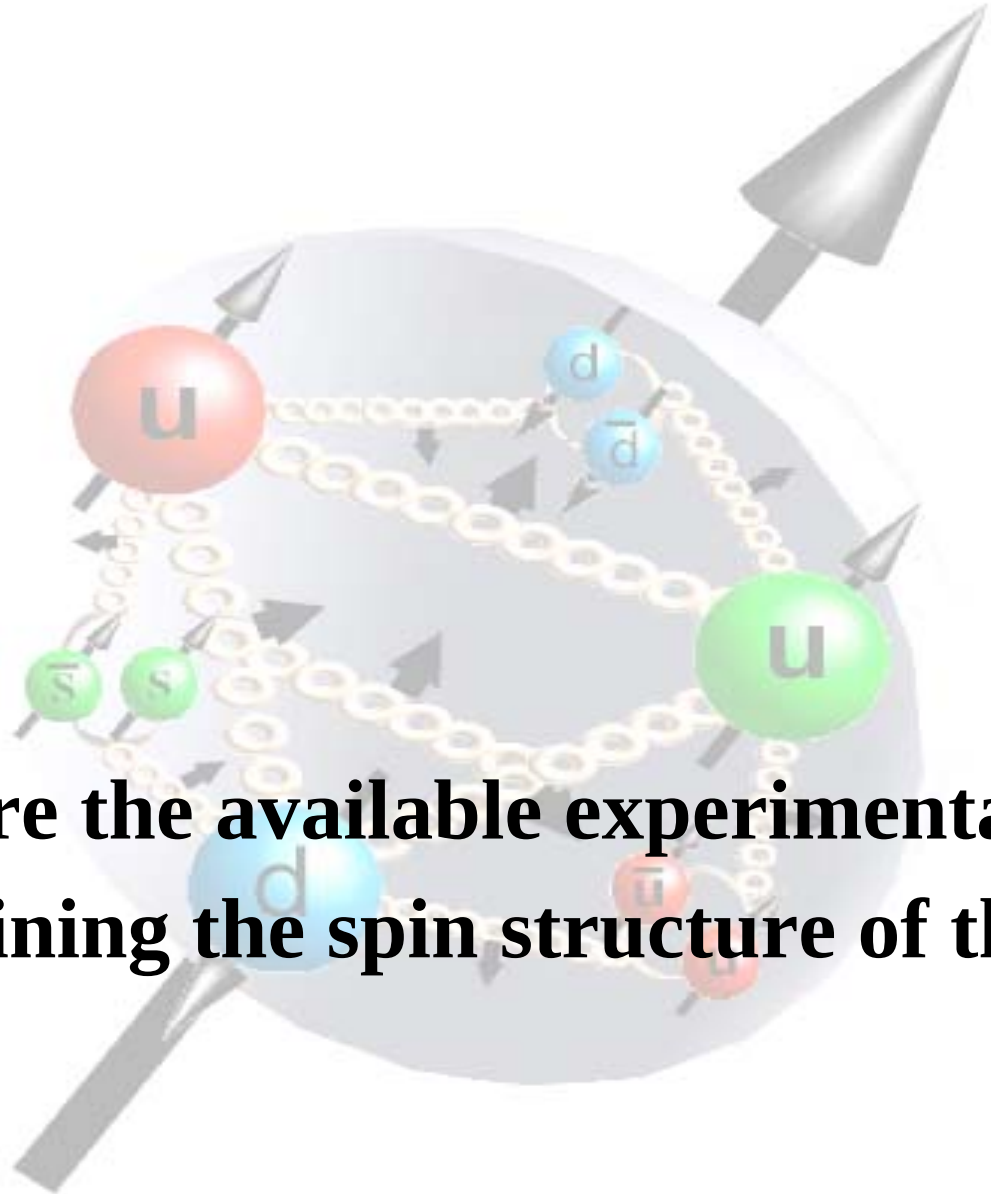


  production of grids much improved recently
  can be all done within a day with new MC sampling techniques

- **method completely general**

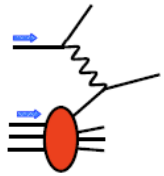
  tested for pp$\to\gamma$X, pp$\to\pi$X, pp$\to$jetX

  (much progress towards 2-jet and jet-π^0 production expected from STAR)

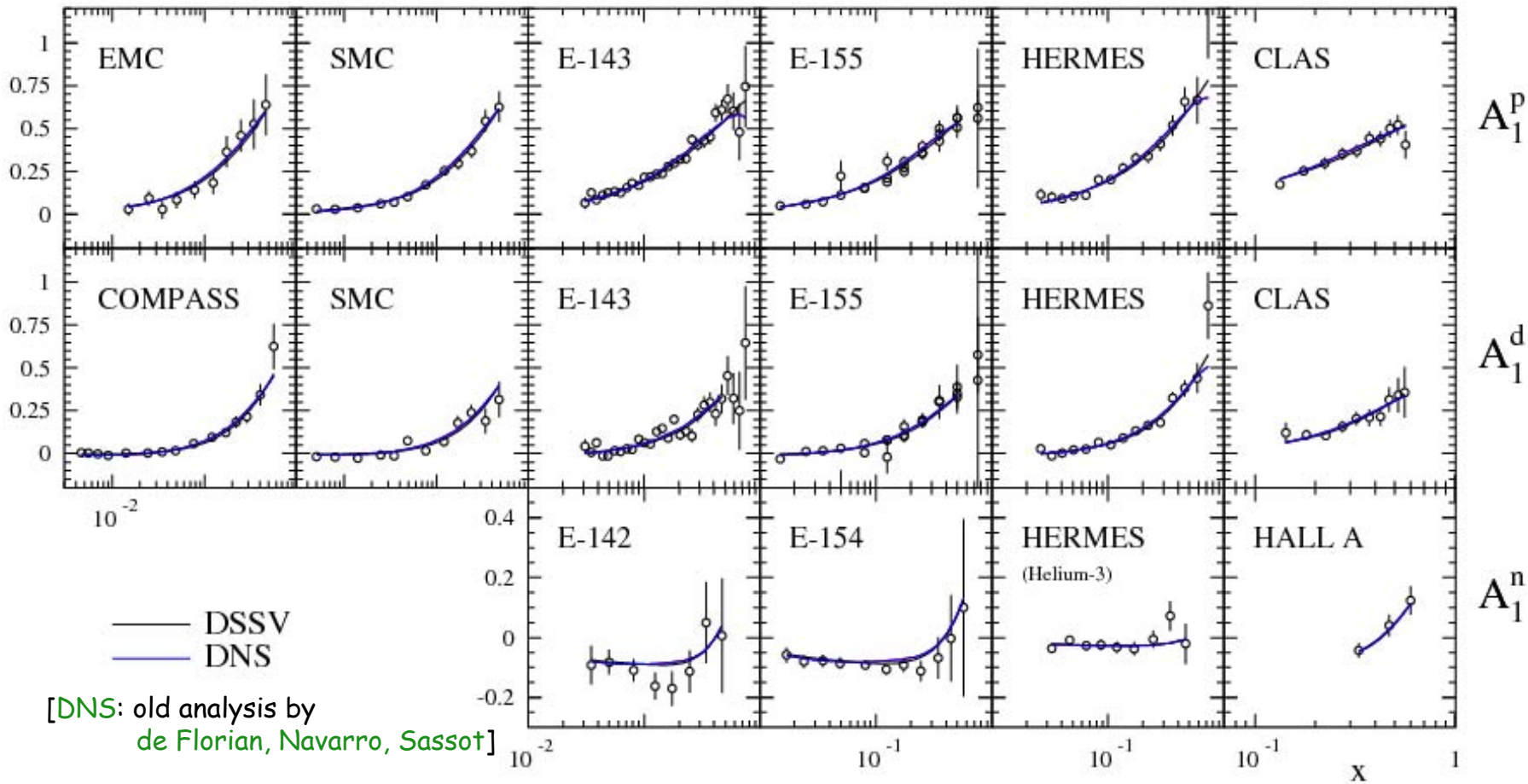


What are the available experimental probes constraining the spin structure of the nucleon ?

spin asymmetries in inclusive DIS



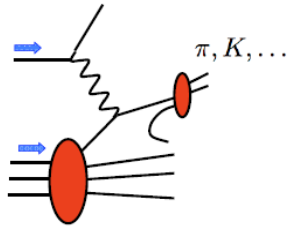
$$A_1 \approx \frac{g_1}{F_1}$$

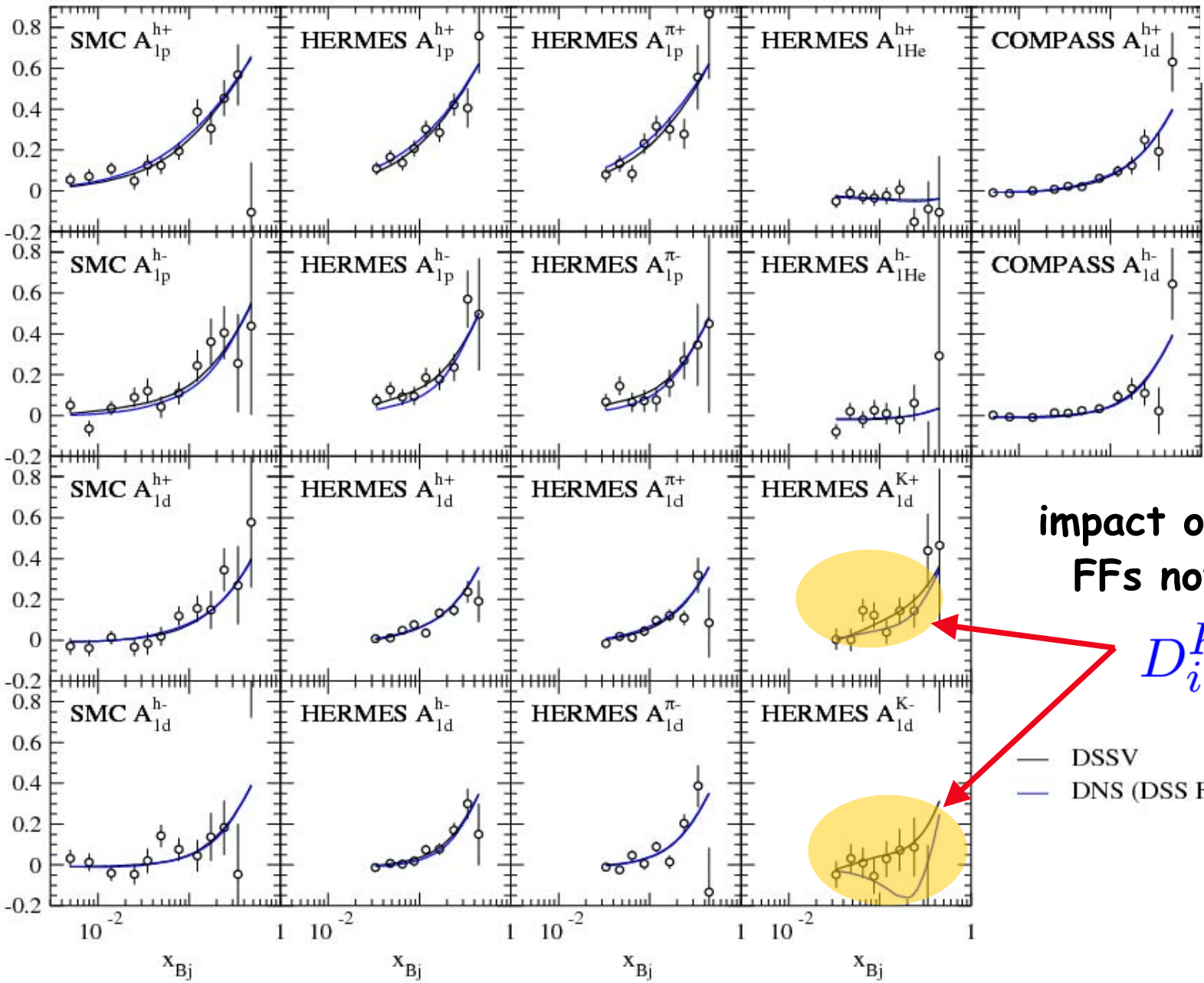


[DNS: old analysis by de Florian, Navarro, Sassot]

- DSSV finds no need for significant dynamical higher twist corrections to A_1 differs from LSS fit to g_1 → **what is the role of HT in A_1 and g_1 ?**
- Accardi, Melnitchouk: new approach to target mass correct.; largely cancel in A_1
  arXiv:0808.2397

spin asymmetries in semi-inclusive DIS

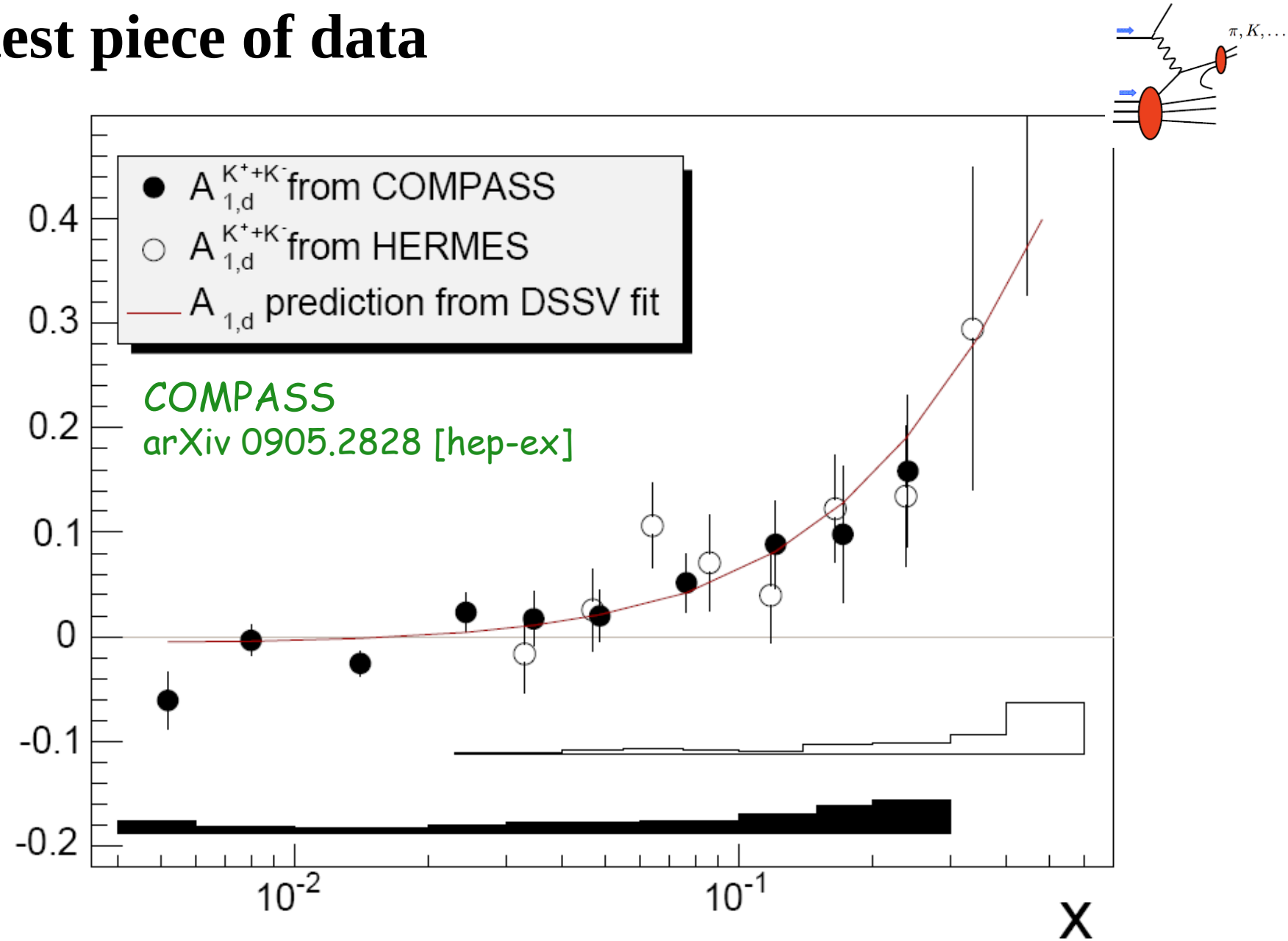




impact of new DSS FFs noticeable!

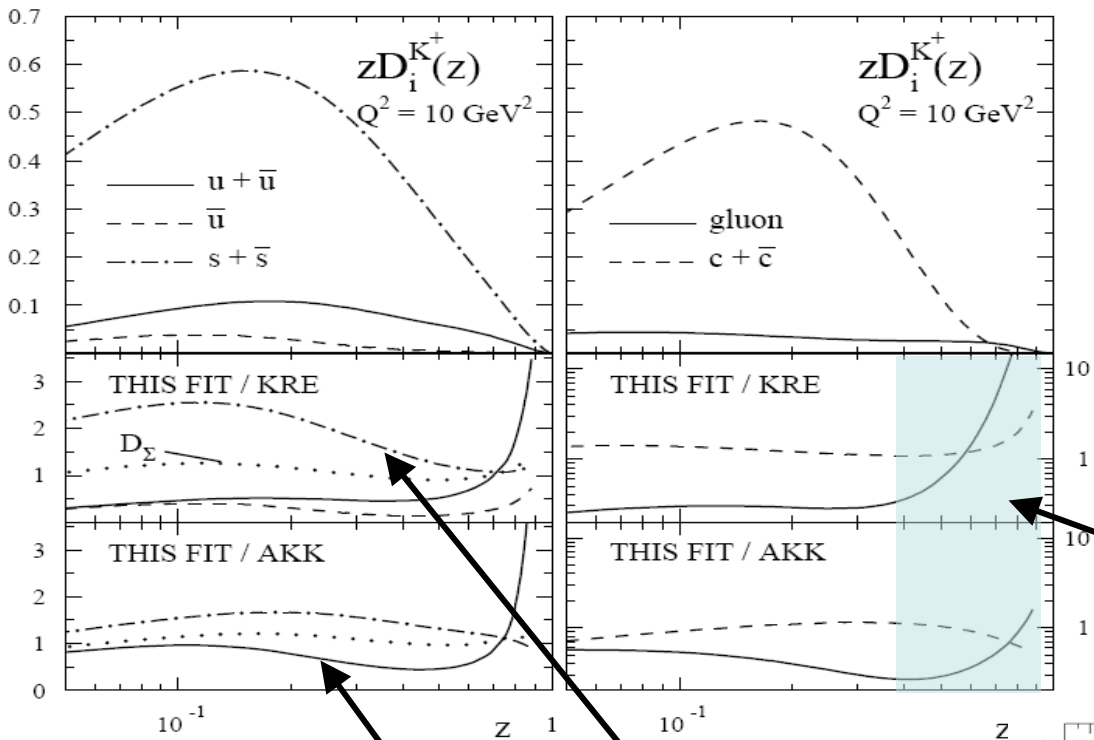
$D_i^{K^\pm}$

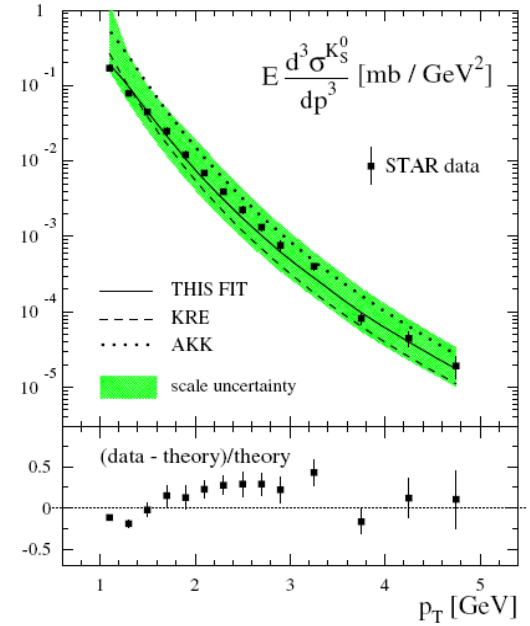
latest piece of data



compatible with HERMES; extended x-range; agrees with DSSV fit

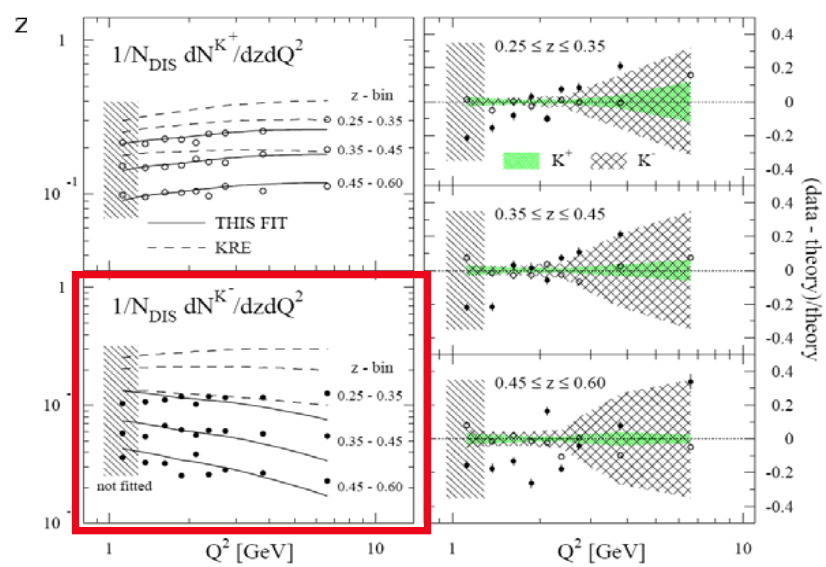
detour: DSS kaon FF's $D_i^K(z)$





RHIC pp data (BRAHMS, STAR) explain different D_g

smaller u & larger s-frag. required by SIDIS

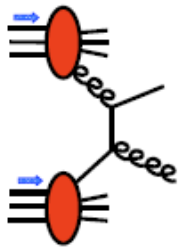


note: some issues with K^- data (slope!)

await eagerly final HERMES data and COMPASS multiplicities

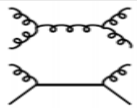
gluons are key players at RHIC

many QCD processes with a **dominant gluon contribution** already at the tree-level:

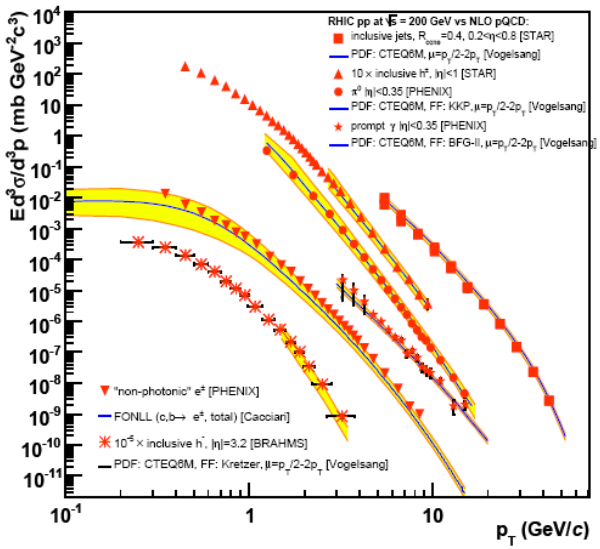


high-p_T jet, pion, heavy quark, …

all available at NLO

	Reaction	Dom. partonic process	probes	LO Feynman diagram	
hadrons	$\vec{p}\vec{p} \to \pi + X$ [61, 62]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg		Jäger, Schäfer, MS, Vogelsang; de Florian
jets	$\vec{p}\vec{p} \to \text{jet(s)} + X$ [71, 72]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	(as above)	Jäger, MS, Vogelsang; Signer et al.
photons	$\vec{p}\vec{p} \to \gamma + X$ $\vec{p}\vec{p} \to \gamma + \text{jet} + X$ $\vec{p}\vec{p} \to \gamma\gamma + X$ [67, 73, 74, 75, 76]	$\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{\bar{q}} \to \gamma\gamma$	Δg Δg $\Delta q, \Delta\bar{q}$		Gordon, Vogelsang; Contogouris et al.
heavy flavors	$\vec{p}\vec{p} \to DX, BX$ [77]	$\vec{g}\vec{g} \to c\bar{c}, b\bar{b}$	Δg		Bojak, MS; Riedl, Schäfer, MS

unpolarized "reference data"
(π, jets, γ, charm)
nicely agree with pQCD @ NLO

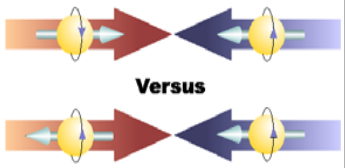


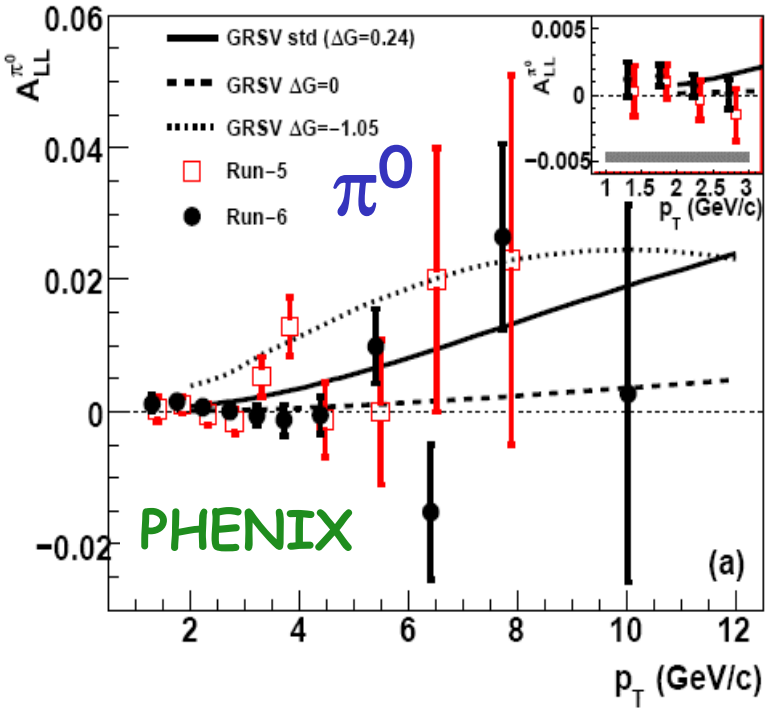
compilation by D.d'Enterria

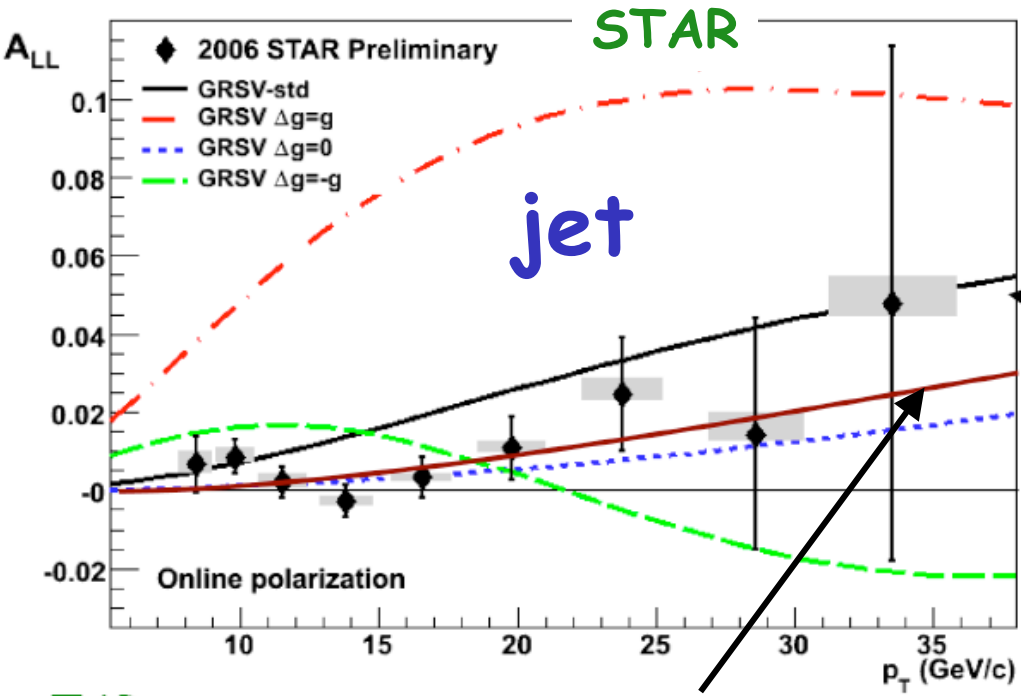
decisive data start to emerge from RHIC

spin asymmetry

A_{LL}







DSSV fit

future focus: particle correlations

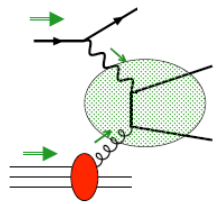
- more directly related to PDFs
- prel. jet-π^0 data (STAR) compare nicely with new NLO calculation using DSSV PDFs

de Florian, arXiv:0904.4402

Δg in lepton-proton scattering

gluons in DIS: a (small) NLO effect [they don't couple directly to the photon]

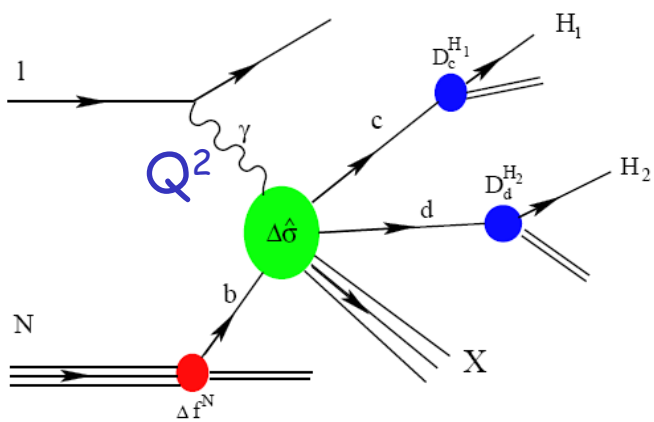
$\rightarrow$ study processes sensitive to **photon-gluon-fusion**



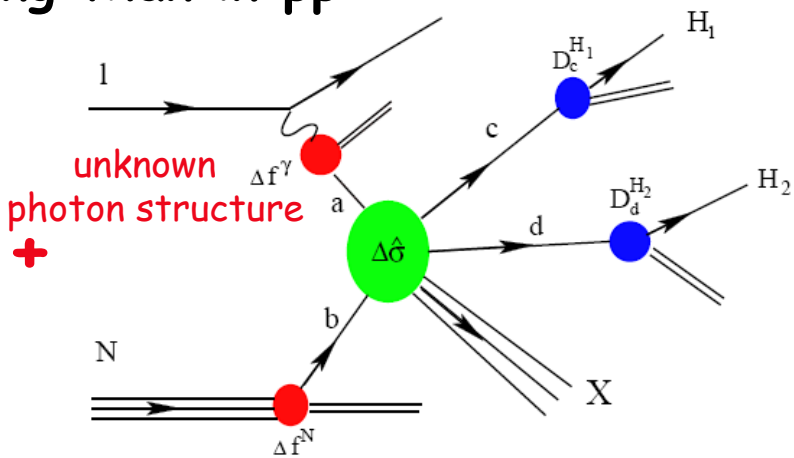
data available for one/two hadron production, charm

COMPASS, HERMES, SMC, E155

theory calculations more challenging than in pp:



Q^2 large: "electroproduction"



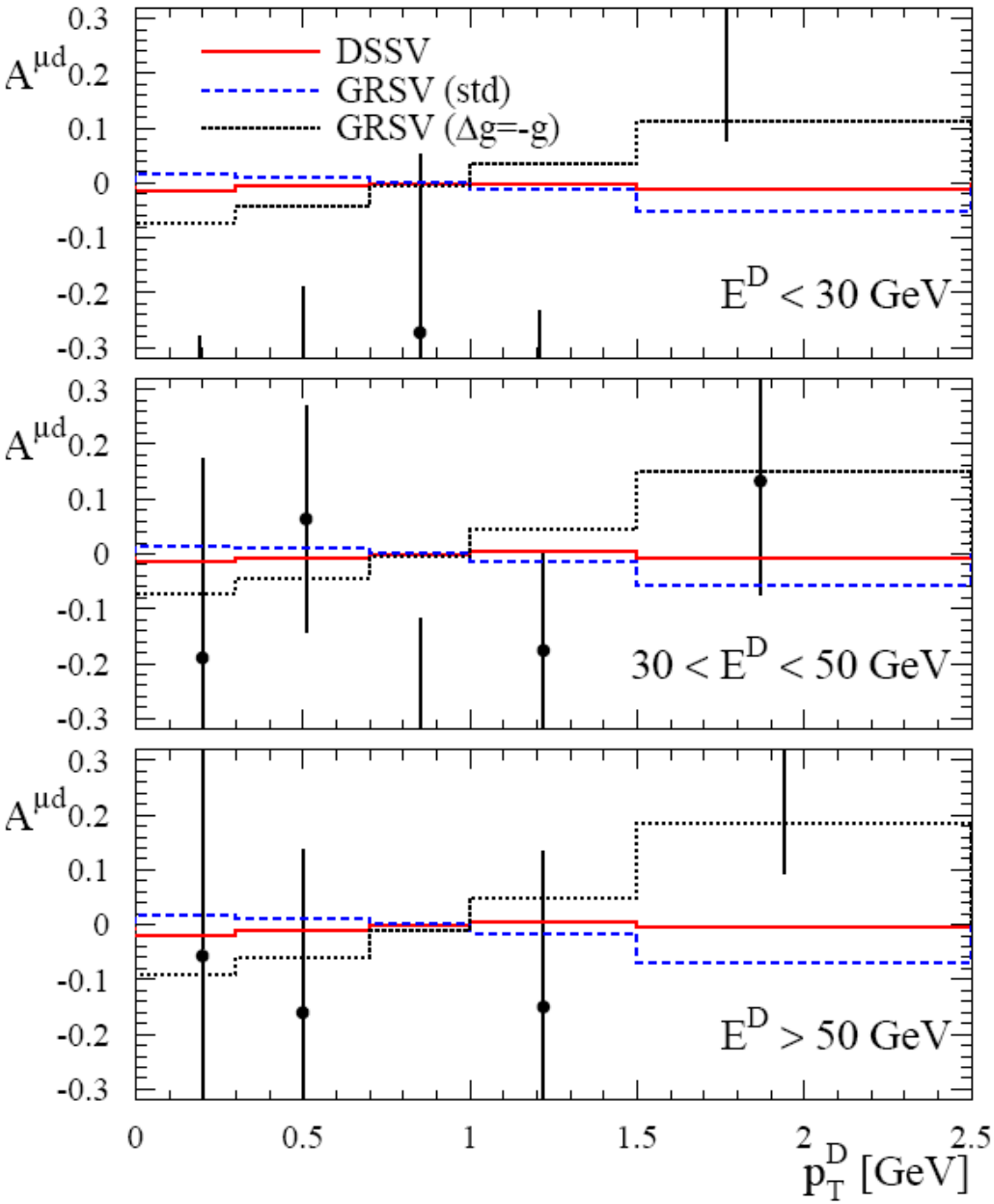
if $Q^2 \simeq 0$: "photoproduction"

NLO results just emerging: [but nothing available for $Q^2 \neq 0$]

Jäger,MS,Vogelsang; Bojak, MS; Riedl, Schäfer, MS; Hendlmeier, Schäfer, MS

sneak preview: charm production at NLO

Riedl, Schäfer, MS



flexible NLO MC code including direct+resolved, $c \to D$, HQ correlations, …

preliminary NLO results:

only direct γ contribution
resolved 10% at most

NLO corrections sizable for spin asymmetry (gets smaller!)

some tension with other data constraining Δg

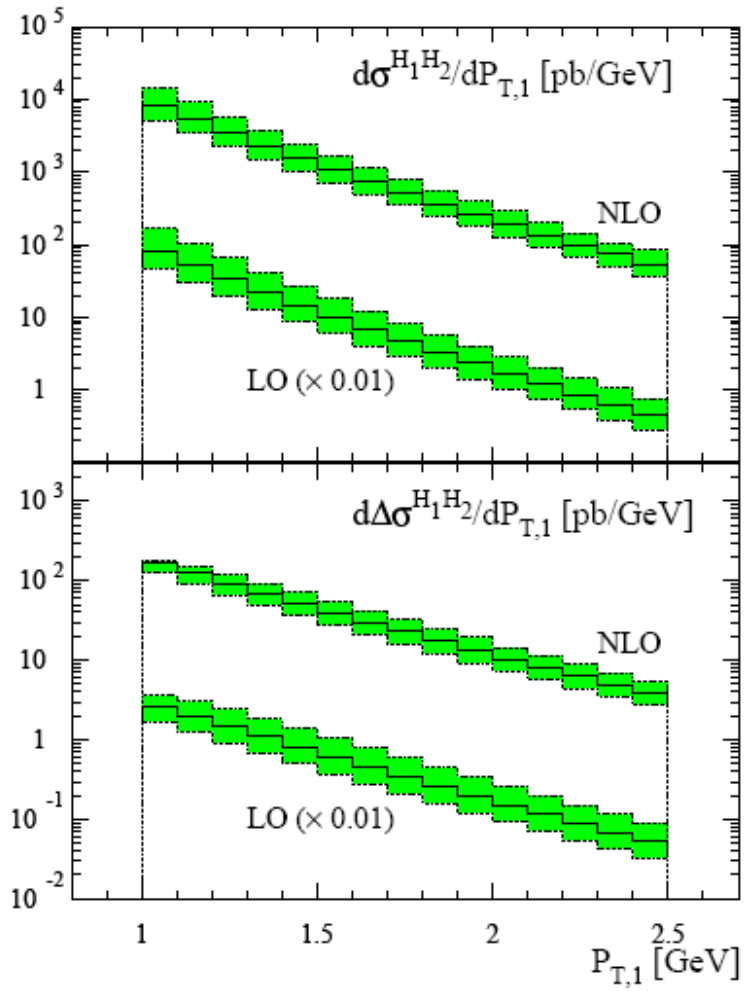
data/bins taken from COMPASS
arXiv:0904.3209 [hep-ex]

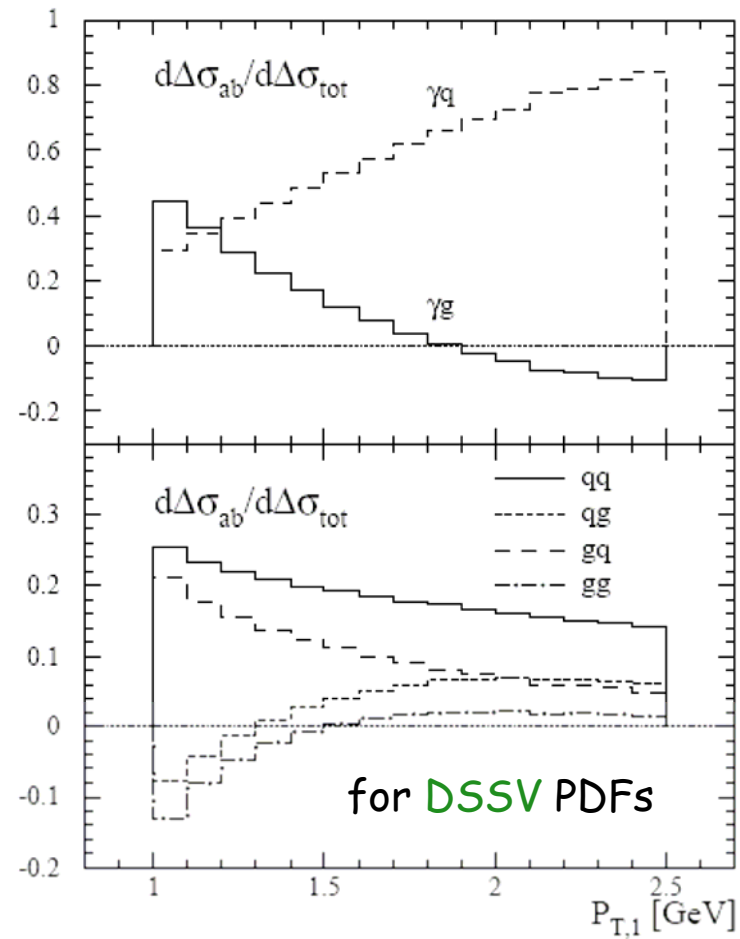
sneak preview II: 2-hadron production at NLO

Hendlmeier, Schäfer, MS

flexible NLO MC code including direct+resolved γ's

for COMPASS kinematics & cuts



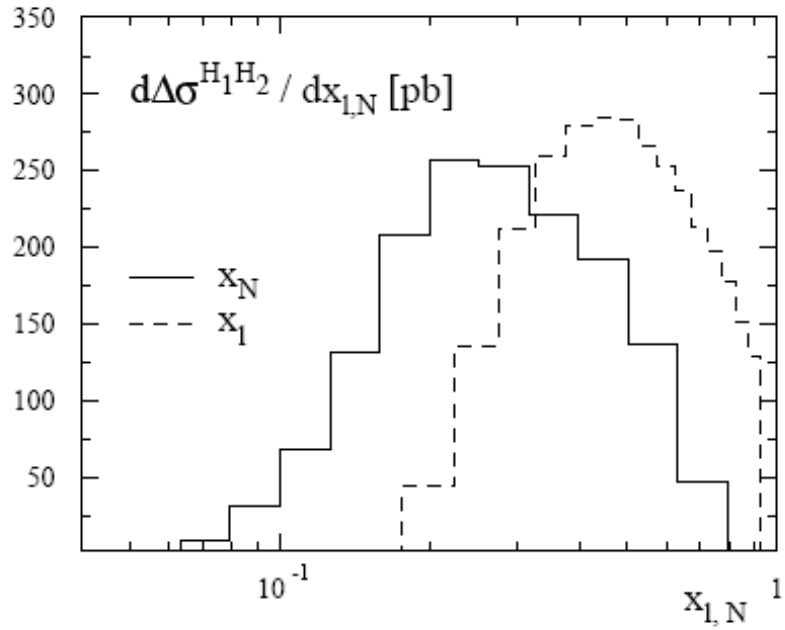


scale uncertainty improves slightly at NLO

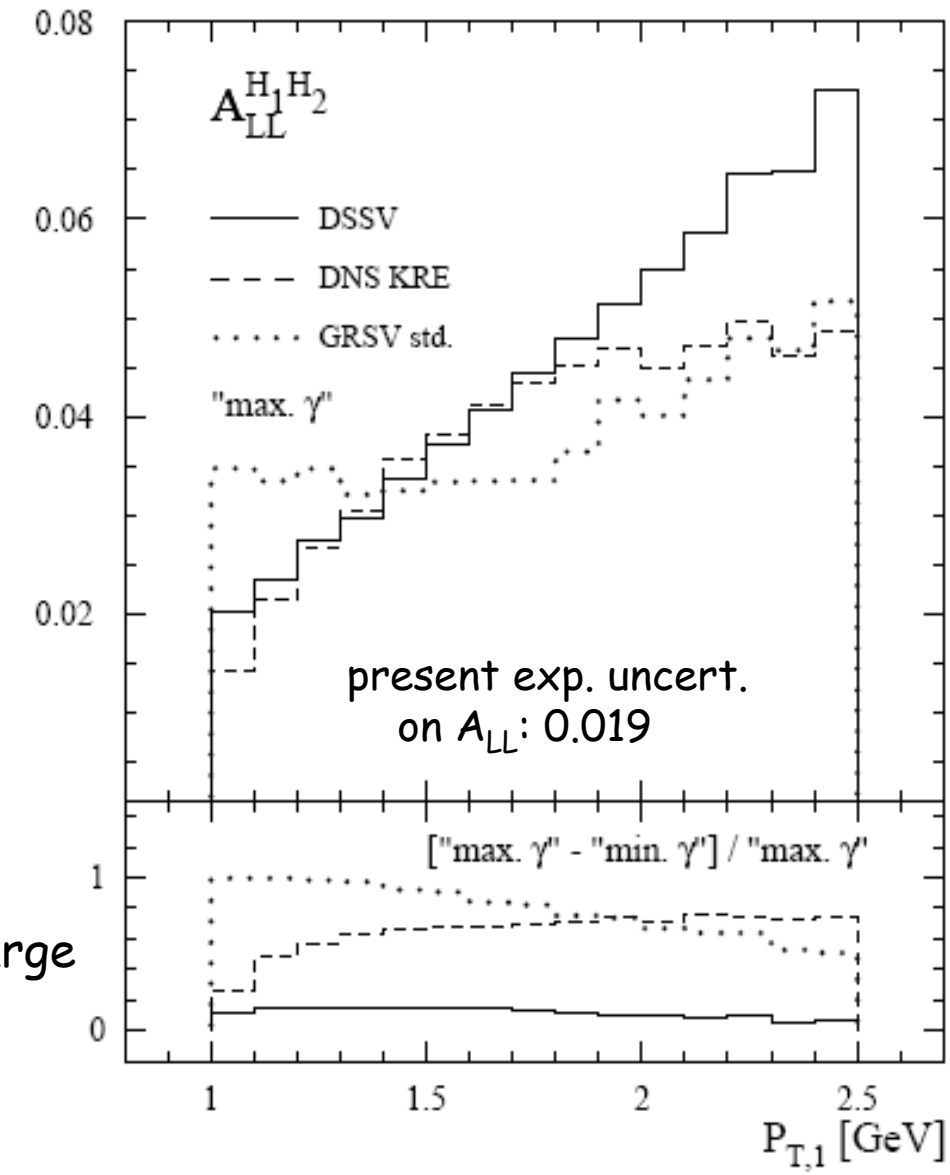
subprocess fractions depend on PDFs
strong cancellations possible

2-hadron production at NLO – cont'd

Hendlmeier, Schäfer, MS

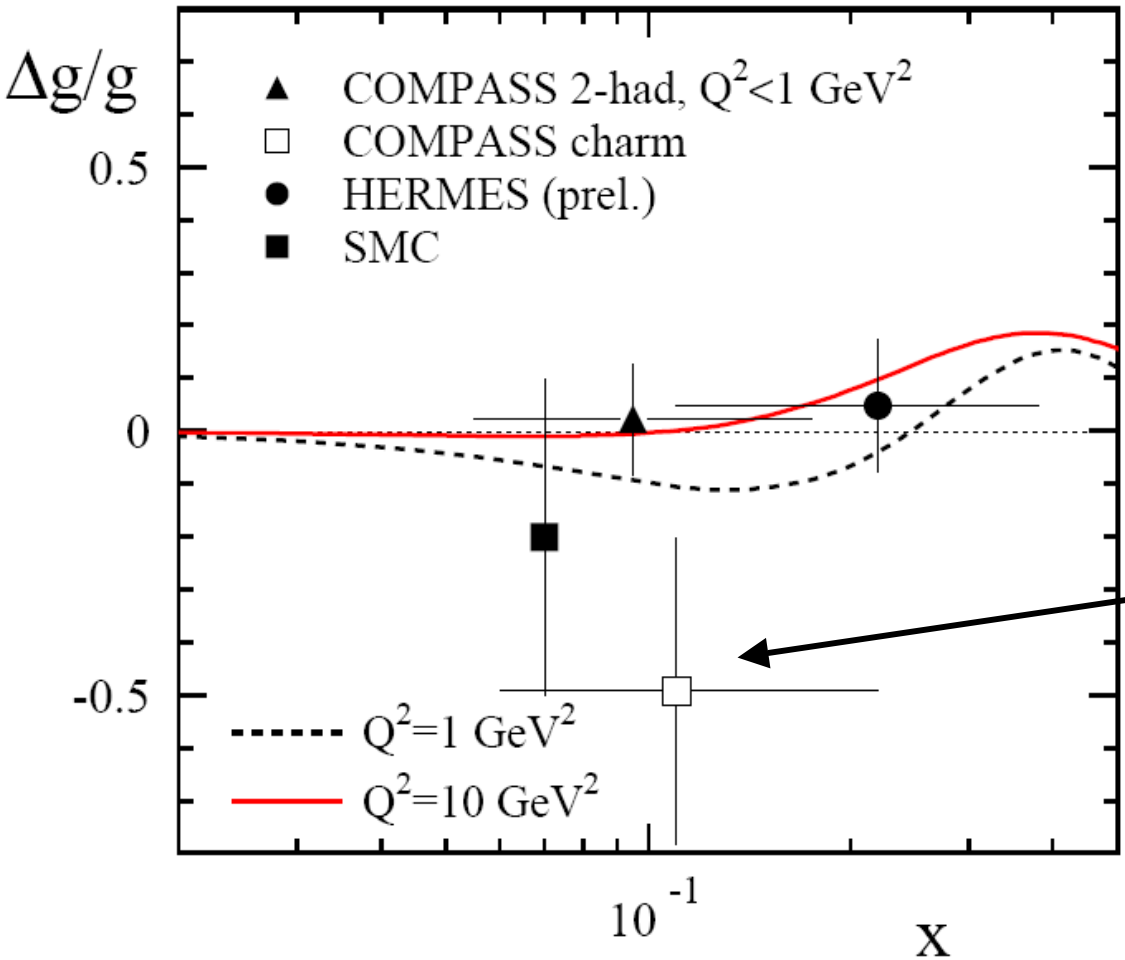


momentum fractions probed
(incl. Weiz.-Will. smearing)



due to cancellations, resolved γ can be large

DSSV gluon agrees well with model-dependent "LO" extractions of $\Delta g/g$



not in global fit
[NLO not available]

latest result:
COMPASS (charm)
arXiv:0904.3209 [hep-ex]

a future global NLO fit will use **measured** A_{LL} not derived $\Delta g/g$

need to check unpolarized cross section as well (like for RHIC)

overall quality of the global fit

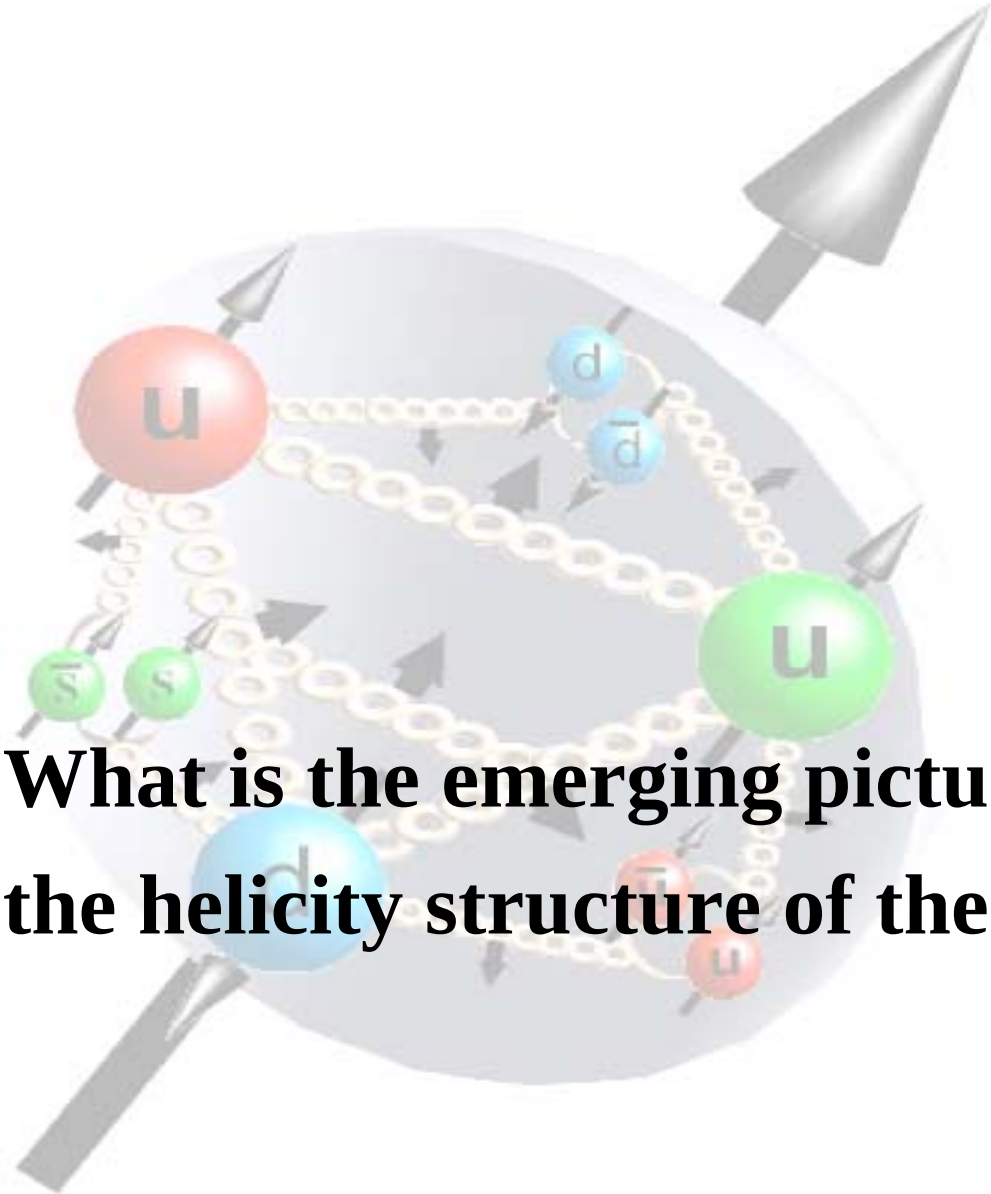
very good!

no significant tension
among different data sets

χ^2/d.o.f. $\simeq$ 0.88

note: for the time being, stat. and syst. errors are added in quadrature

experiment	process	$N_{\rm data}$	χ^2
EMC [2]	DIS (p)	10	3.9
SMC [3]	DIS (p)	12	3.4
SMC [3]	DIS (d)	12	18.4
COMPASS [4]	DIS (d)	15	8.1
E142 [5]	DIS (n)	8	5.6
E143 [6]	DIS (p)	28	19.3
E143 [6]	DIS (d)	28	40.8
E154 [7]	DIS (n)	11	4.5
E155 [8]	DIS (p)	24	22.6
E155 [9]	DIS (d)	24	17.1
HERMES [10]	DIS (He)	9	6.3
HERMES [11]	DIS (p)	15	10.5
HERMES [11]	DIS (d)	15	16.9
HALL-A [12]	DIS (n)	3	0.2
CLAS [13]	DIS (p)	10	5.9
CLAS [13]	DIS (d)	10	2.5
SMC [14]	SIDIS (p, h^+)	12	18.7
SMC [14]	SIDIS (p, h^-)	12	10.6
SMC [14]	SIDIS (d, h^+)	12	7.3
SMC [14]	SIDIS (d, h^-)	12	14.1
HERMES [15]	SIDIS (p, h^+)	9	6.4
HERMES [15]	SIDIS (p, h^-)	9	4.9
HERMES [15]	SIDIS (d, h^+)	9	11.4
HERMES [15]	SIDIS (d, h^-)	9	4.5
HERMES [10]	SIDIS (He, h^+)	9	4.7
HERMES [10]	SIDIS (He, h^-)	9	6.9
HERMES [15]	SIDIS (p, π^+)	9	9.6
HERMES [15]	SIDIS (p, π^-)	9	4.9
HERMES [15]	SIDIS (d, π^+)	9	9.4
HERMES [15]	SIDIS (d, π^-)	9	19.5
HERMES [15]	SIDIS (d, K^+)	9	6.2
HERMES [15]	SIDIS (d, K^-)	9	5.8
HERMES [15]	SIDIS (d, $K^+ + K^-$)	9	3.4
COMPASS [16]	SIDIS (d, h^+)	12	6.2
COMPASS [16]	SIDIS (d, h^-)	12	12.0
PHENIX [22]	pp (200 GeV, π^0)	10	14.2
PHENIX [23]	pp (200 GeV, π^0)	10	7.1 [13.8] [a]
PHENIX [24]	pp (62 GeV, π^0)	5	3.1 [2.8] [a]
STAR [25]	pp (200 GeV, jet)	10	8.8
STAR (prel.) [26]	pp (200 GeV, jet)	9	6.9
TOTAL:		467	392.6

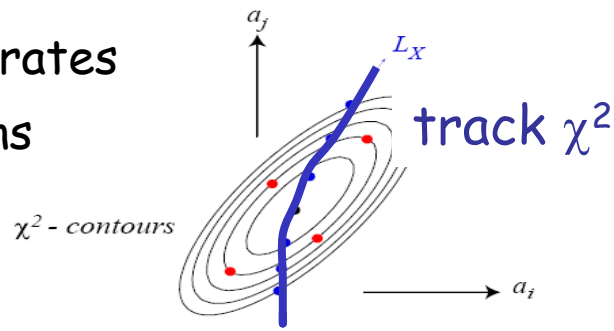


What is the emerging picture for the helicity structure of the nucleon ?

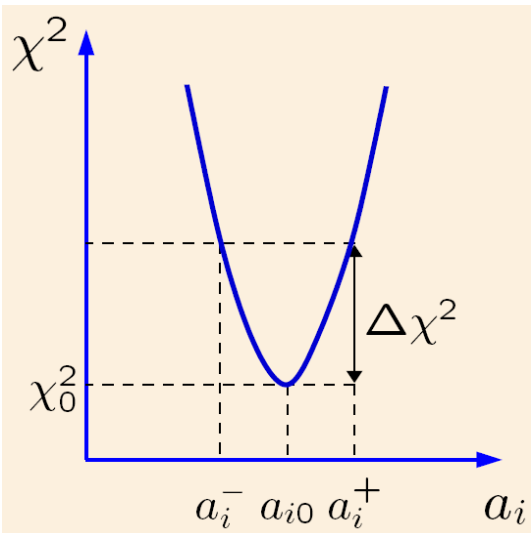
estimating PDF uncertainties

mainly two methods in use: [reshaped for PDF analyses by J. Pumplin and CTEQ]

- **Hessian method:** classic tool, explores vicinity of χ^2-minimum in quadratic approx.; often unstable for multi-parameter PDF analyses
- **Lagrange multiplier:** track how the fit deteriorates when PDFs are forced to give different predictions for selected observables; explores the full paramater space indep. of approximations



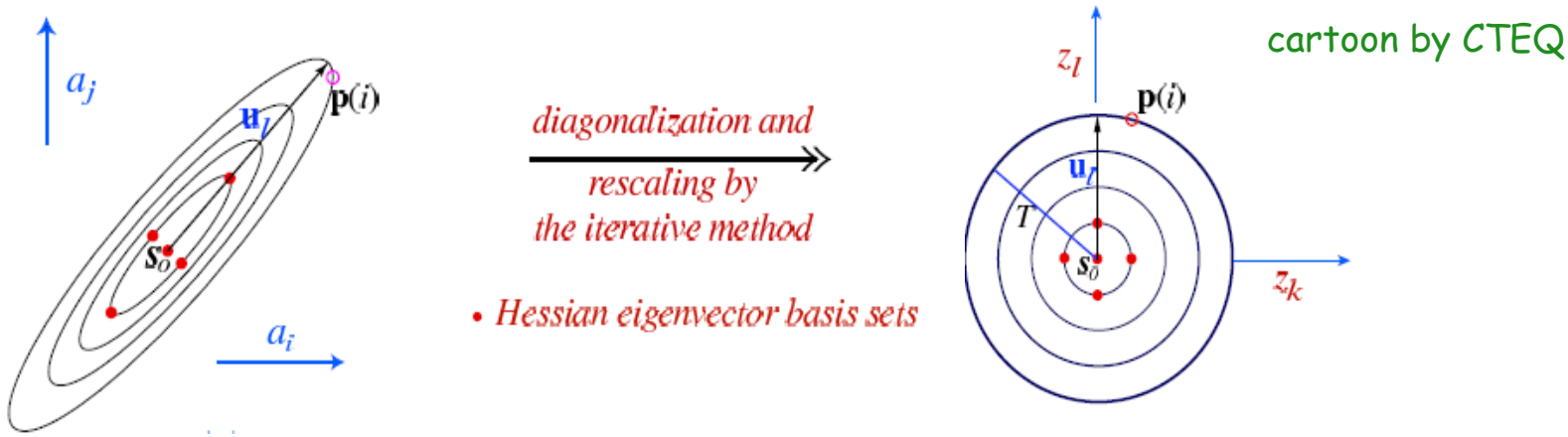
issue: what value of $\Delta\chi^2$ (tolerance) defines a 1-σ error?



- non Gaussian errors, χ^2 "landscape" not parabolic
- uncertainties with diverse characteristics
- theor. errors correlated and poorly known
- data sets often marginally consistent for $\Delta\chi^2=1$

we present uncertainties bands for both $\Delta\chi^2$ = 1 and a more pragmatic 2% increase in χ^2

Hessian eigenvector PDF basis sets



- eigenvectors provide an optimized orthonormal basis near the minimum
- construct $2N_{par}$ eigenvector basis sets $S_k^{\pm}$ by displacing each z_k by ± 1
- the "coordinates" are rescaled such that $\Delta\chi^2 = \sum_k z_k^2$
- sets $S_k^{\pm}$ can be used to calculate uncertainties of observables O_i

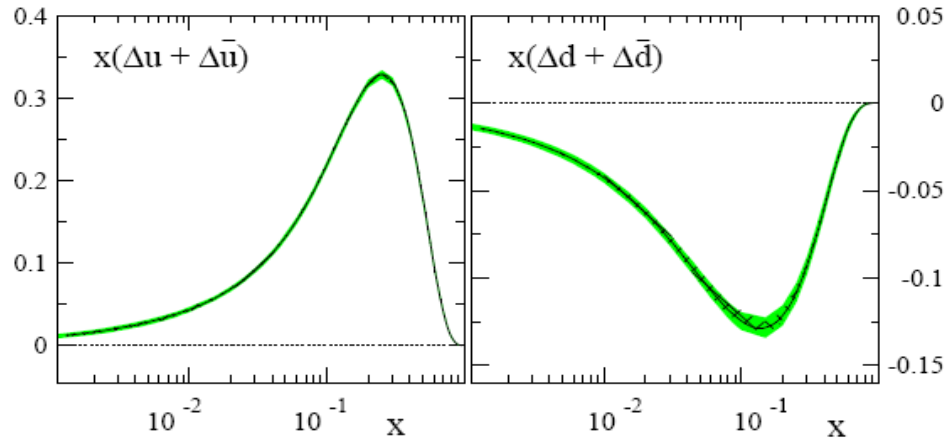
$$\Delta\mathcal{O}_i = \frac{1}{2}\left(\sum_{k=1}^{N_{\rm par}}\left[\mathcal{O}_i(S_k^+) - \mathcal{O}_i(S_k^-)\right]^2\right)^{1/2}$$

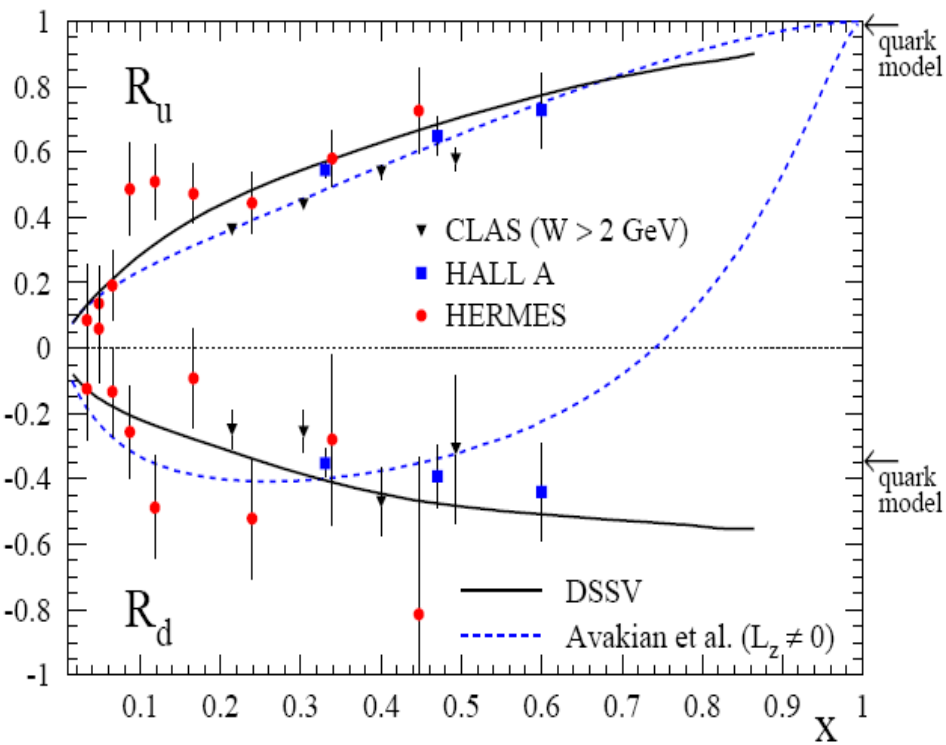
38 DSSV eigenvector sets are available from ribf.riken.jp/~marco/DSSV

(very soon)

DSSV valence quark polarizations

- **best determined**
  - uncertainty bands very narrow
  - agrees well with previous "DIS-only" fits
    GRSV, BB, LSS, AAC, DNS, …





- **large-x frontier**

$$R_q(x) \equiv \frac{\Delta q(x) + \Delta \bar{q}(x)}{q(x) + \bar{q}(x)}$$

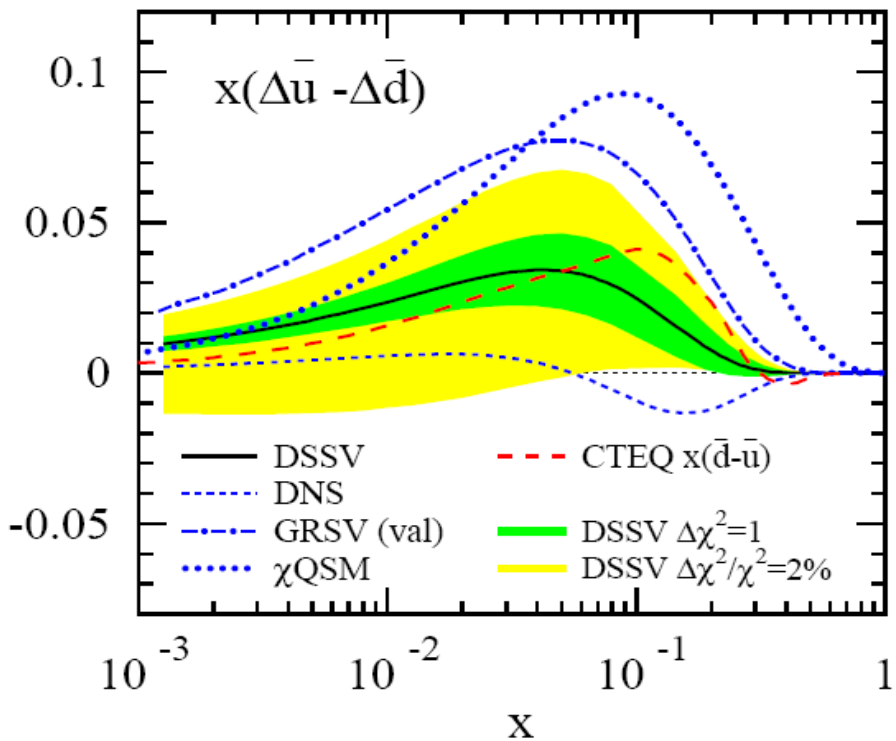
- $R_u(x \to 1) \to 1$ as expected
- $R_d(x \to 1)$ remains negative
- counting rules + helicity retention + nonzero OAM: expect $R_d(x \to 1) \to 1$

  Avakian, Brodsky, Deur, Yuan

what happens as $x \to 1$?

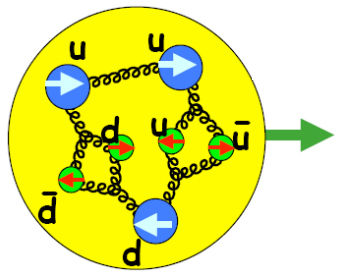
DSSV: tantalizing hints at non-trivial sea polarizations

- indications for an SU(2) breaking of light polarized u,d sea



$$\Delta\bar{u} > 0$$
$$\Delta\bar{d} < 0$$

- similar size than in unpol. case
- driven by SIDIS $h^{\pm}$, $\pi^{\pm}$ data



- many models give comparable results

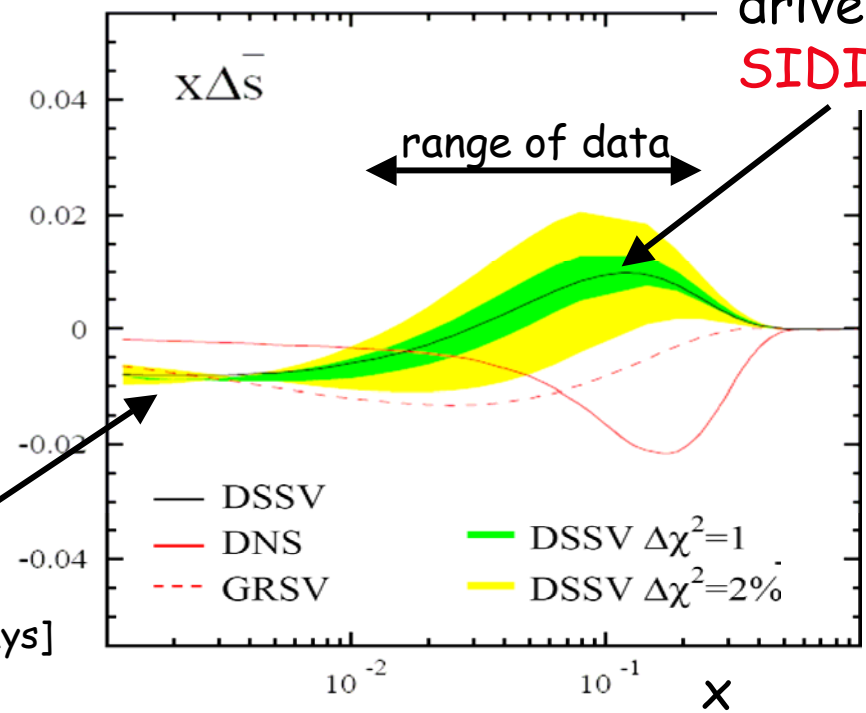
  large-N_C, chiral quark models, meson cloud, Pauli blocking, …

  Thomas, Signal, Cao; Holtmann, Speth, Fassler; Diakonov, Polyakov, Weiss; Schafer, Fries; Kumano; Wakamatsu; Gluck, Reya; Bourrely, Soffer, …

DSSV sea polarizations – cont'd

- **strangeness conundrum**
  - $\Delta s(x)$ always thought to be negative, but …

driven by
SIDIS $K^{\pm}$



driven by
SU(3) constraint
on $\int_0^1 \Delta s(x)\, dx$
[F,D values from hyperon decays]

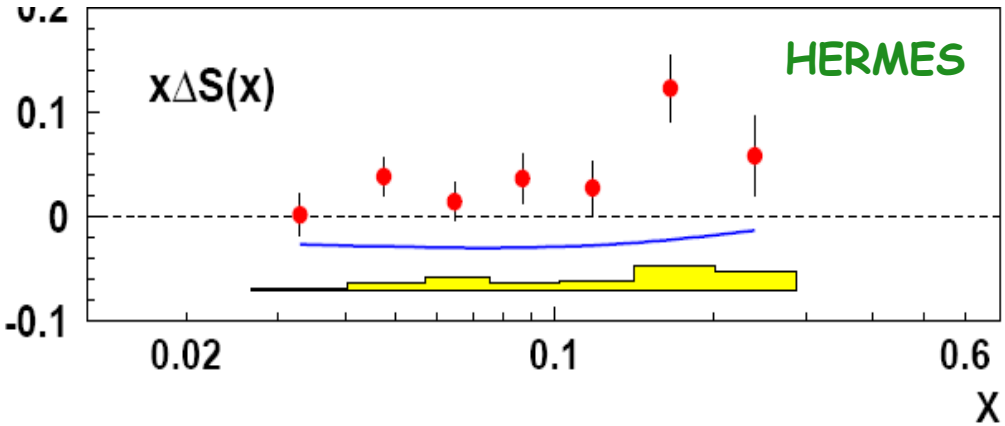
striking result, but relies on

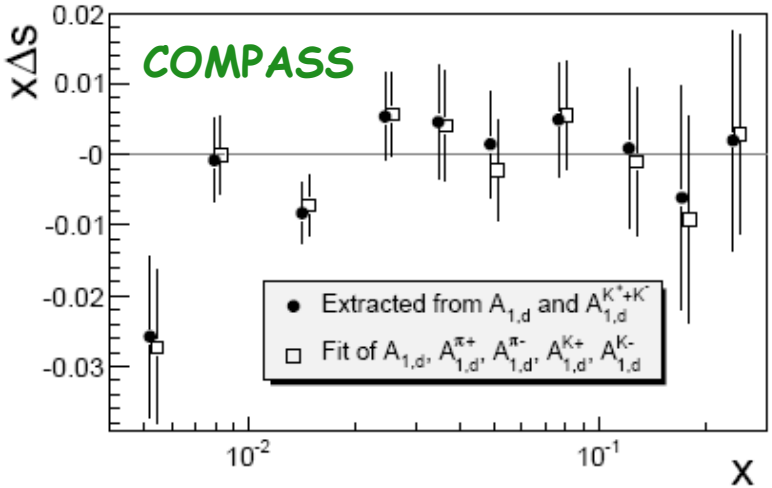
- kaon fragmentation – **how reliable ?**
  more data available soon (BELLE, …)
- unpolarized PDFs – **how well do we know s(x) ?**
  HERMES result for s(x) does not agree well with CTEQ
- SU(3) breaking uncertainties – **sizable ?**
  Lipkin; Zhu, Puglia, Ramsey-Musolf; …

needs further studies exp. & theory !

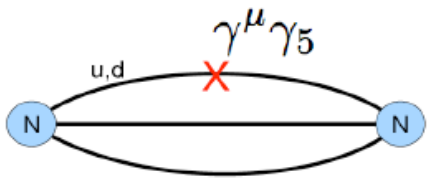
strangeness conundrum – cont'd

- **LO extractions** by HERMES & COMPASS agree well with DSSV

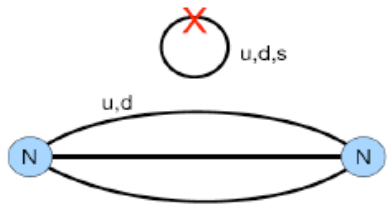




- **lattice QCD result:** Bali, Collins, Schafer, arXiv:0811.0807v2



connected



disconnected

find: $\frac{1}{2}\int_0^1[\Delta s+\Delta\bar{s}](x)\,dx = -0.01 \ldots 0.01$ (95%CL) [DSSV w/ SU(3): -0.06 !!]

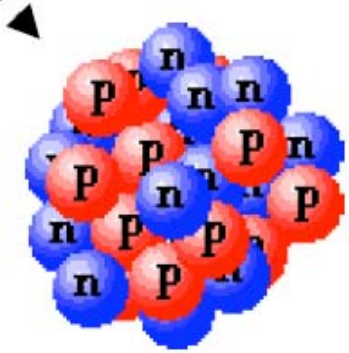
issues: not renormalized yet, continuum extrapolation, …

very small value → **SU(3) strongly broken ? perhaps $\Delta s = -\Delta\bar{s}$?**

Δs of interest not only for nucleon structure enthusiasts:

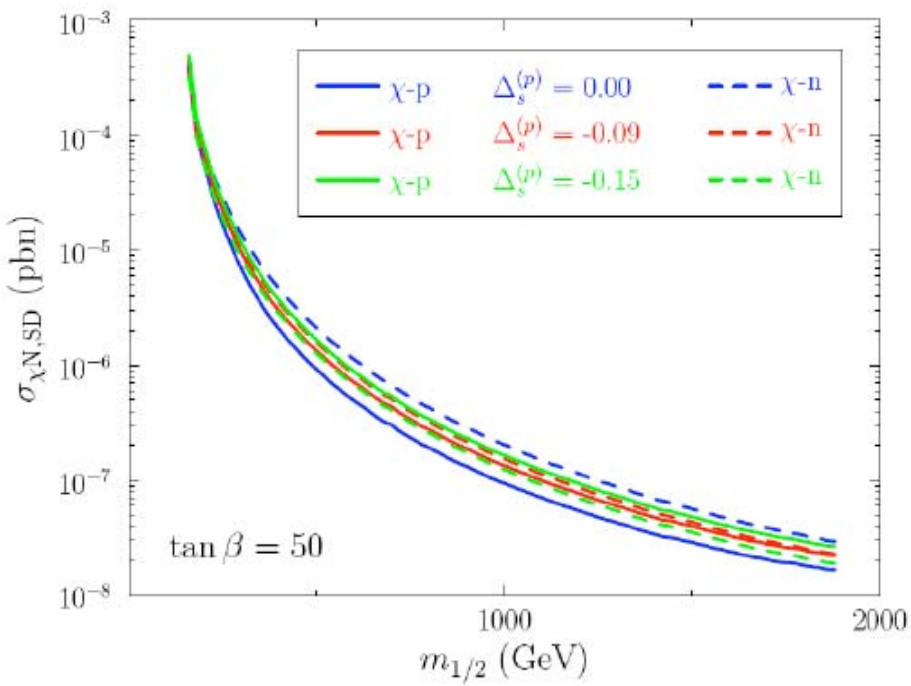
e.g. elastic scattering of SUSY dark matter

neutralino χ



$$\mathcal{L} = \alpha_i \, (\bar{\chi}\,\chi)\,(\bar{q}_i\, q_i) \; + \; \beta_i \, (\bar{\chi}\,\gamma^\mu\gamma^5\,\chi)\,(\bar{q}_i\,\gamma_\mu\gamma^5\, q_i)$$

gives spin-dep. cross sec.

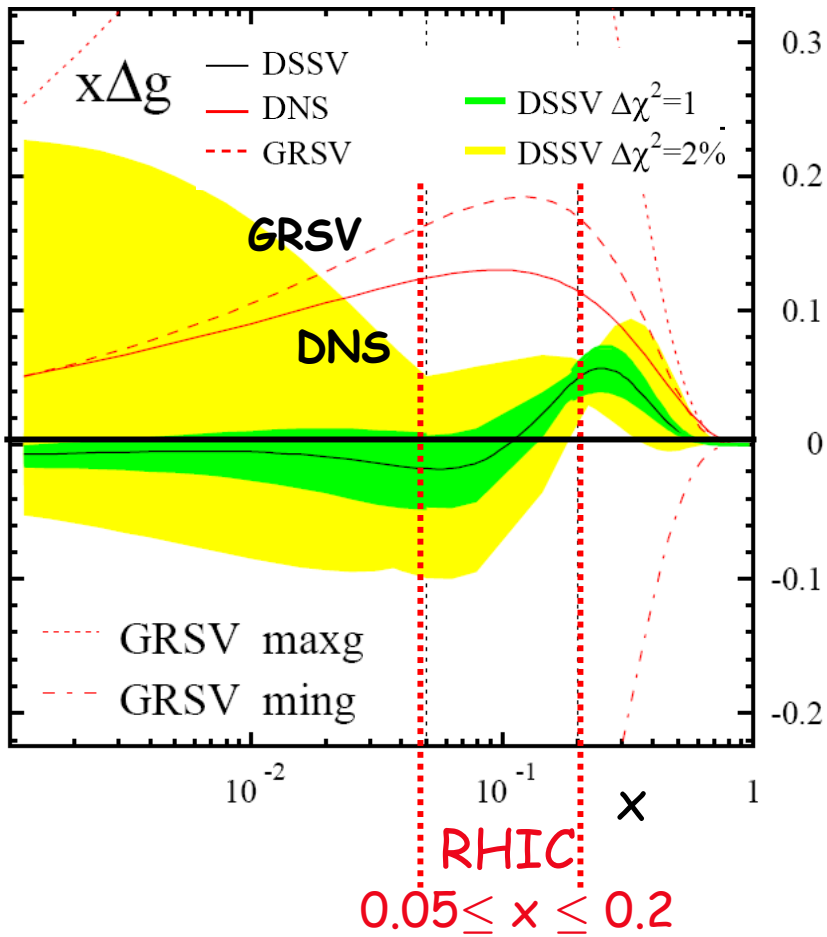


Δs largest single uncertainty

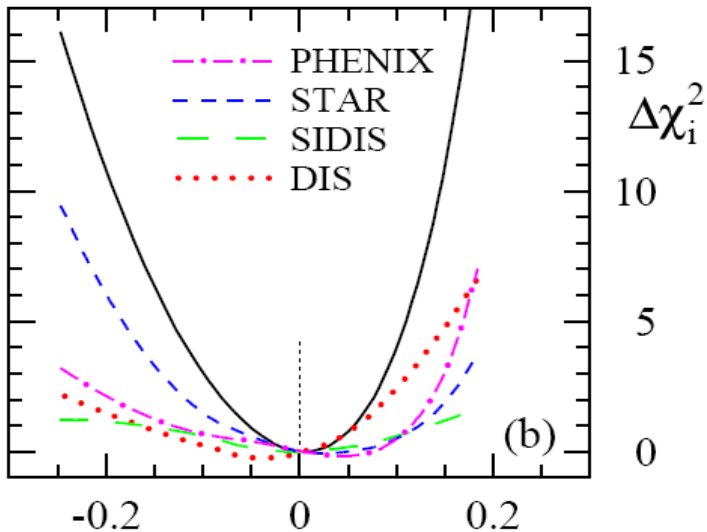
Ellis, Olive, Savage arXiv:0801.3656

also: Savage, Gondolo, Freese; Bertone, Cerdeno, Collar, Odom

DSSV gluon polarization



- best fit has a node at $x \simeq 0.1$
- $\Delta g(x)$ very small at medium x
  (vanishing integral)



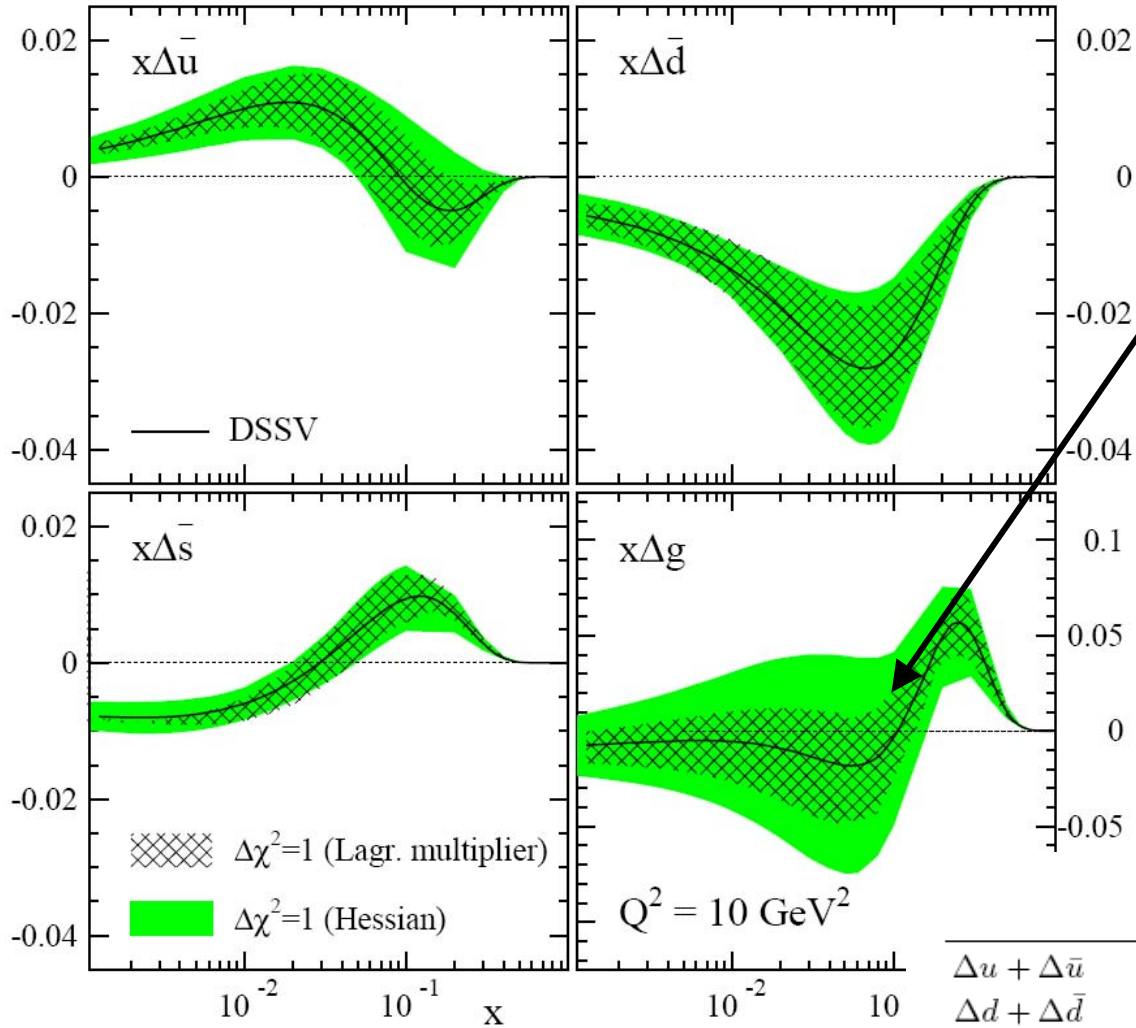
$$\delta g \equiv \int_{0.05}^{0.2} \Delta g(x, 10\,\mathrm{GeV}^2)\,dx$$

- huge uncertainties at small x
  a significant polarization is still possible, even opposite to the nucleon spin

could the gluons be paired to spin-0 at around μ = 1 GeV ?

Kharzeev, Levin, Tuchin hep-ph:0809.3794

comparison with uncertainties from Lagrange multipliers



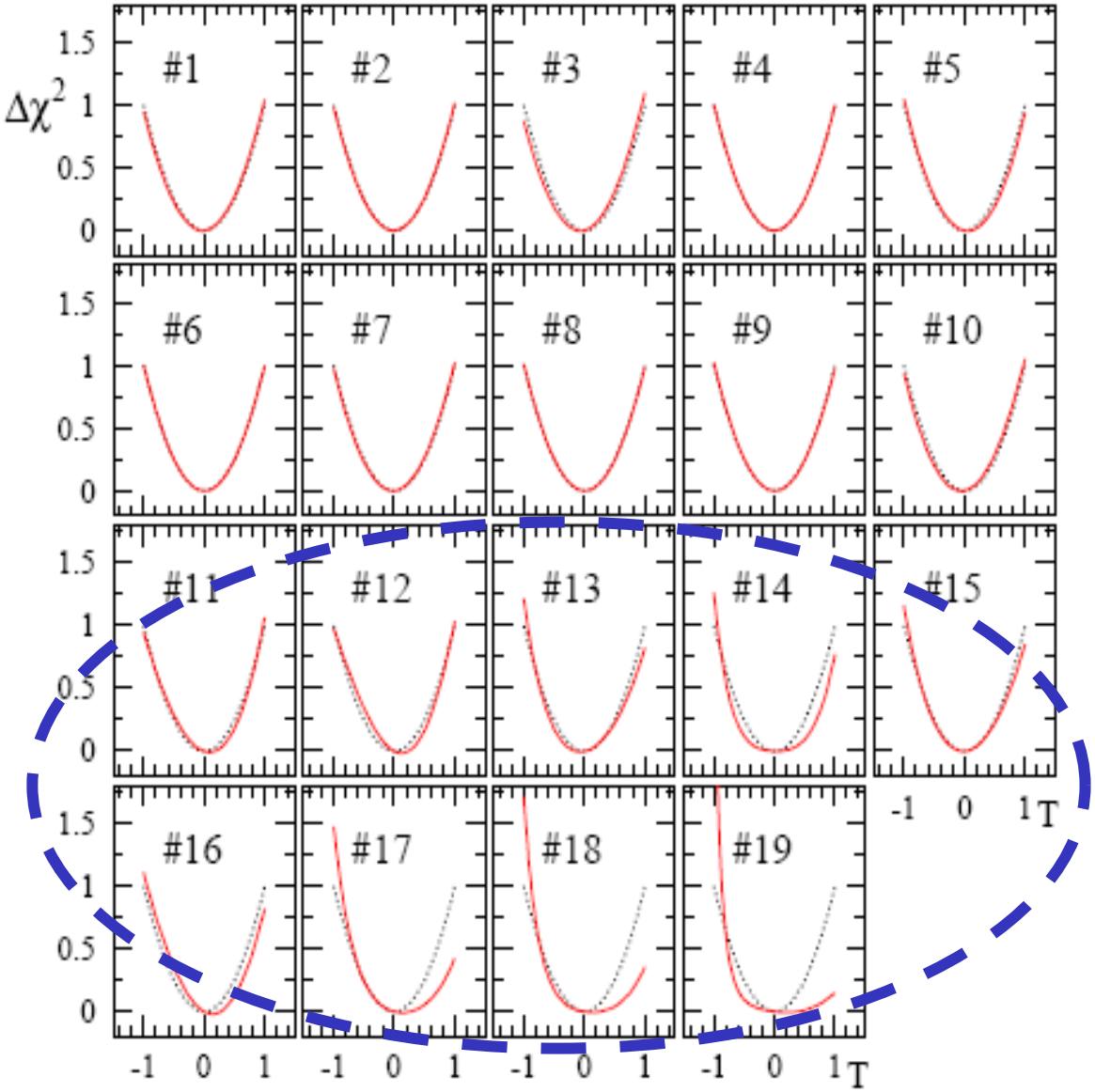
- tend to be a bit larger for Hessian, in particular for $\Delta g(x)$
- Hessian method goes crazy if asking for $\Delta\chi^2>1$

$x_{\min} = 0.001$

	Lagr. mult. $\Delta\chi^2 = 1$	Hessian $\Delta\chi^2 = 1$
$\Delta u + \Delta\bar{u}$	$0.793\ ^{+0.011}_{-0.012}$	$0.793\ \pm 0.012$
$\Delta d + \Delta\bar{d}$	$-0.416\ ^{+0.011}_{-0.009}$	$-0.416\ \pm 0.011$
$\Delta\bar{u}$	$0.028\ ^{+0.021}_{-0.020}$	$0.028\ \pm 0.022$
$\Delta\bar{d}$	$-0.089\ ^{+0.029}_{-0.029}$	$-0.089\ \pm 0.029$
$\Delta\bar{s}$	$-0.006\ ^{+0.010}_{-0.012}$	$-0.006\ \pm 0.011$
Δg	$0.013\ ^{+0.106}_{-0.120}$	$0.013\ \pm 0.182$
$\Delta\Sigma$	$0.366\ ^{+0.015}_{-0.018}$	$0.366\ \pm 0.017$

- uncertainties of truncated moments for $\Delta\chi^2=1$ agree well except for Δg

χ^2 profiles of eigenvector directions (of Hessian)



#1: largest eigenvector
(steep direction in χ^2)

...

#19: smallest eigenvector
(shallow direction in χ^2)

significant deviations from assumed quadratic dependence

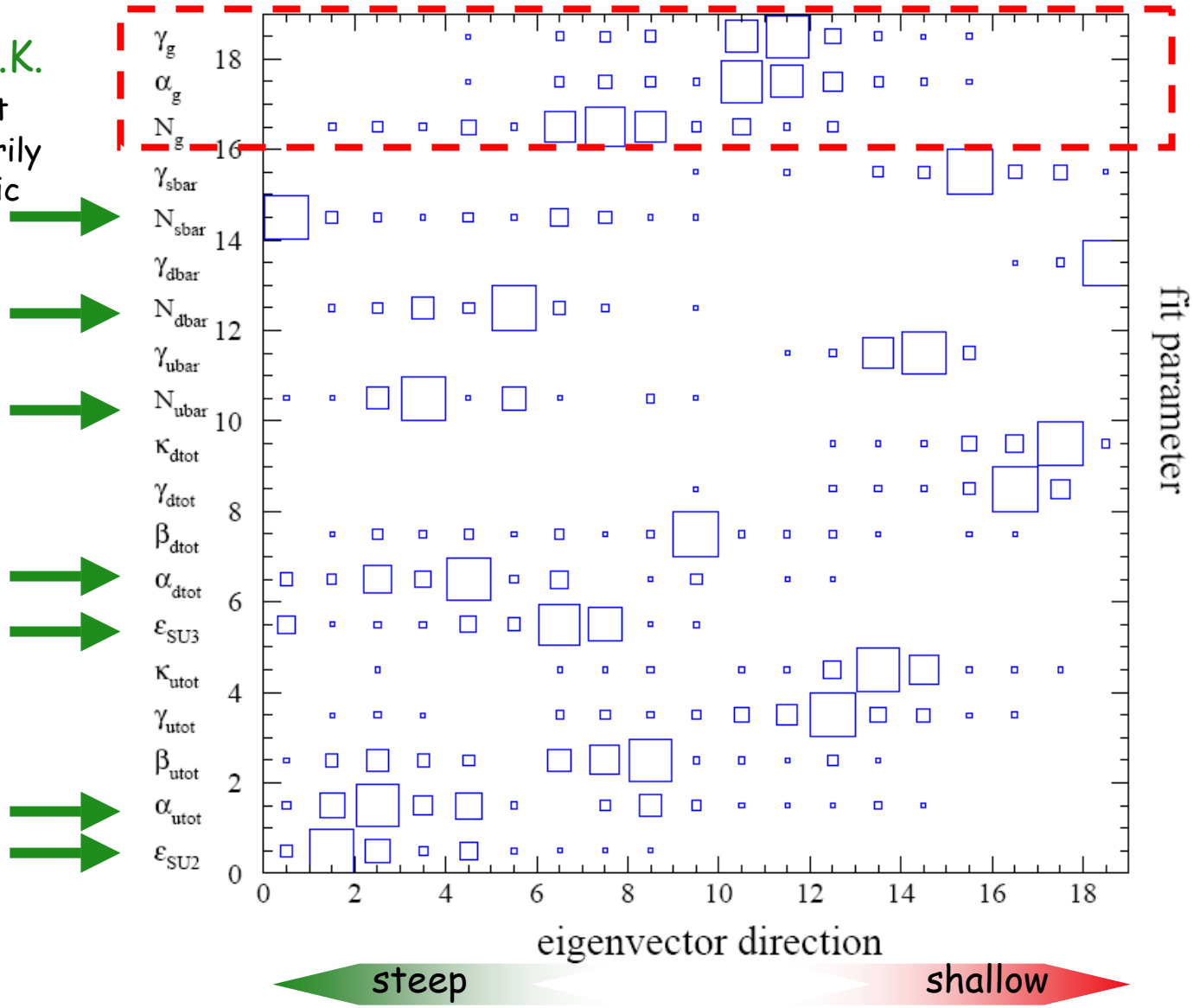
for a somewhat simplified
DSSV fit with 19 parameters

worse for fit parameters: mix with all e.v. (steep & shallow)

$$x\Delta f_j(x, 1\,\text{GeV}) = N_j\, x^{\alpha_j}\,(1-x)^{\beta_j}\left[1+\kappa_j\sqrt{x}+\gamma_j x\right]$$

look O.K.

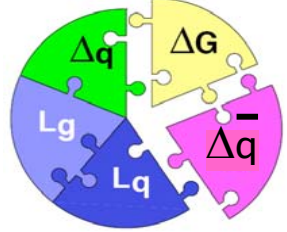
but not necessarily parabolic



Δg

mixed bag

spin audit: 1st moments & the spin of the proton



"helicity sum rule"

Jaffe, Manohar; Ji; ...

$$\frac{1}{2}\hbar = \langle P, \frac{1}{2} | J^z_{\rm QCD} | P, \frac{1}{2} \rangle = \sum_q \frac{1}{2} S^z_q + S^z_g + \sum_q L^z_q + L^z_g$$

$A^+ = 0$ gauge, IMF
partonic interpretation

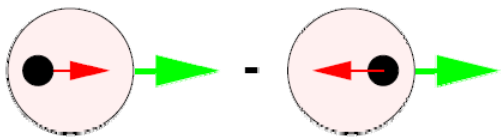
total u+d+s quark spin | gluon spin | orbital angular momentum

"quotable" properties of the nucleon !

momentum fraction

$$\Delta f(x) \equiv f_+^{N_+}(x) - f_-^{N_+}(x)$$

helicity parton densities



x-moment $\xrightarrow{\int_0^1 dx}$ **total spin polarizations S_q and S_g !**

$$S_q \equiv \Delta\Sigma(Q^2) = \int_0^1 dx [\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}](x, Q^2)$$

$$S_g \equiv \Delta g(Q^2) = \int_0^1 dx \Delta g(x, Q^2)$$

numerical results (DSSV)

$Q^2 = 10\ GeV^2$

$$\Delta f^{1,[x_{\min},1]} \equiv \int_{x_{\min}}^{1} \Delta f(x)dx$$

	$x_{\min} = 0$	$x_{\min} = 0.001$	
	best fit	$\Delta\chi^2 = 1$	$\Delta\chi^2/\chi^2 = 2\%$
$\Delta u + \Delta\bar{u}$	0.813	$0.793\ ^{+0.011}_{-0.012}$	$0.793\ ^{+0.028}_{-0.034}$
$\Delta d + \Delta\bar{d}$	-0.458	$-0.416\ ^{+0.011}_{-0.009}$	$-0.416\ ^{+0.035}_{-0.025}$
$\Delta\bar{u}$	0.036	$0.028\ ^{+0.021}_{-0.020}$	$0.028\ ^{+0.059}_{-0.059}$
$\Delta\bar{d}$	-0.115	$-0.089\ ^{+0.029}_{-0.029}$	$-0.089\ ^{+0.090}_{-0.080}$
$\Delta\bar{s}$	-0.057	$-0.006\ ^{+0.010}_{-0.012}$	$-0.006\ ^{+0.028}_{-0.031}$
Δg	-0.084	$0.013\ ^{+0.106}_{-0.120}$	$0.013\ ^{+0.702}_{-0.314}$
$\Delta\Sigma$	0.242	$0.366\ ^{+0.015}_{-0.018}$	$0.366\ ^{+0.042}_{-0.062}$

- Δs, $\Delta\Sigma$ receive large negative contribution at small x
- Δg: huge uncertainties

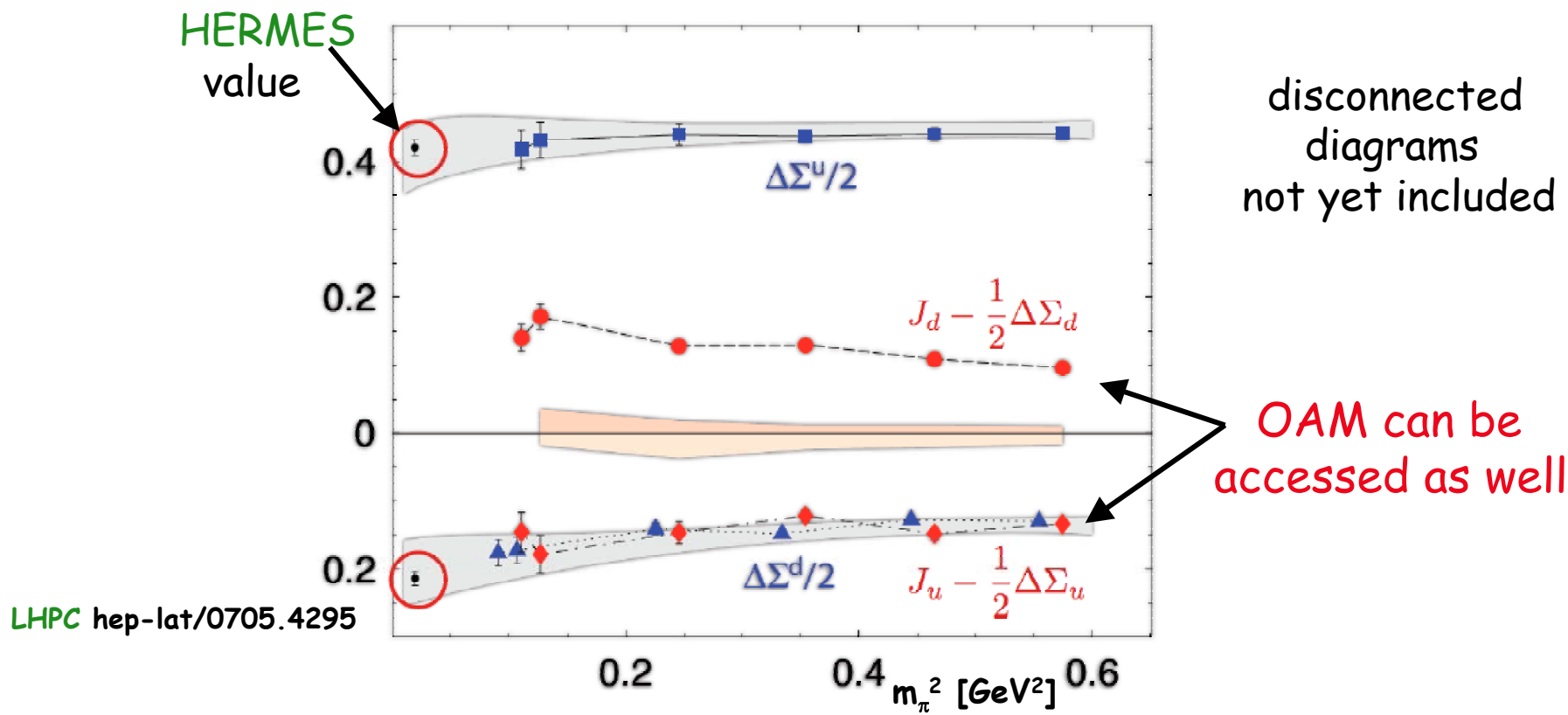
very difficult to give reliable estimates for full moments

issues:

- small x extrapolation
- validity of SU(3) "constraints"

http://www.sciencecartoonsplus.com/index.htm

1st moments can be computed on the lattice …



find: $\Delta u > 0$ and $L_u < 0$; $\Delta d < 0$ and $L_d > 0$ but in any quark model $L_u > 0$, $L_d < 0$

sign due to strong scale evolution of L_q ? Myhrer, Thomas

$L_u + L_d \simeq 0$ contribution from disconnected diagrams?

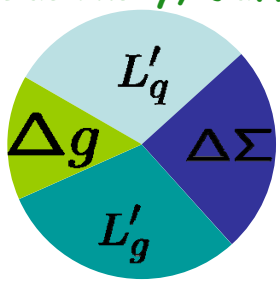
if $\Delta g \simeq 0$, does this leave us with gluon OAM as culprit in spin audit?

note: using AdS/CFT nucleon spin comes entirely from OAM Hatta, Ueda, Xiao arXiv:0905.2493

complication: "different" spin sum rules

ambiguities arise when decomposing proton spin in gauge theories

Jaffe, Manohar;
Bashinsky, Jaffe

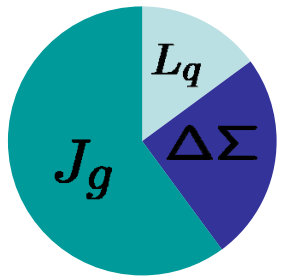


reshuffling of ang. momentum

between matter and gauge degrees

only $\Delta\Sigma$ unchanged

Ji



intuitive; partonic interpretation

Δg, $L'_{q,g}$ local only in $A^+ = 0$ gauge

how to determine $L'_{q,g}$ experimentally ?

manifest gauge invariant local operators

contain interactions $\rightarrow$ interpretation ?

$L_q + \Delta q/2$, $J_g \leftrightarrow$ GPDs (DVCS)

- lattice results for L_q are for Ji's sum rule and cannot be mixed with Δg
- num. difference between L_q and L_q' can be sizable Burkardt, Hikmat arXiv:0812.1605

latest twist: Chen, Lu, Sun, Wang, Goldman arXiv:0806.3166; 0904.0321

3rd decomposition: like Jaffe, Manohar but w/ manifest gauge inv. operators

physical interpretation? requires new def. of PDFs – relation to experiment?

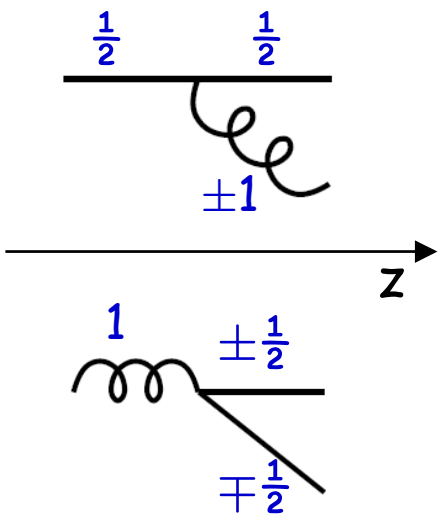
scale evolution and orbital angular momentum

determining OAM quantitatively is challenging (GPDs ↔ DVCS , TMDs, …)

that **OAM must play a role** is obvious, however, from DGLAP evolution

total angular momentum conservation in parton-parton splittings necessarily *implies* presence of orbital angular momentum: Ratcliffe

e.g.

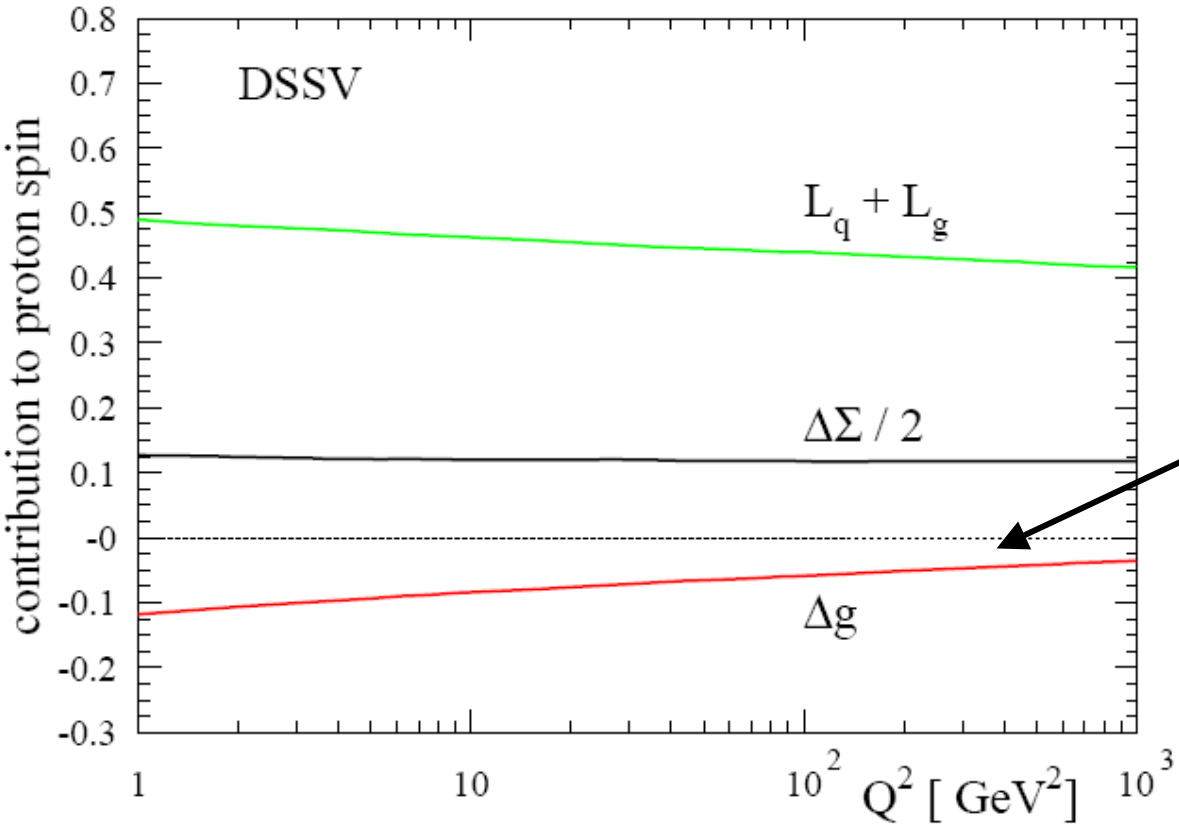


$$\frac{1}{2} = \frac{1}{2} \boxed{\pm 1}_{\Delta g} \boxed{\mp 1}_{L_z}$$

$$1 = \pm\frac{1}{2} \mp \frac{1}{2} \boxed{+1}_{L_z}$$

and analogously for the other "splittings"

Q^2 dependence of helicity sum rule



DSSV Δg is close to **"static solution"**

$\Delta g \simeq -0.15$

where $d\Delta g/d\ln\mu = 0$

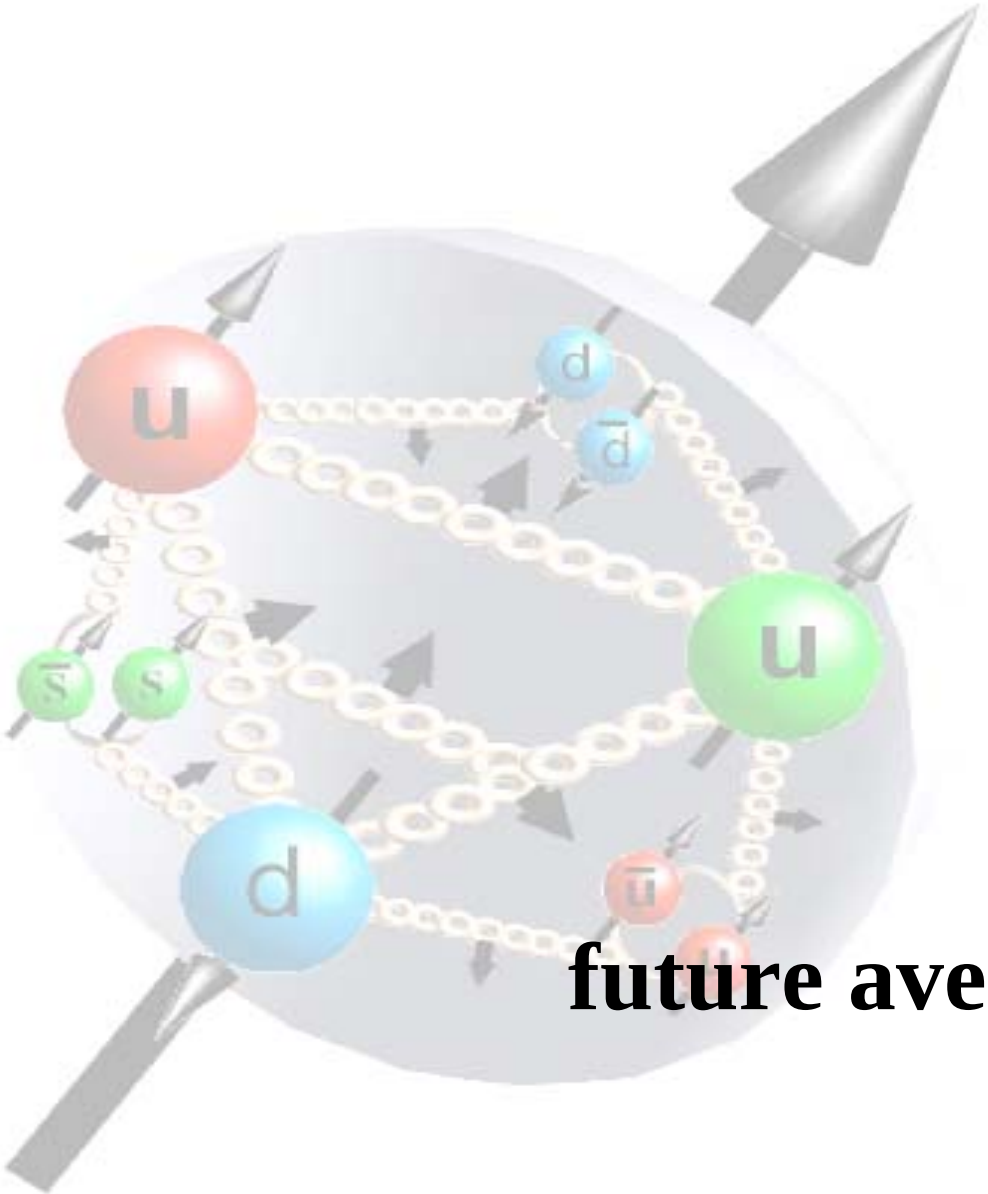
any deeper reason for that ?

recall (at LO):

$$\frac{d}{d\ln(\mu^2)}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}0 & 0\\ \frac{3}{2}C_F & \frac{1}{2}\beta_0\end{pmatrix}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix}$$

$\Delta\Sigma$ scale invariant; NLO($\overline{\text{MS}}$) mild decrease with Q^2

Δg evolves logarithmically: $\alpha_s(\mu^2)\Delta g(\mu^2) \to$ const as $\mu^2 \to \infty$



future avenues

getting ready to analyze new types of data

- **milestone:** RHIC has just completed the 1st 500 GeV run
- hope for O(50pb^{-1}) with 60% pol. from current 200 GeV run

expect:

- significant improvement of existing inclusive jet + π^0 data sample
- charged pion data

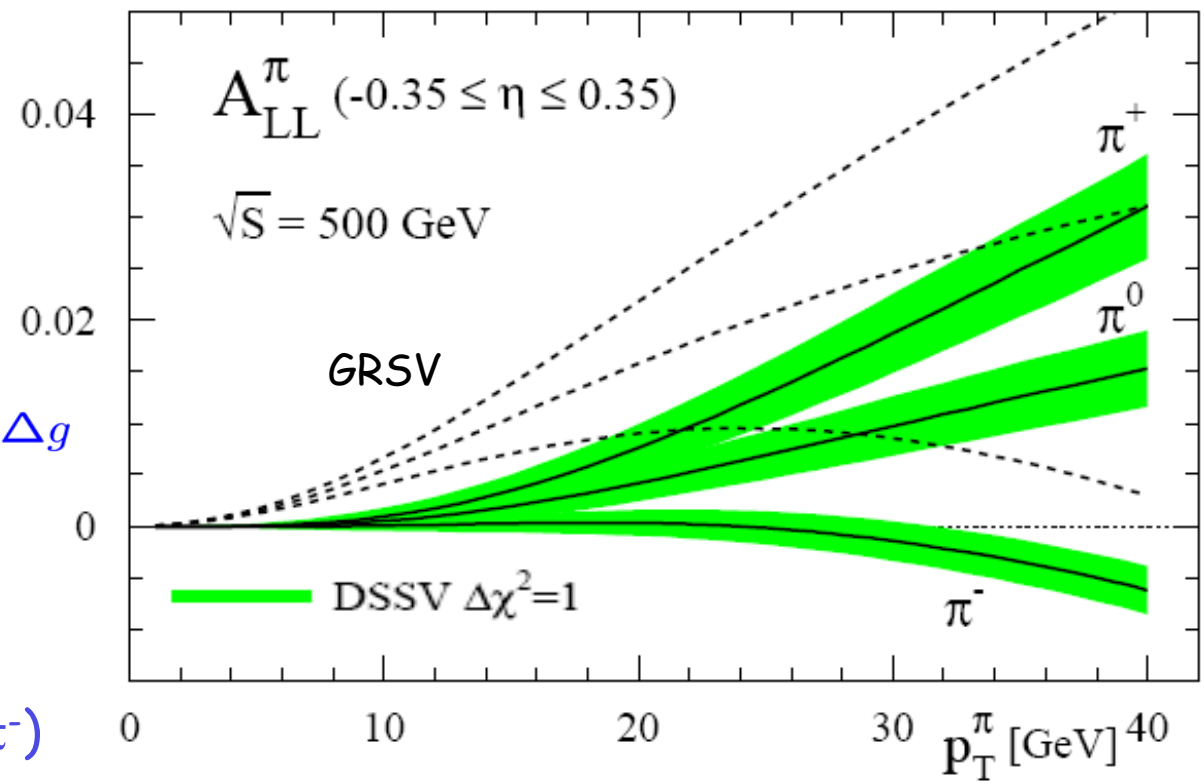
  at large p_T driven by

  - qg scattering

    $[\Delta u \otimes D_u^\pi + \Delta\bar{u} \otimes D_{\bar{u}}^\pi + \Delta d \otimes D_d^\pi + \Delta\bar{d} \otimes D_{\bar{d}}^\pi] \otimes \Delta g$

  - sign of Δg

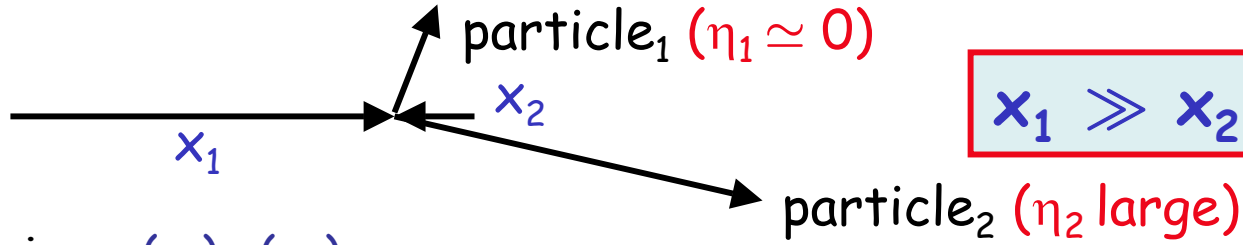
$A_{LL}(\pi^+) > A_{LL}(\pi^0) > A_{LL}(\pi^-)$



o going beyond single-inclusive measurements: **particle correlations**

idea:

forward-central correlations



$x_1 \gg x_2$

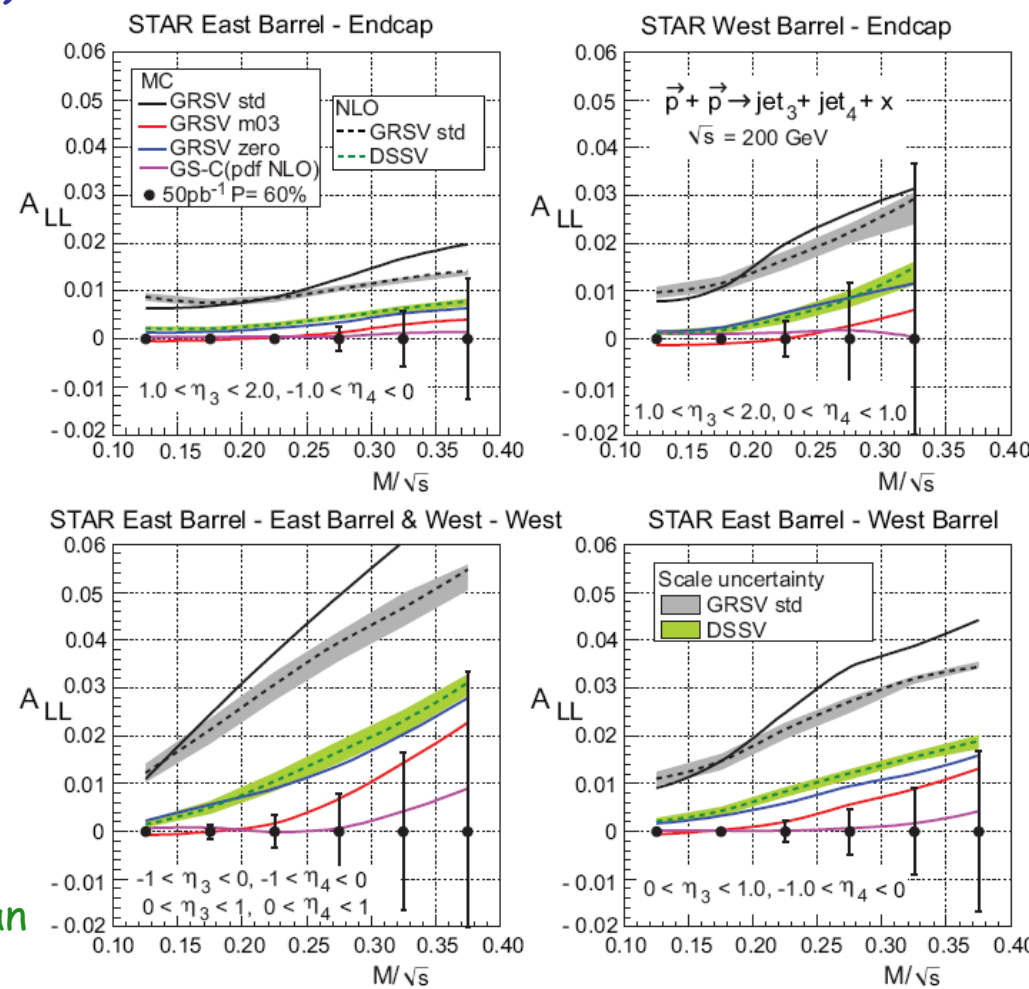
→ mainly qg-scattering: $q(x_1)\, g(x_2)$

di-jet simulation from STAR

→ more precise mapping of $\Delta g(x)$

✓ the Mellin technique is basically in place to analyze also particle correlations

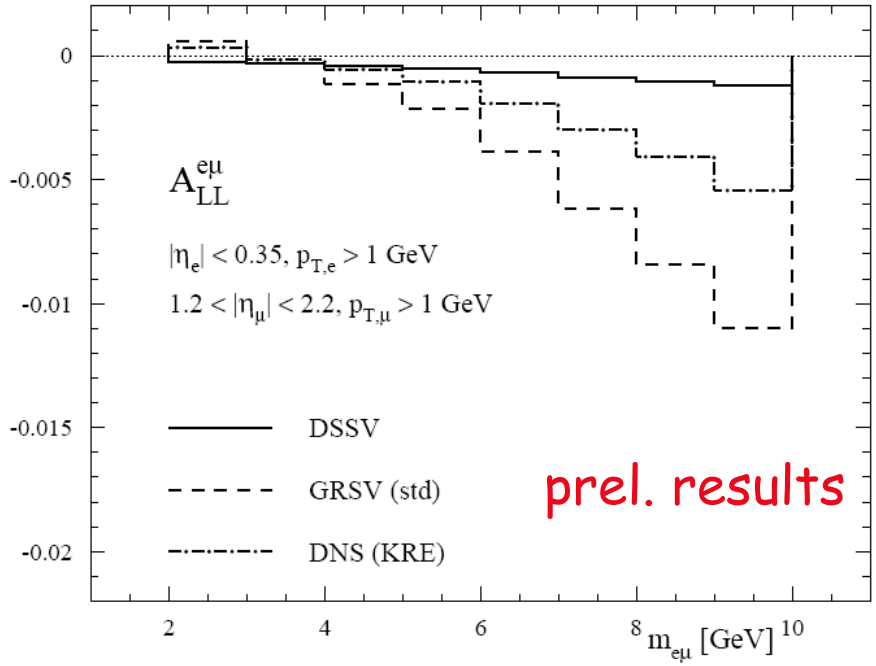
challenge: much slower MC-type codes in NLO than for 1-incl.



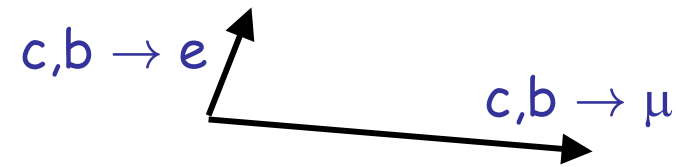
taken from 2008 RHIC spin plan

… also **heavy flavor correlations** are of interest

probe rather different hard scattering dynamics than jets and hadrons

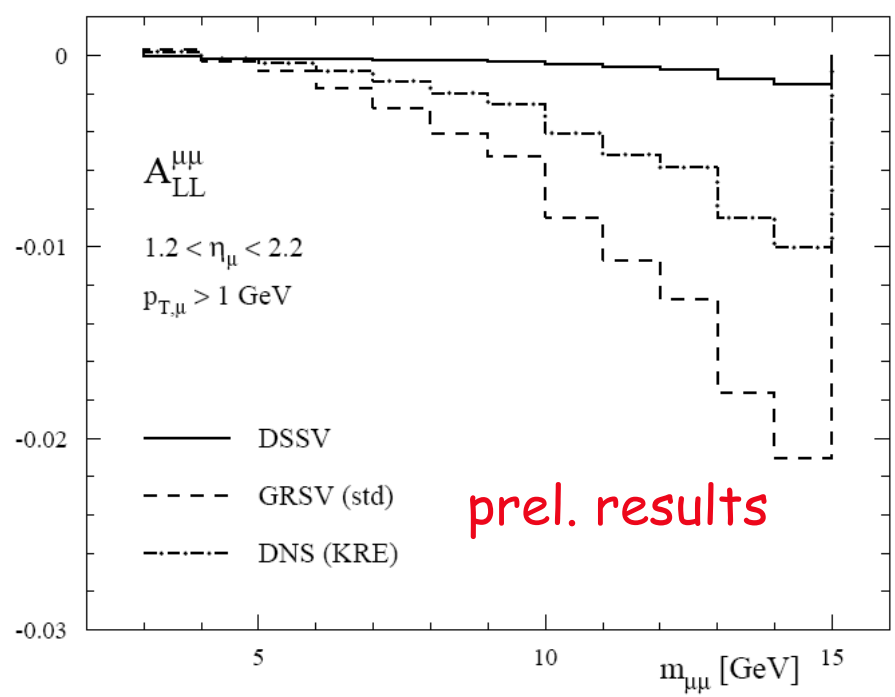


forward-central e-μ coincidences



note: single-e & single-μ have tiny A_{LL}'s

forward-backward μ-μ coincidences

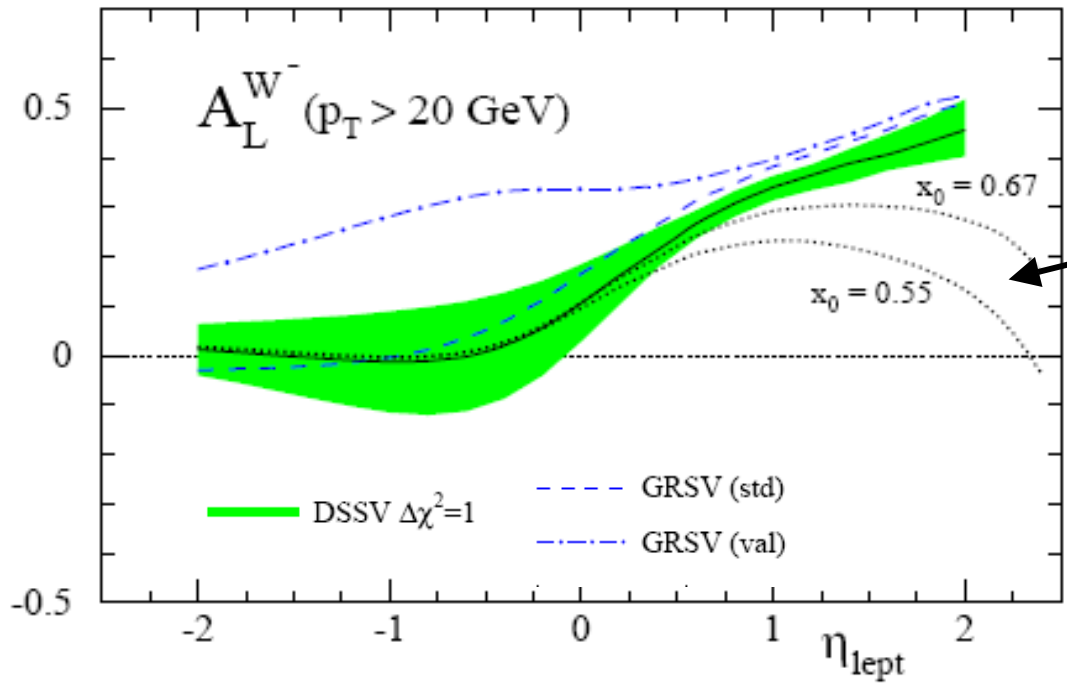


obtained with new flexible NLO MC code
Riedl, Schafer, MS

o **main goal of 500 GeV program:** W boson production

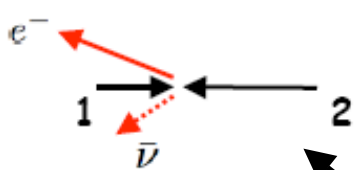
flavor separation from parity-viol. single-spin asymmetry $A_L \equiv \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$

example: A_L for W^-



two sets enforcing $\Delta d/d \to 1$ as $x \to 1$

$\frac{\Delta\bar{u}}{\bar{u}}$

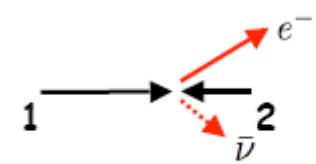


$\theta \to \pi$

$x_2 \gg x_1$

$\theta \to 0$

$x_1 \gg x_2$



$\frac{\Delta d}{d}$

$$\Delta\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 - \Delta d(x_1)\bar{u}(x_2)(1+\cos\theta)^2$$

prospects on Δs

- final HERMES data sets for SIDIS & DIS multiplicities;
  more from COMPASS; can we distinguish Δs and $\Delta\bar{s}$ in the future?

- notoriously difficult in pp:
  two channels: W+charm (extremely rare probe)
  polarized Λ production

  issues to be addressed for Λ production:

  - reliable NLO sets of D_i^Λ *and* ΔD_i^Λ

    DSV: de Florian, MS, Vogelsang, PRD 57 (1998) 5811
    updated global analysis required, D_g too small (STAR data)

    AKK: Albino et al., arXiv:0803.2768v2

    DSV: de Florian, MS, Vogelsang, PRD 57 (1998) 5811
    sparse data; 3 models considered; update desirable

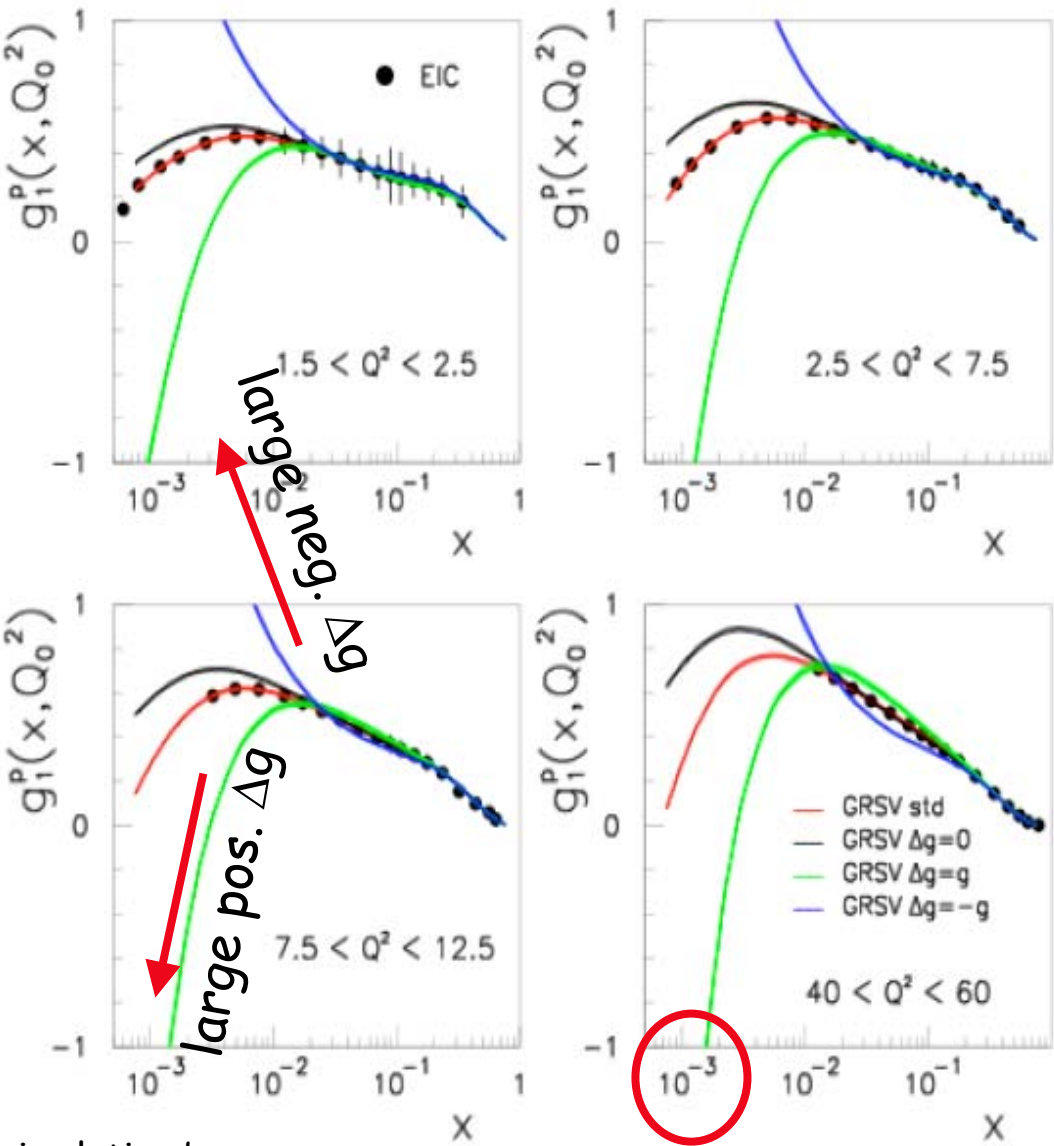
  - feed-down from hyperon weak decays; effect on polarization?

  - compute helicity-transfer subprocesses at NLO (work in progress)

further improving on uncertainties

- Lagrange multipliers more reliable than Hessian with present data
- Hessian method perhaps useful for $\Delta\chi^2 = 1$ studies, beyond ??
- include experimental error correlations if available
  work started together with help from the RHIC Spin Collaboration
  (aiming at a CTEQ-like collaboration of theory and experiment)

in the future we have to take up HERA's legacy: eRHIC !



simulation by
A. Brull, R. Ent

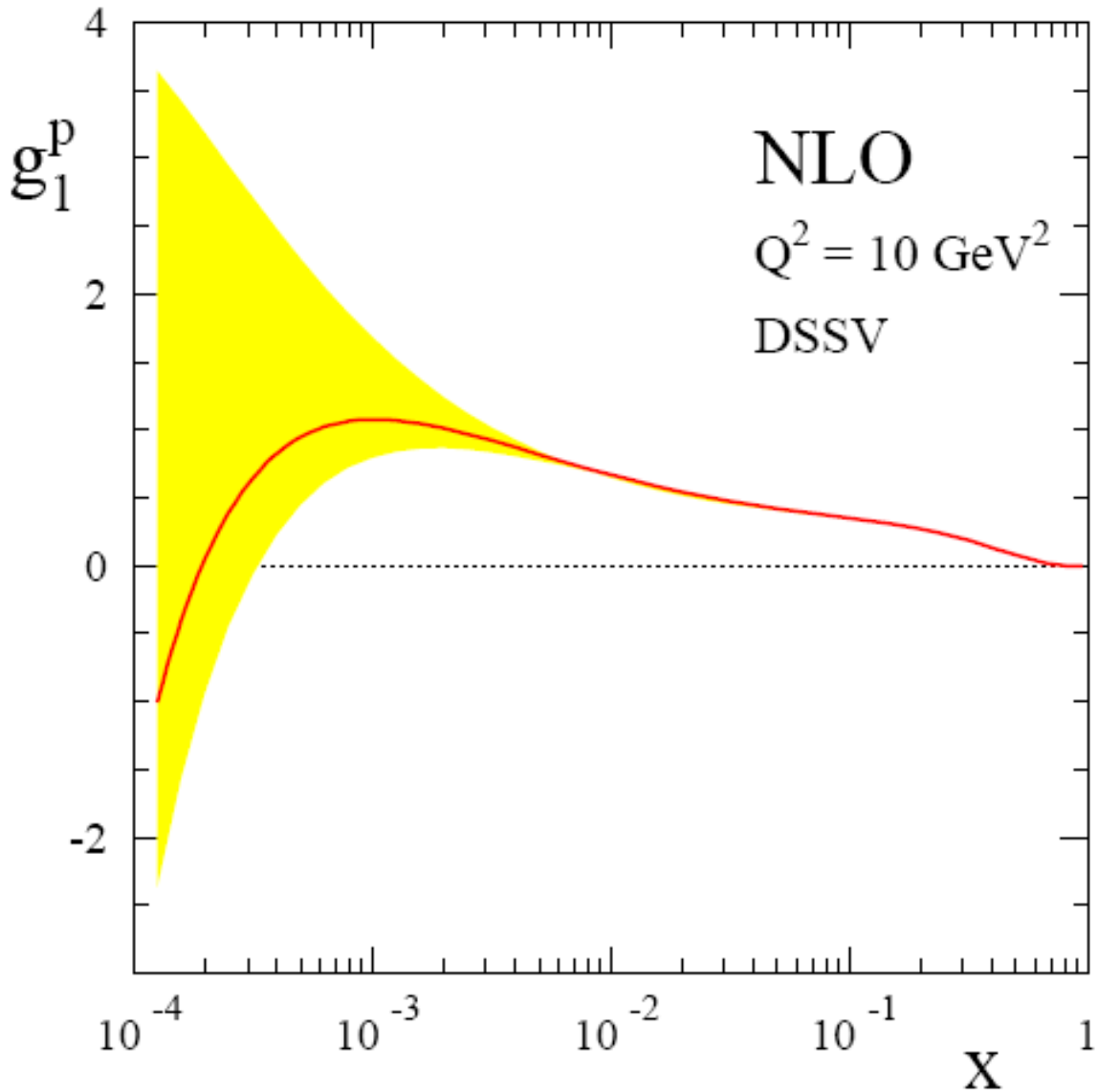
- RHIC will map spin structure perhaps down to $x \simeq 0.005$
  $\rightarrow$ sufficient for $\int \Delta g(x)\, dx$?

most natural solution:
repeat HERA program
but now with spin

- scaling violations most effective *below* $x \simeq 0.01$
  gluon "drives" g_1 at small x
  $\rightarrow$ **Δg down to $x \simeq 0.0005$ accessible at eRHIC**

power tool !

small x uncertainty in DSSV



Can we locate the missing proton spin?

Yes We Can!

many crucial measurements from RHIC still to come

W-boson program, jet-correlations, prompt photons,...

→ better constraints on helicity PDFs, extended x-range

ongoing and future fixed target experiments

COMPASS, JLab-12GeV → large-x frontier

plus Lattice QCD and progress on GPDs

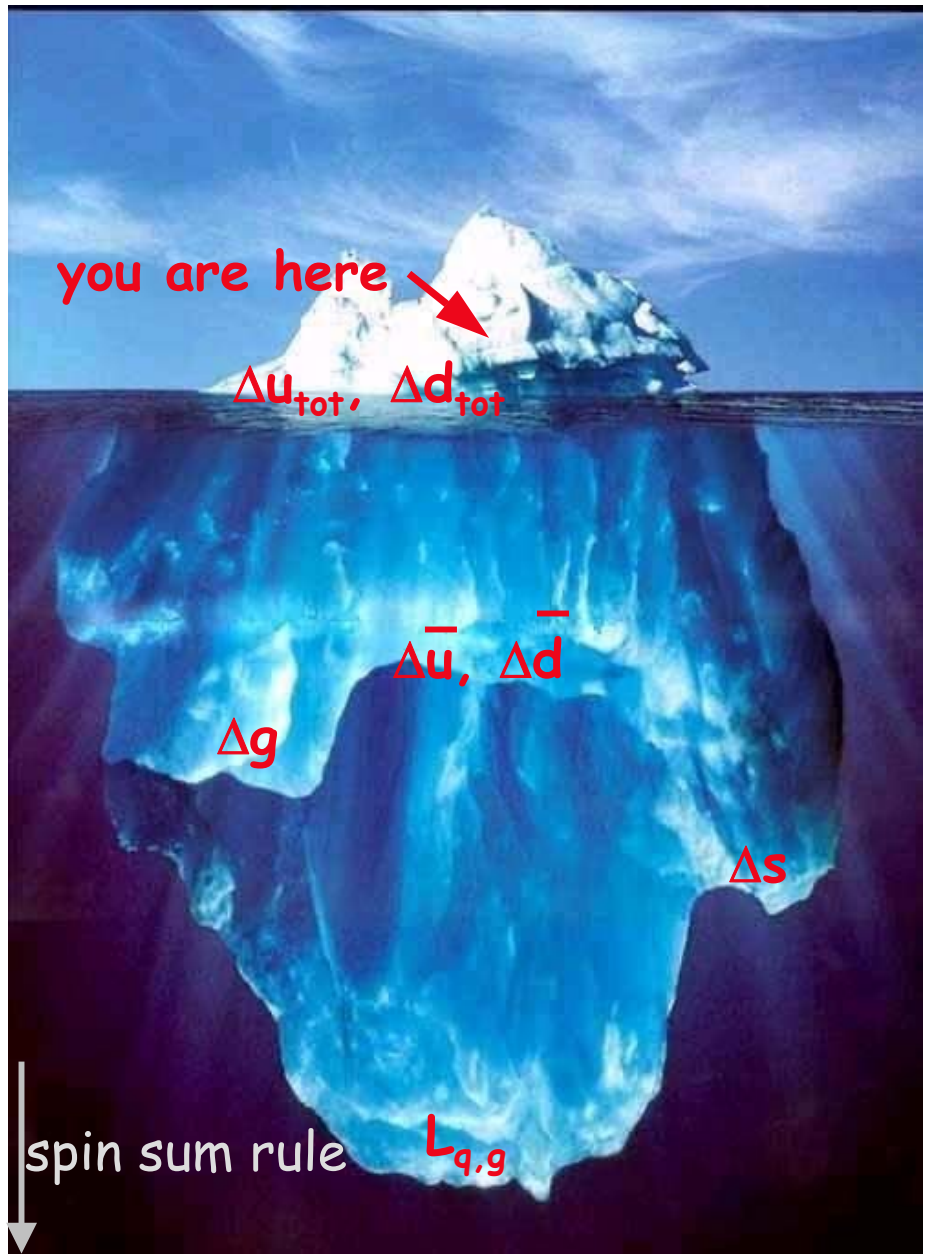
however, history (DESY-HERA) suggests that extrapolations towards small-x are notoriously unreliable

we need to go to smaller x to settle this issue

→ case for a high-energy polarized ep-collider

conclusions

we have just explored the tip of the iceberg

Δg and sea polarizations start to surface

many avenues for further important measurements and theoretical developments