1 Interactions of a Nearby Supernova

a) Taking

$$nkT = \frac{3}{16}\rho_{\rm csm} v_{\rm sh}^2 \tag{1}$$

and the fact $\mu m_p n = \rho$, where μ is the mean atomic weight. We see that

$$T = \frac{3}{16}\frac{\mu m_p v_{\rm sh}^2}{k} \approx 3 \times 10^9 \tag{2}$$

This is quite hot. We know that $10^4 K$ is roughly 1 eV, so this corresponds to 300 keV. The optically thin free-free spectrum (in frequency units) is roughly flat up until the cutoff frequency at kT. Therefore in the Chandra bands around 1 keV, we expect a fairly flat spectrum.

b) The optically thin emission per unit volume due to free-free from fully ionized gas is, integrated over all frequencies,

$$\epsilon = A\rho^2 T^{1/2} \tag{3}$$

where A is a constant, specified on the webpage. The CSM density is given by

$$\rho_{\rm csm} = \frac{\dot{M}_w}{4\pi r^2 v_w} \tag{4}$$

after the shock, the material is compressed by a factor of 4, so $\rho = 4\rho_{\rm cam}$.

The luminosity from free-free is determined by integrating over the volume of the shell

$$L = \int_{r_{\rm in}}^{r_{\rm out}} 4\pi r^2 A\rho^2 T^{1/2} dr = 4\pi \int_{r_{\rm in}}^{r_{\rm out}} r^2 \left(\frac{4\dot{M}_w}{4\pi r^2 v_w}\right)^2 A T^{1/2} dr \tag{5}$$

$$L = \frac{4\dot{M}_w^2}{\pi v_w^2} A T^{1/2} \int_{r_{\rm in}}^{r_{\rm out}} r^{-2} dr = \frac{4\dot{M}_w^2}{\pi v_w^2} A T^{1/2} (r_{\rm in}^{-1} - r_{\rm out}^{-1}) \tag{6}$$

Using the relation $r = vt$ we can write this as

$$L = \frac{4\dot{M}_w^2}{\pi v_w^2}\frac{AT^{1/2}}{t}(v_{\rm in}^{-1} - v_{\rm out}^{-1}) \tag{7}$$

which scales as t^{-1}. Inverting this expression gives a limit for $\dot{M}_w$ in terms of L.

To calculate the optical depth we use

$$\tau = \int_{r_{\rm in}}^{r_{\rm out}} \alpha_{\rm ff} dr \tag{8}$$

where the extinction coefficient is

$$\alpha_{\rm ff} = B\rho^2 T^{-3/2}\nu^{-2} \tag{9}$$

where B is a constant and we have used the limit $h\nu \ll kT$ for the Chandra observations. Integrating and plugging in numbers, one finds that the shell is very optically thin at these x-ray wavelengths.

2 Powering Radio Lobes

a) The observed spectrum looks like a power-law $F \propto \nu^{-\alpha}$, where $\alpha = 0.8$. From synchrotron theory we know the spectrum is given by $\alpha = (p-1)/2$, so we find $p = 2\alpha + 1 = 2.6$.

b) The energy of an electron is $\gamma m_e c^2$ so we write the energy density as

$$u_e = \int_{\gamma_{\min}}^{\gamma_{\max}} \gamma m_e c^2 n d\gamma = \int_{\gamma_{\min}}^{\gamma_{\max}} m_e c^2 C \gamma^{-p+1} d\gamma \tag{10}$$

Integrating we find

$$u_e = \frac{C}{p-2} m_e c^2 \gamma_{\min}^{-p+2} \tag{11}$$

where we take $\gamma_{\max}$ to be essentially infinity. So we see

$$u_e \propto \gamma_{\min}^{-p+2} \tag{12}$$

c) The expression for the synchrotron specific luminosity is

$$L \propto \frac{V u_B}{\nu_{\rm cyc}} (\nu/\nu_{\rm cyc})^{-(p-1)/2} \tag{13}$$

We know $\nu/\nu_{\rm cyc} = \gamma^2$, so we can write this at a the minimum frequency, ν_m, as

$$L_m \propto \frac{V u_B}{\nu_{\rm cyc}} \gamma_{\min}^{-p+1} \tag{14}$$

We now replace $\gamma_{\min}$ with the electron energy density. Rewrite the expression as

$$L_m \propto \frac{V u_B}{\nu_{\rm cyc}} \gamma_{\min}^{-p+2} \gamma_{\min}^{-1} \propto \frac{V u_B u_e}{\nu_{\rm cyc}} \gamma_{\min}^{-1} \propto \frac{V u_B u_e}{\nu_{\rm cyc}} (\nu_{\rm cyc}/\nu_m)^{1/2} \tag{15}$$

Since $U_B \propto B^2$ and $\nu_{\rm cyc} \propto B$, we see

$$L_m \propto V B^{3/2} u_e v_m^{-1/2} \tag{16}$$

and solving for u_e

$$u_e = A L_m V^{-1} B^{-3/2} v_m^{1/2} \tag{17}$$

where A is some constant.

d) The total energy density is

$$u_{\rm tot} = u_e + u_B = A_1 B^{-3/2} + A_2 B^2 \tag{18}$$

where A_1, A_2 are some constants. To minimize with respect to the magnetic field we take the derivative

$$\frac{\partial u_{\rm tot}}{\partial B} = -3/2 A_1 B^{-5/2} + 2 A_2 B = -\frac{3}{2} u_e/B + 2 u_B/B = 0 \tag{19}$$

Solving we see that

$$u_B = \frac{3}{4} u_e \tag{20}$$