# *Ab initio* calculations for p-shell nuclei using effective field theory interactions with the similarity renormalization group method

The George Bertsch Symposium

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# Ab initio approaches to nuclear physics

Goal: Answer the question whether it is possible to describe the basic properties of atomic nuclei (structure and reactions) from the point of view of point-like nucleons with “fundamental” inter-nucleon interactions.

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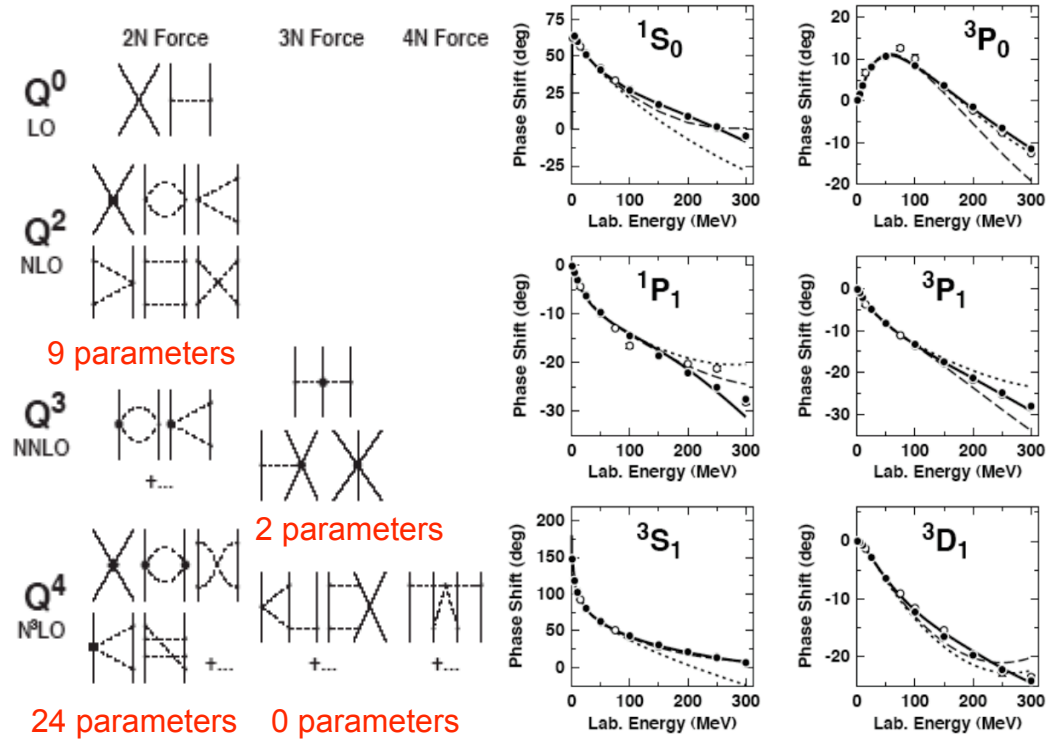
High-performance computing and new theoretical methods have been the source of a renaissance in nuclear physics

## Be careful how you solve your problem

"You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!" – Steven Weinberg

# "Realistic" Interactions

- There is no **fundamental** NN interaction
  - The underlying physics is QCD – quarks and gluons
  - But we want to treat nuclei as a collection of nucleons
  - We build phenomenological potentials built on the NN and NNN systems
    - Argonne potentials
    - Effective Field Theory (EFT)

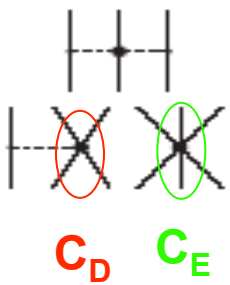



Cutoff Parameters: $Q/\Lambda$

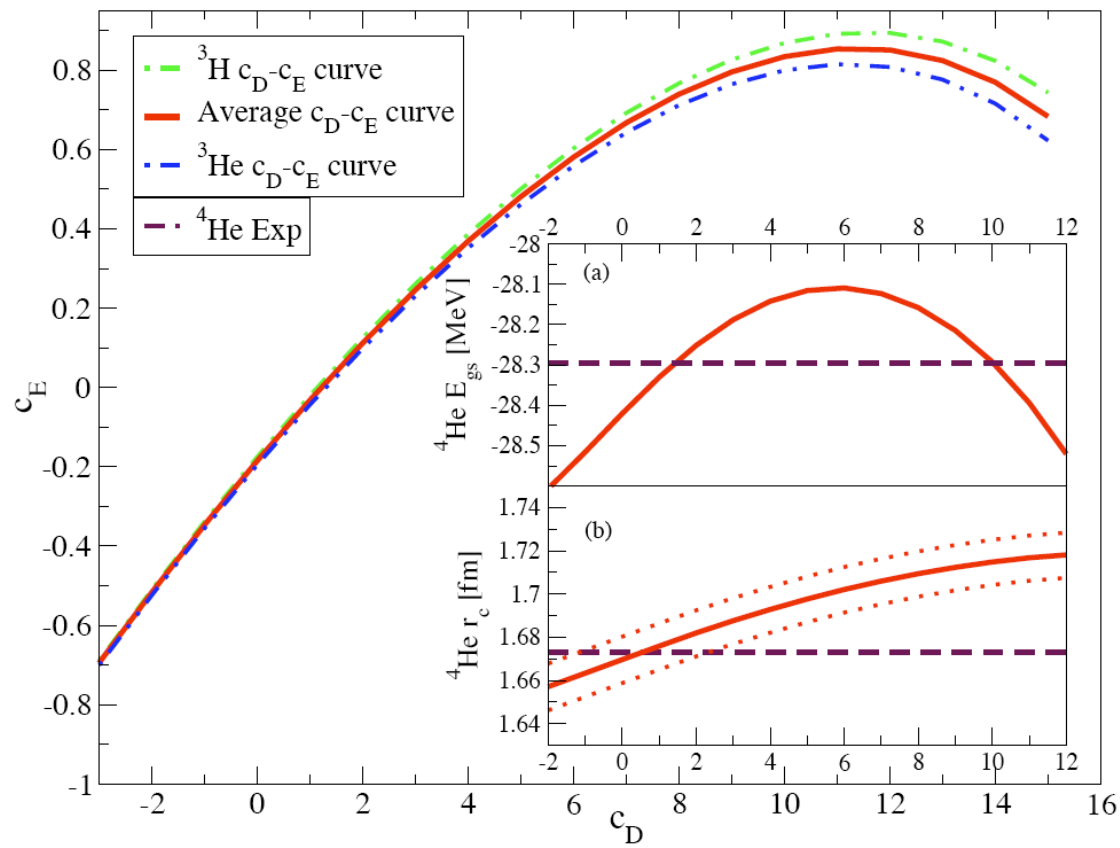
EFT- two-body N³LO, $\chi^2/\nu \sim 1$:
Entem et al., PRC **68**, 041001 (2003)

# Three-body interactions

- $N^2LO$ three-body



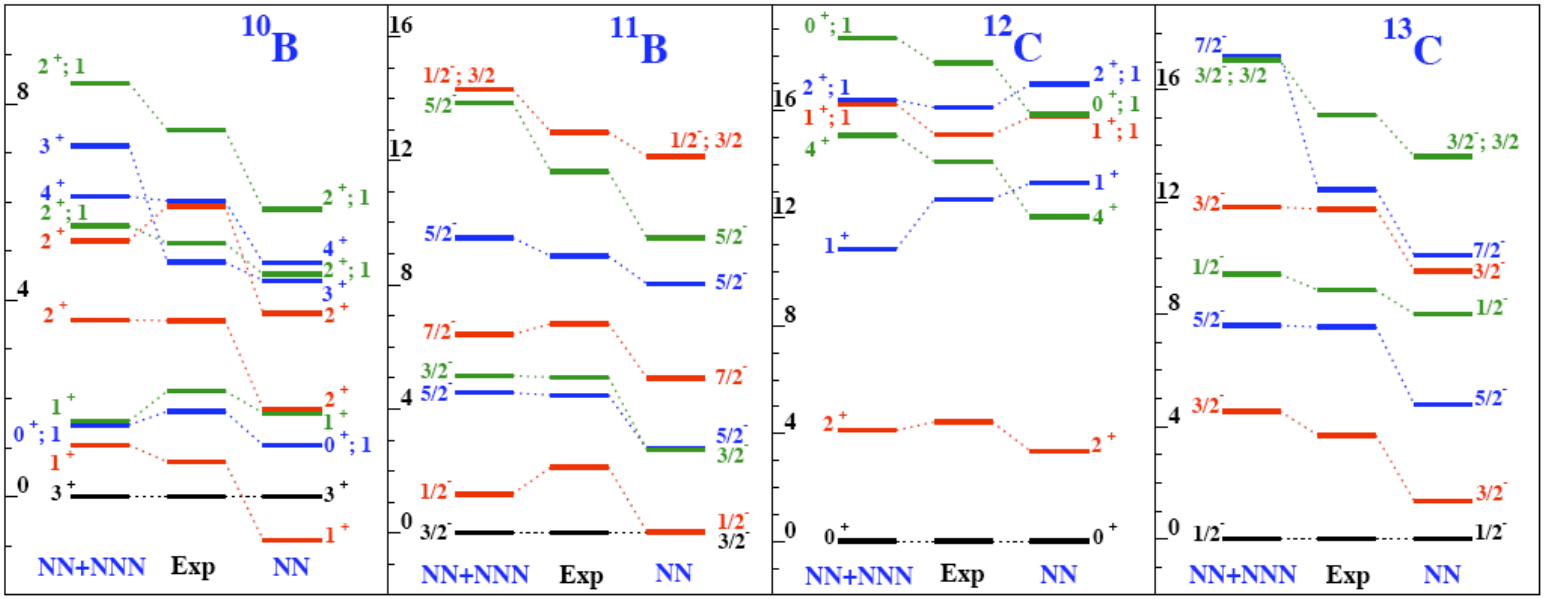

- Fine tune $C_D$ with Tritium beta-decay lifetime (Gazit, Quaglioni, Navratil)



# The three-nucleon interaction is important

- Spin-orbit physics comes from

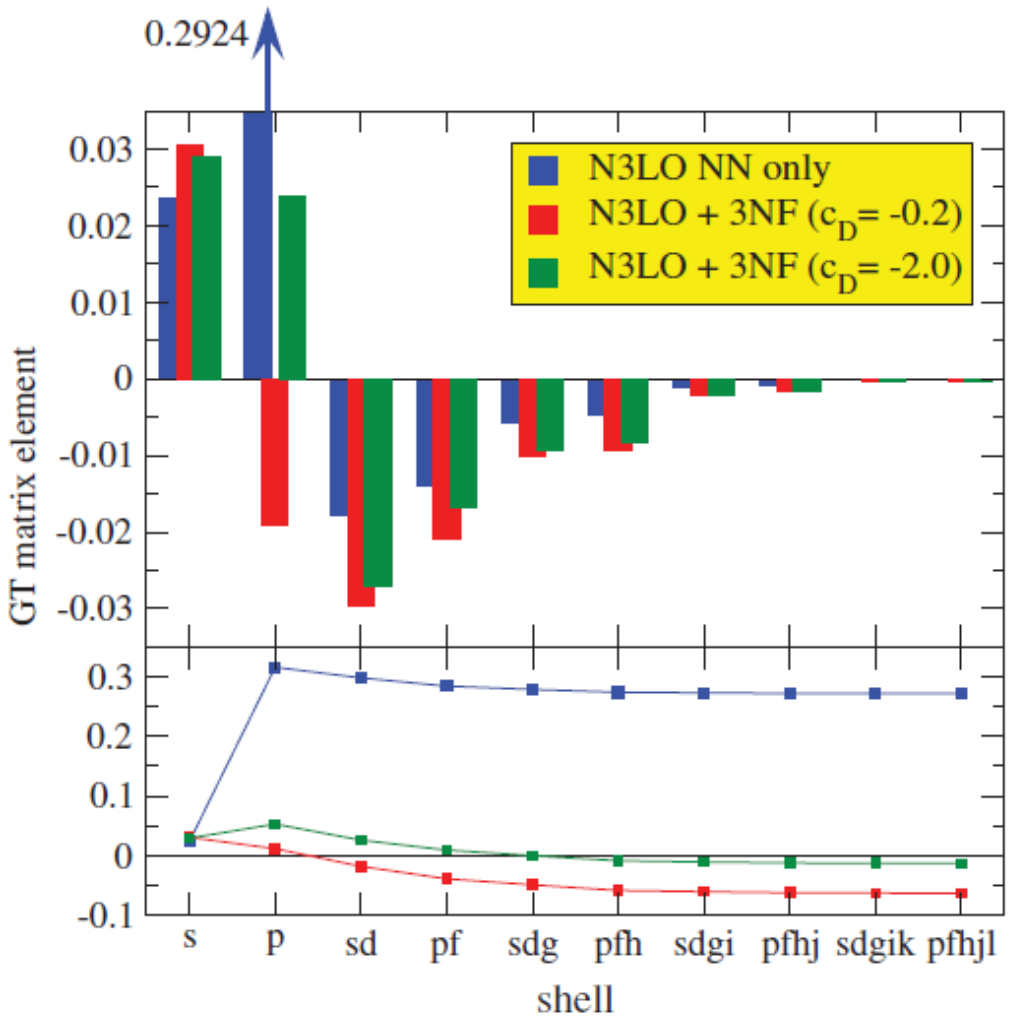
- While these prevent collapse




**3NF improves the description of nuclear states**

P. Navrátil, V. G. Gueorguiev, J. P. Vary, W. E. Ormand, and A. Nogga
Phys. Rev. Lett. **99**, 042501

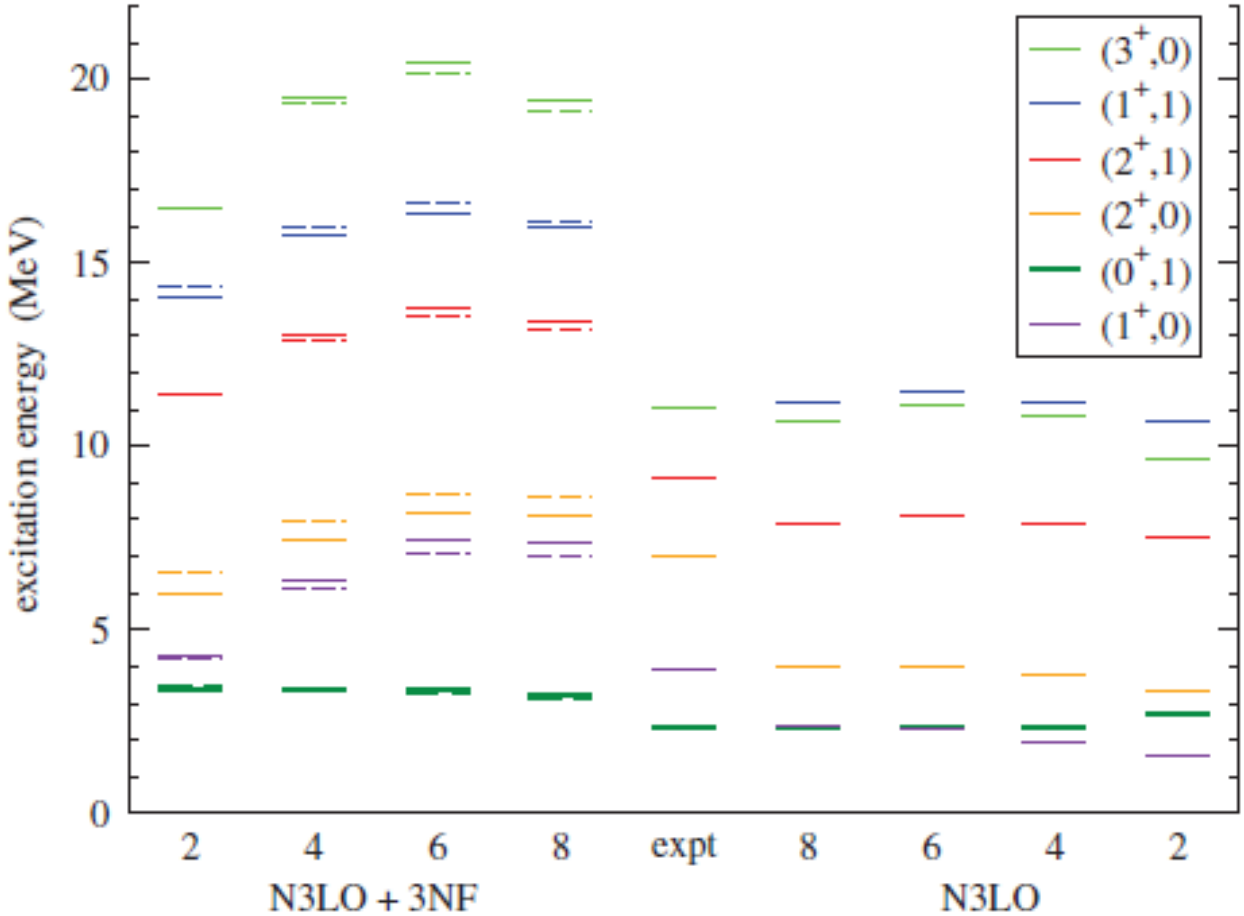
# Anomalously long lifetime of $^{14}C$




Largest *ab initio* CI calculations performed, 10M CPU hr on JAGUAR

P. Maris, J. P. Vary, P. Navrátil, W. E. Ormand, H. Nam, and D. J. Dean
Phys. Rev. Lett. 106, 202502

# Anomalously long lifetime of $^{14}C$

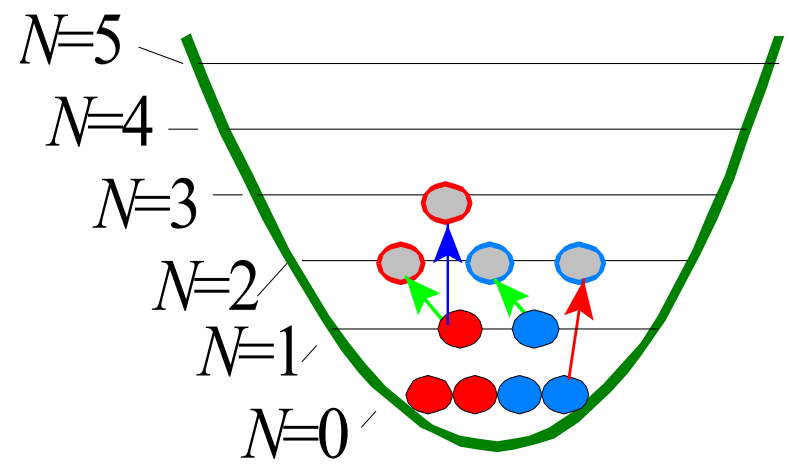



But the spectrum could be better

# No-Core Shell Model

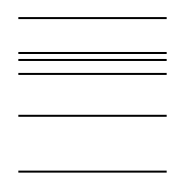
- Ab initio approach based on “realistic” NN, NNN, and possibly NNNN interactions
- Configuration-interaction method with Harmonic-oscillator basis
- Effective interactions
  - Okubo-Lee-Suzuki (OLS)
  - Similarity-Renormalization Group method (SRG)

**Cutoff Parameters:**



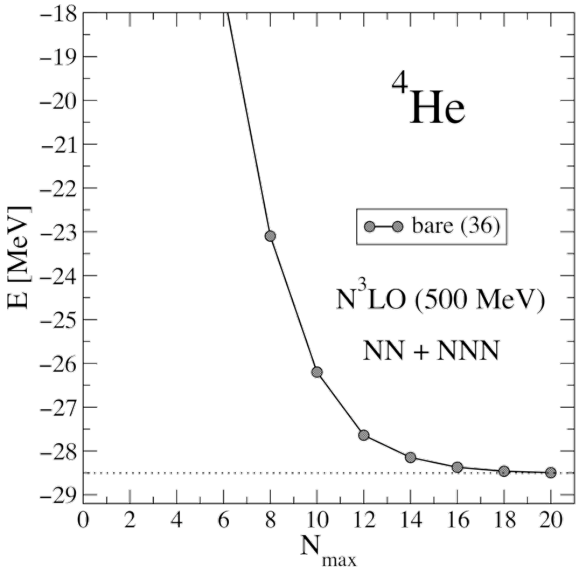

$$\phi = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_i(\mathbf{r}_1) & \phi_i(\mathbf{r}_2) & \cdots & \phi_i(\mathbf{r}_A) \\ \phi_j(\mathbf{r}_1) & \phi_j(\mathbf{r}_2) & & \phi_j(\mathbf{r}_A) \\ & \vdots & \ddots & \vdots \\ \phi_l(\mathbf{r}_1) & \phi_l(\mathbf{r}_2) & \cdots & \phi_l(\mathbf{r}_A) \end{vmatrix} = a_l^+ \cdots a_j^+ a_i^+ |0\rangle$$

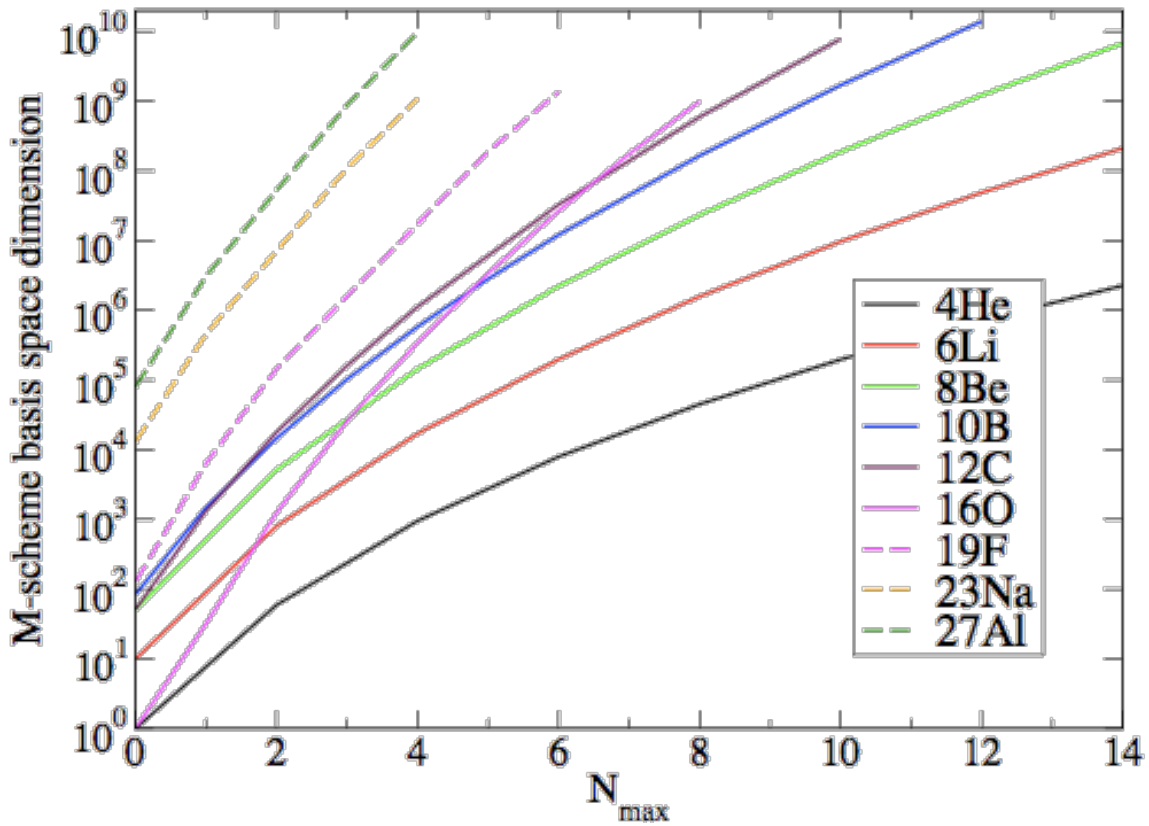
$$H_{ij} = \langle \phi_j | H | \phi_i \rangle \longrightarrow \begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1N} \\ H_{21} & H_{22} & & \\ \vdots & & \ddots & \\ H_{N1} & & \cdots & H_{NN} \end{pmatrix} \longrightarrow$$



# Convergence in the oscillator basis

- Oscillator basis scaling is harsh
- $^4$He is hard
- $^{12}$C is intractable




- Need substantial reduction in required basis size

➔ need smarter way to solve this problem!

# SRG to the rescue – effective interactions

- Similarity-Renormalization group (SRG)
  - Variational with model space size $N_{max}$
  - Study behaviors as a function of $\lambda$
  - Somewhat easier to use
  - Induced NNN, NNNN, etc terms

$$H_s = U_s H U_s^+ \equiv T_{rel} + V_s$$

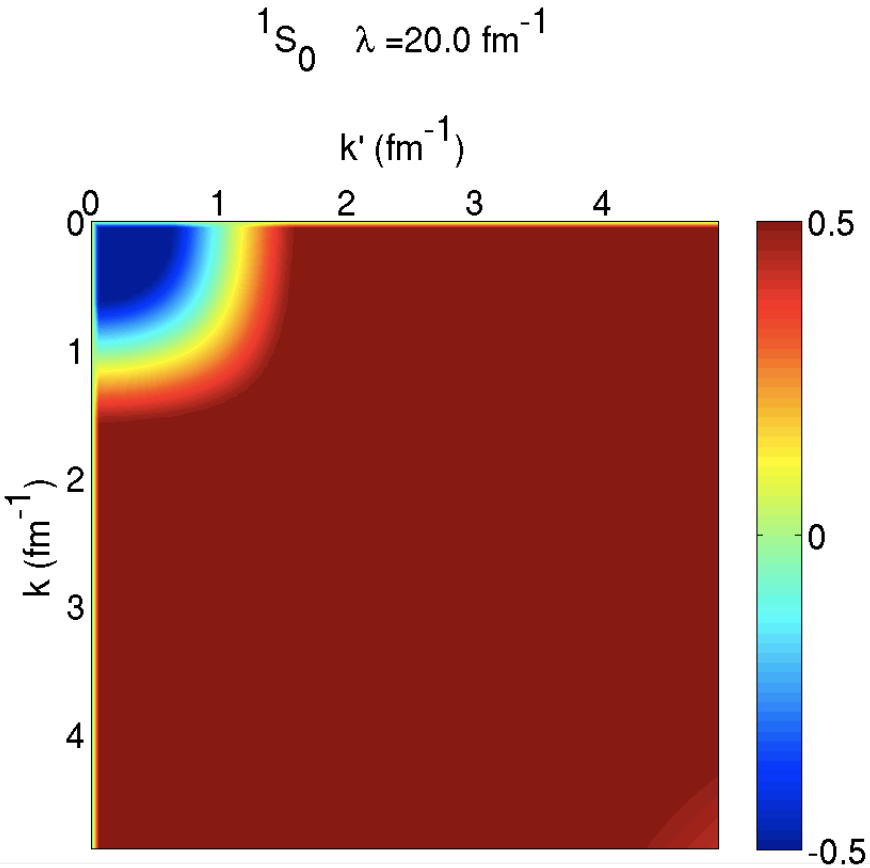
$$\frac{dH_s}{ds} = \left[\left[G_s, H_s\right], H_s\right]$$

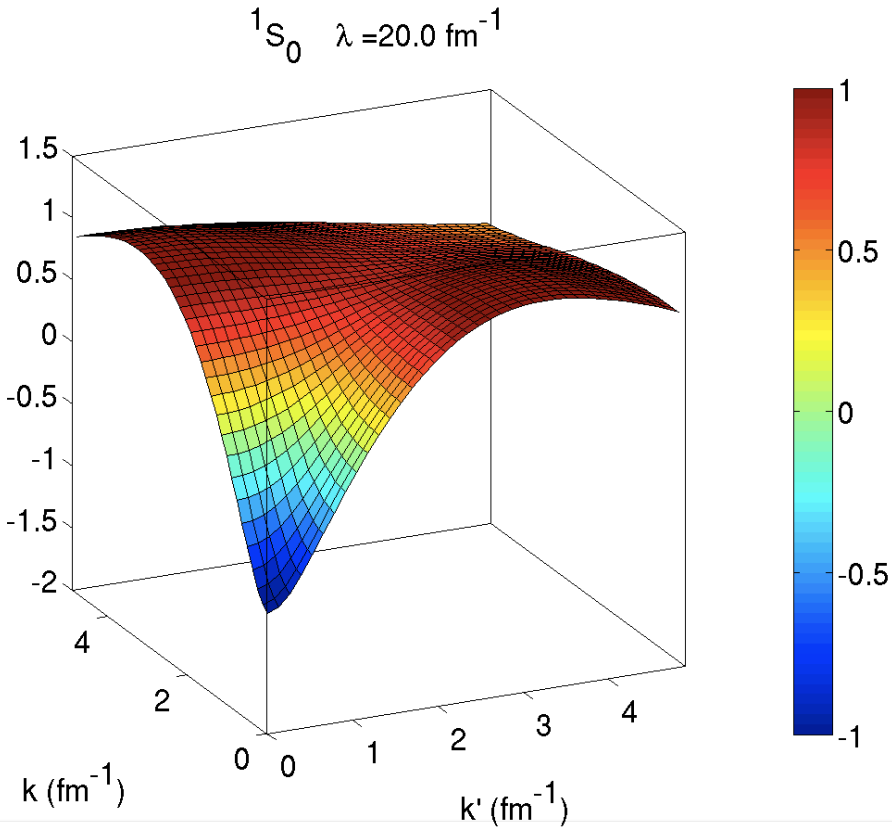
$$G_s = T_{rel}, H_{diag}, H_{BD}, \exp[-T], \cdots$$

Cutoff Parameters: $s=1/\lambda^4$

# SRG evolves Hamiltonians unitarily

$$H_s = U_s H U_s^+ \Rightarrow \frac{dH_s}{ds} = \left[\left[T, H_s\right], H_s\right] \qquad \left(s = 1/\lambda^4\right)$$



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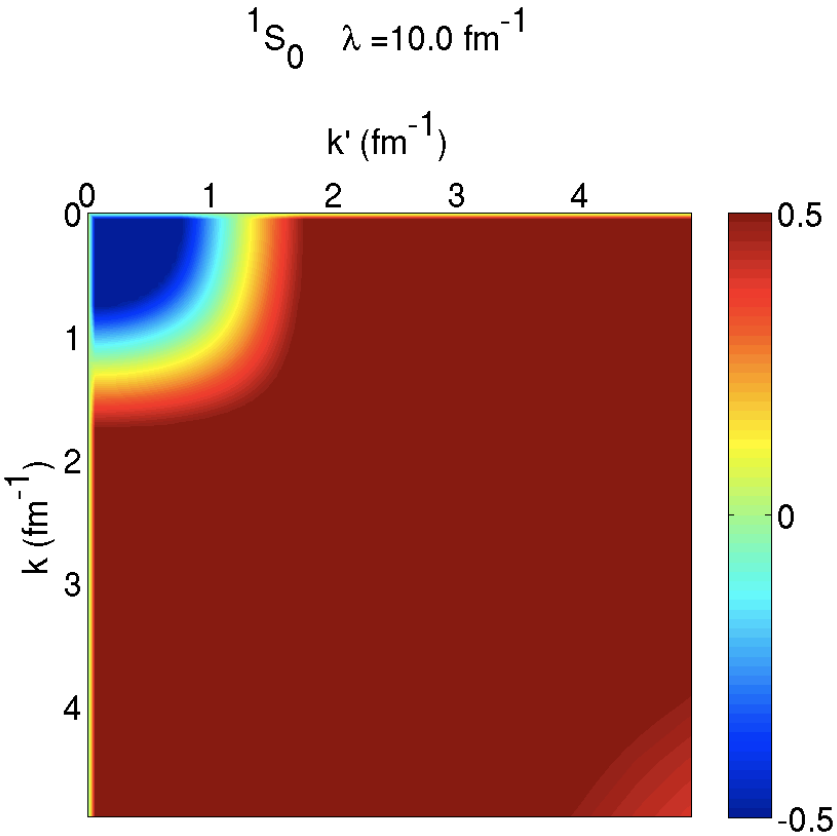


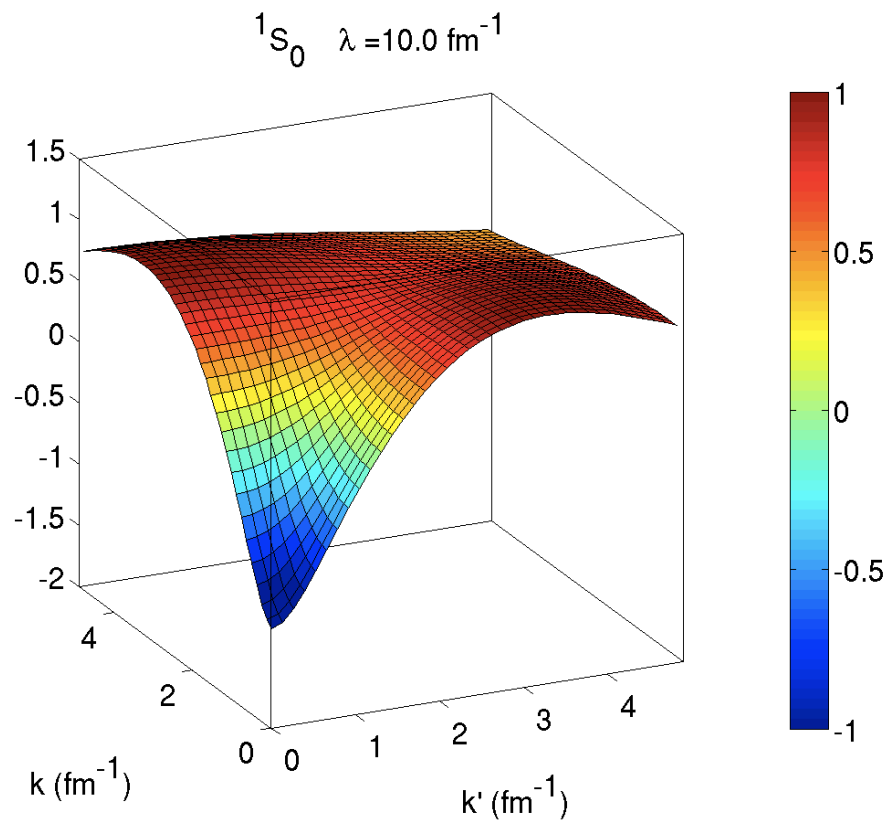

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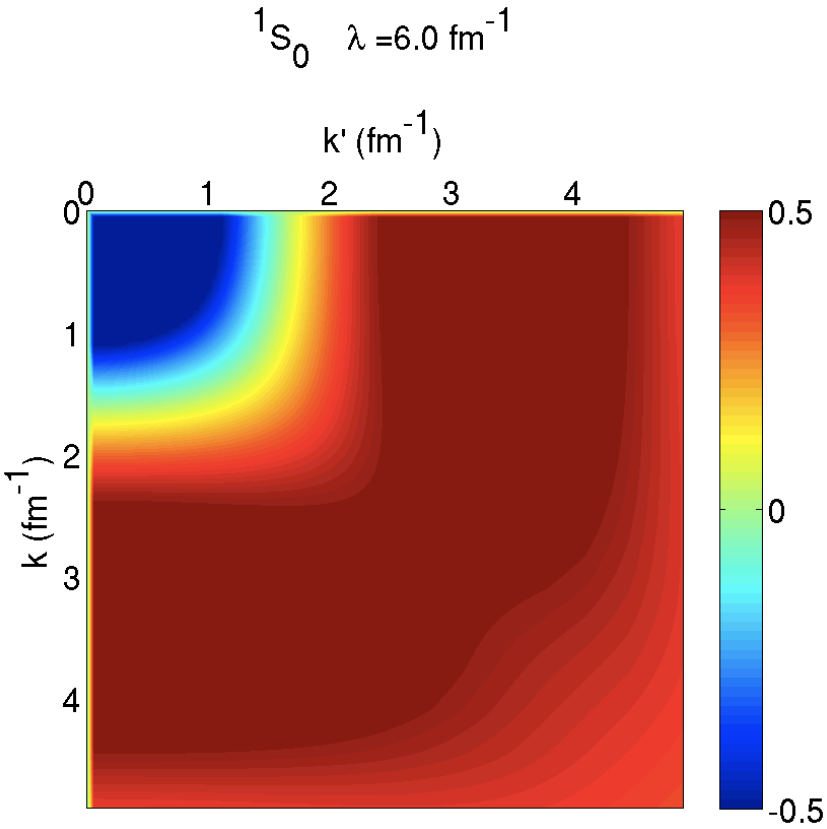


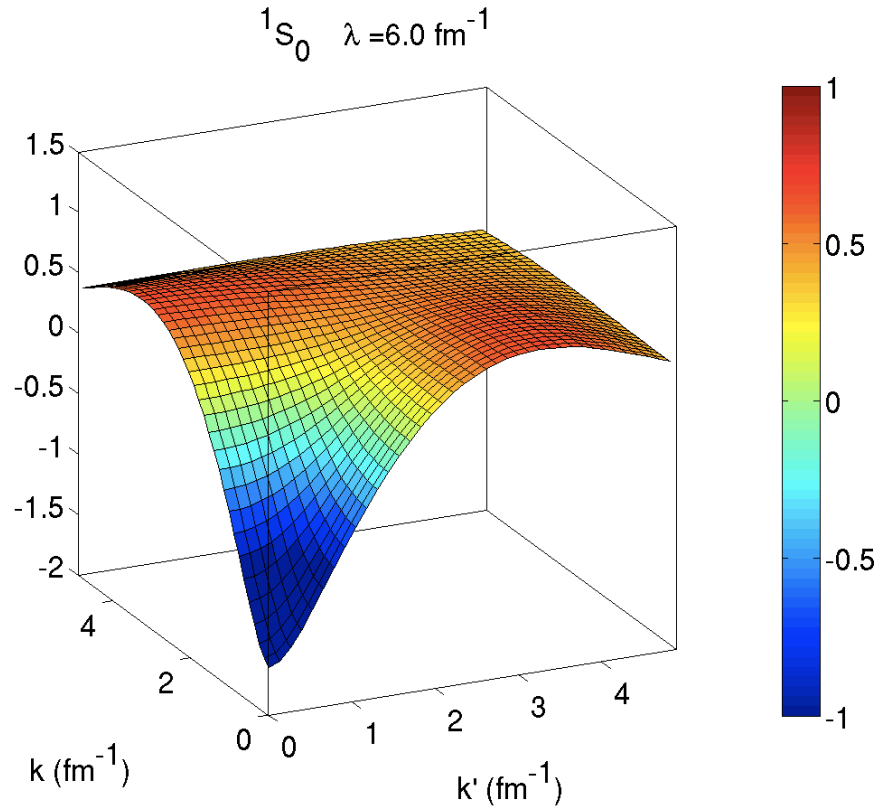

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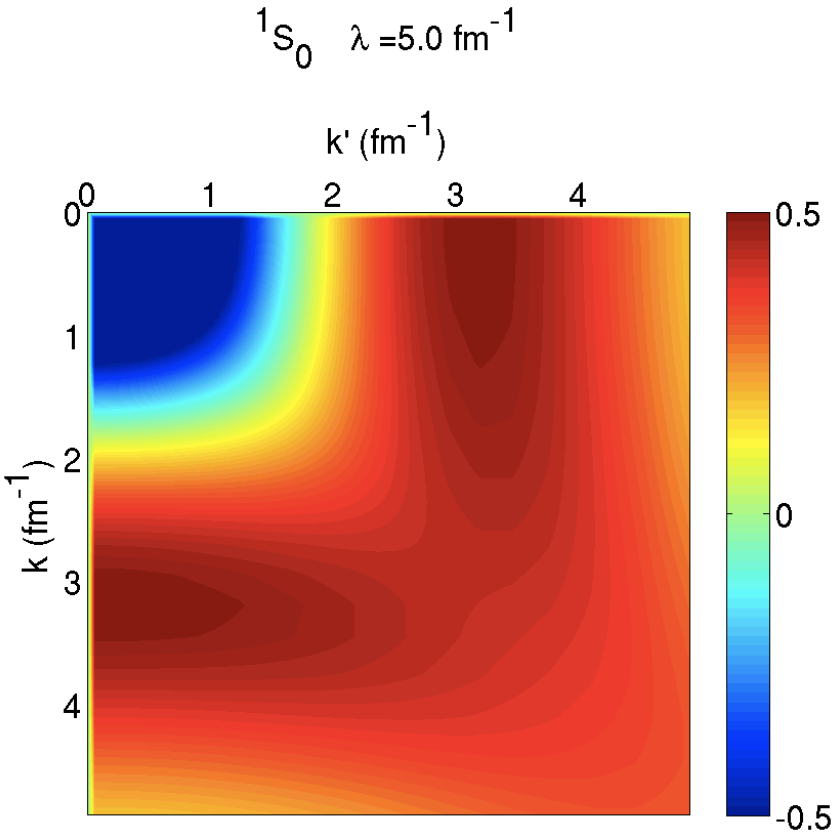


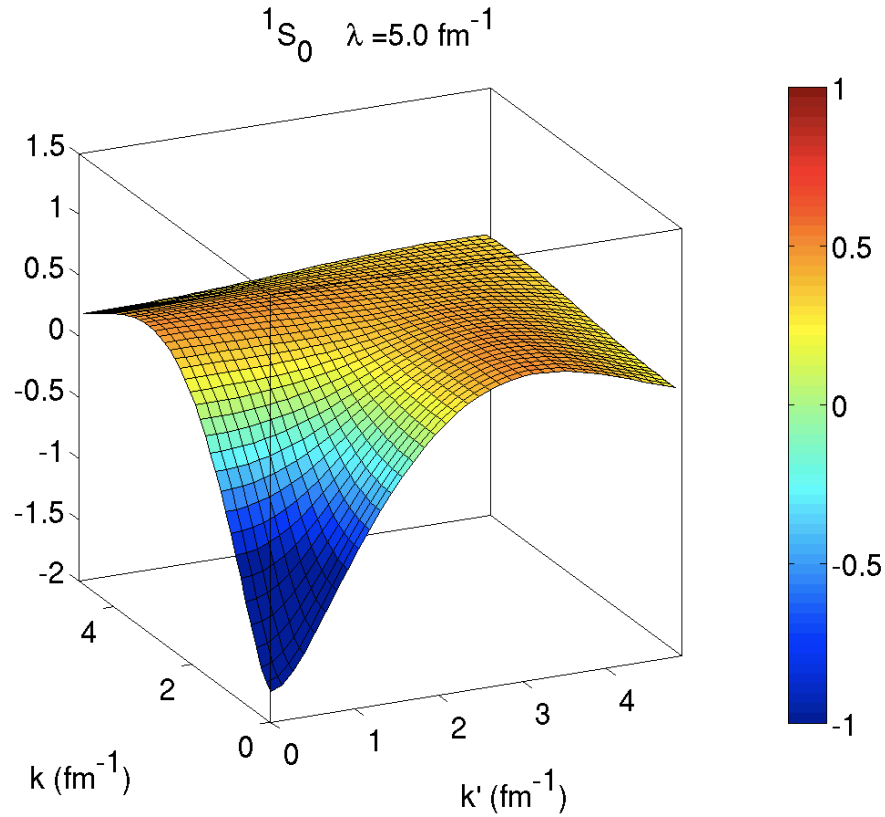

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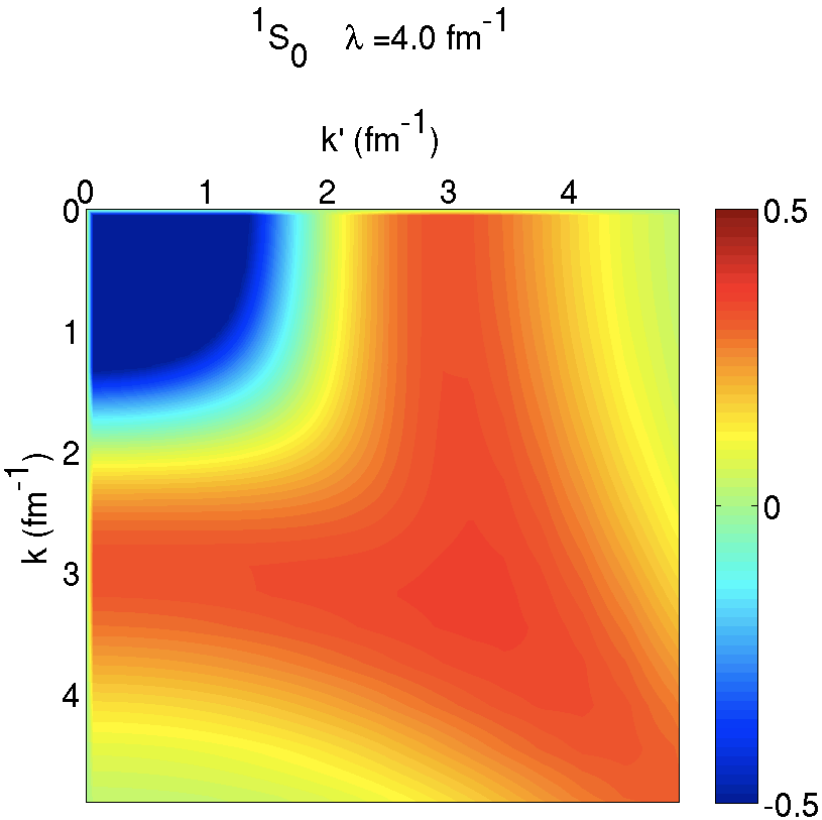


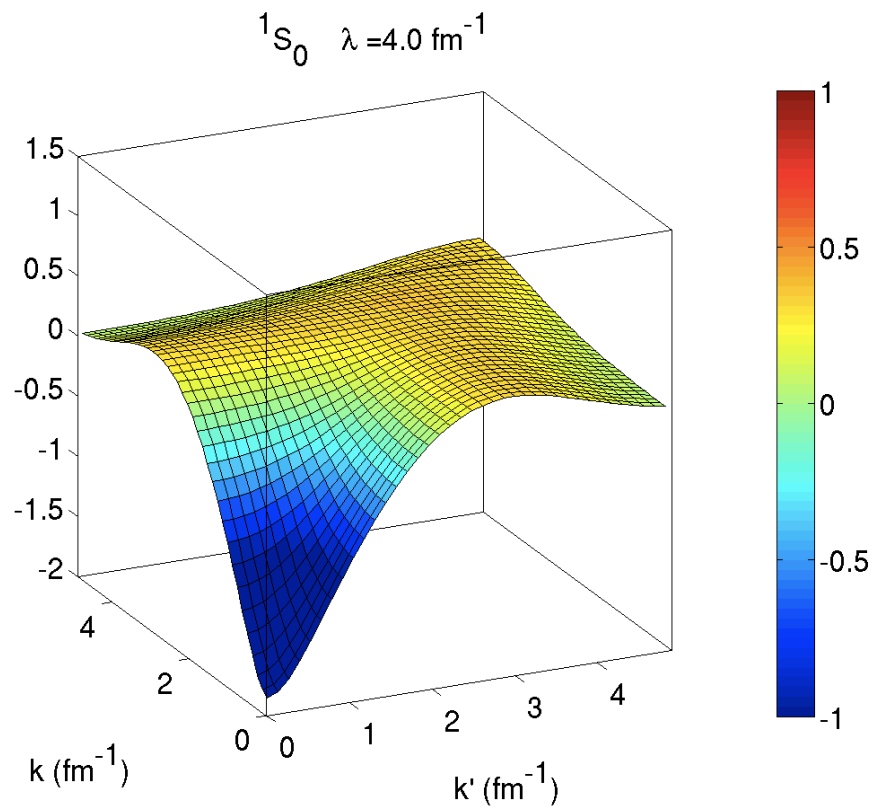

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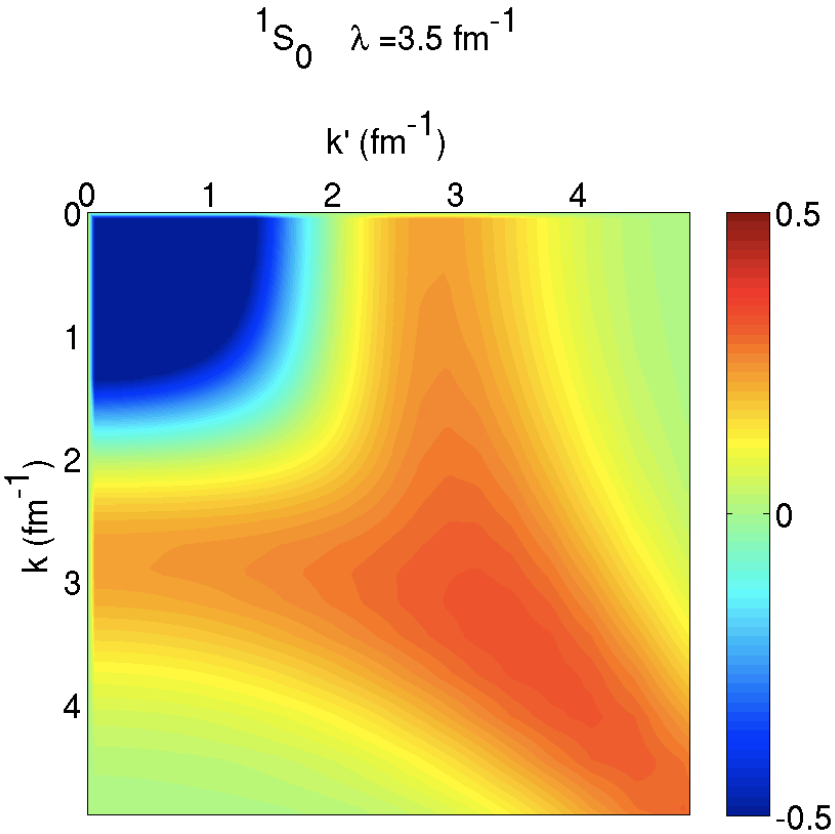


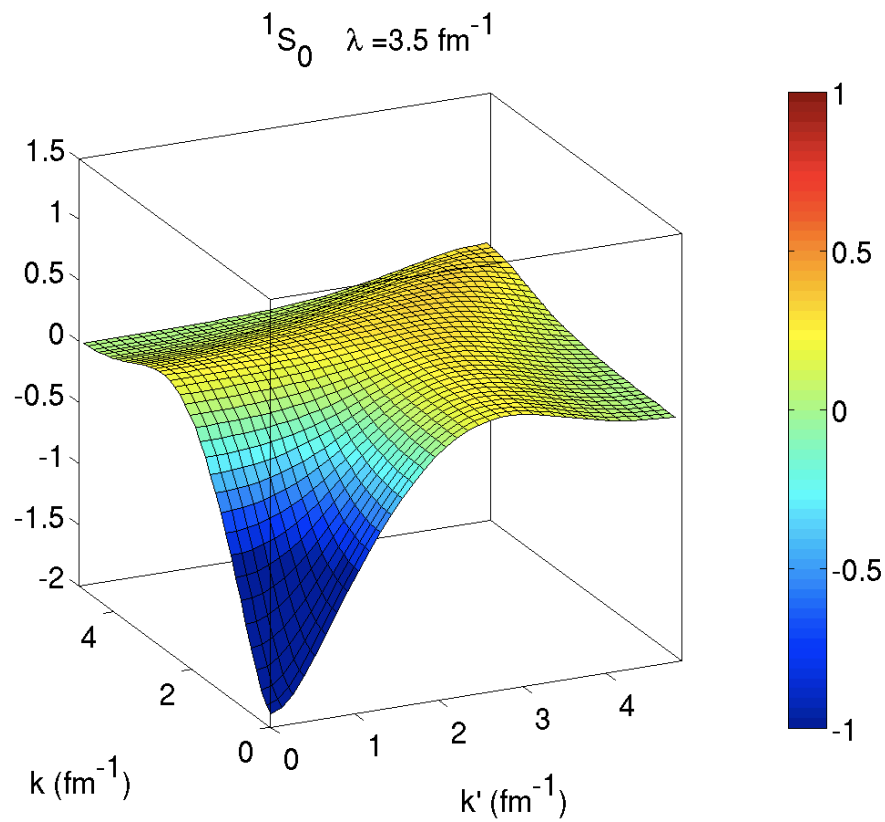

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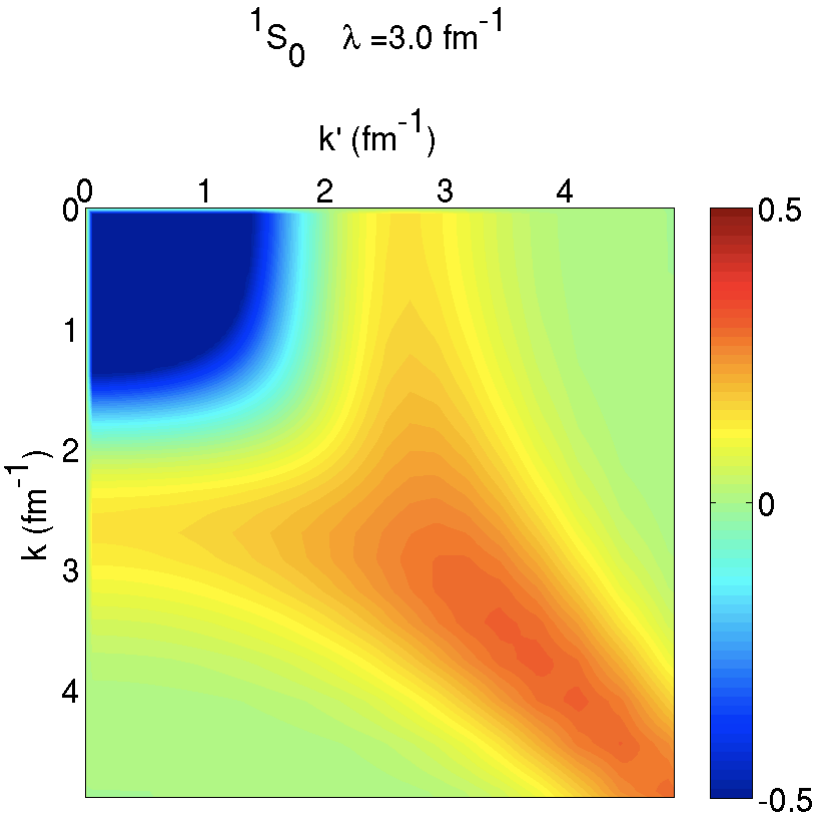


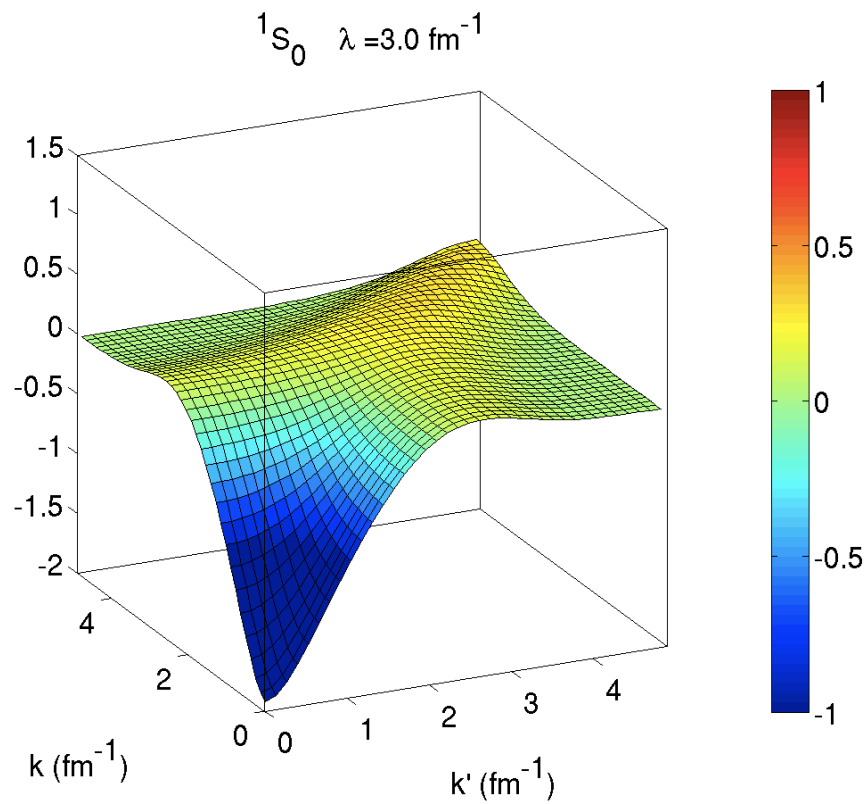

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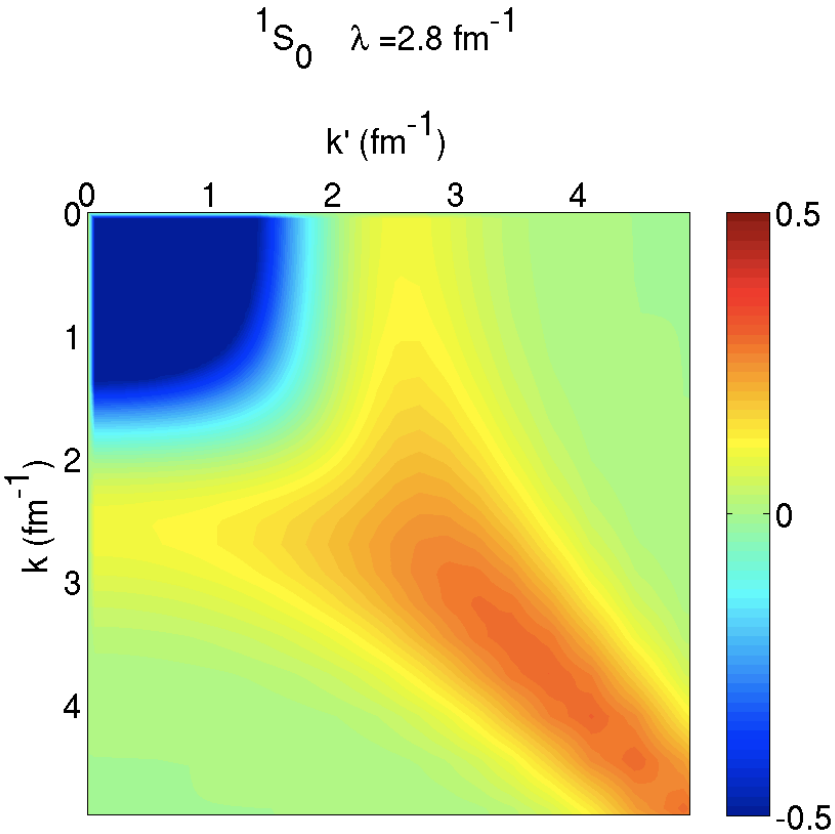


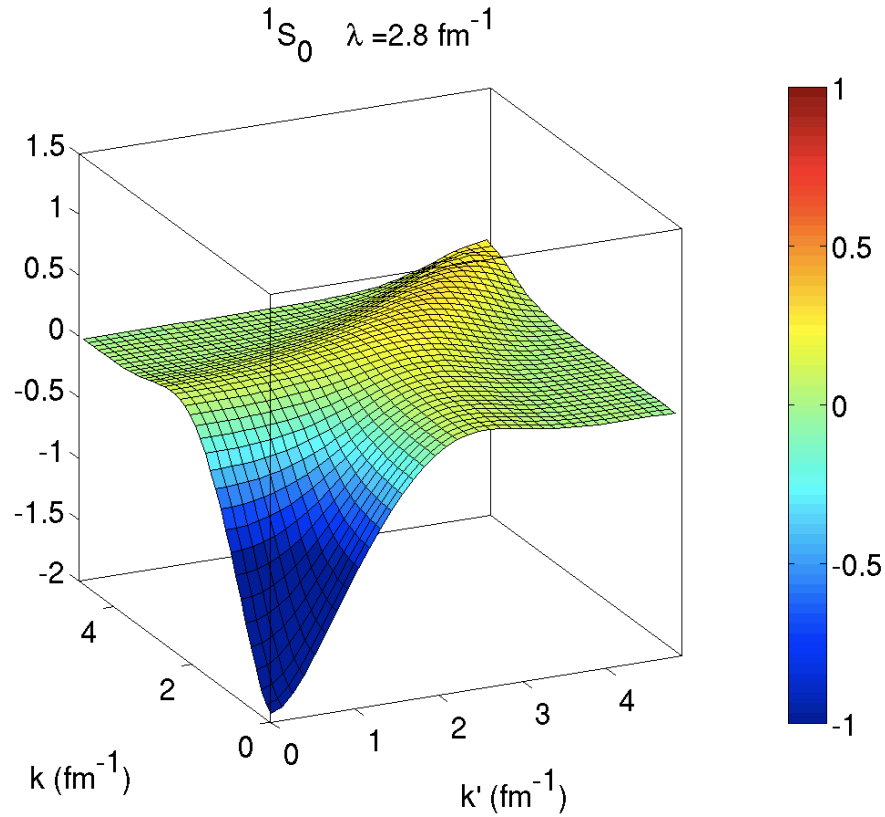

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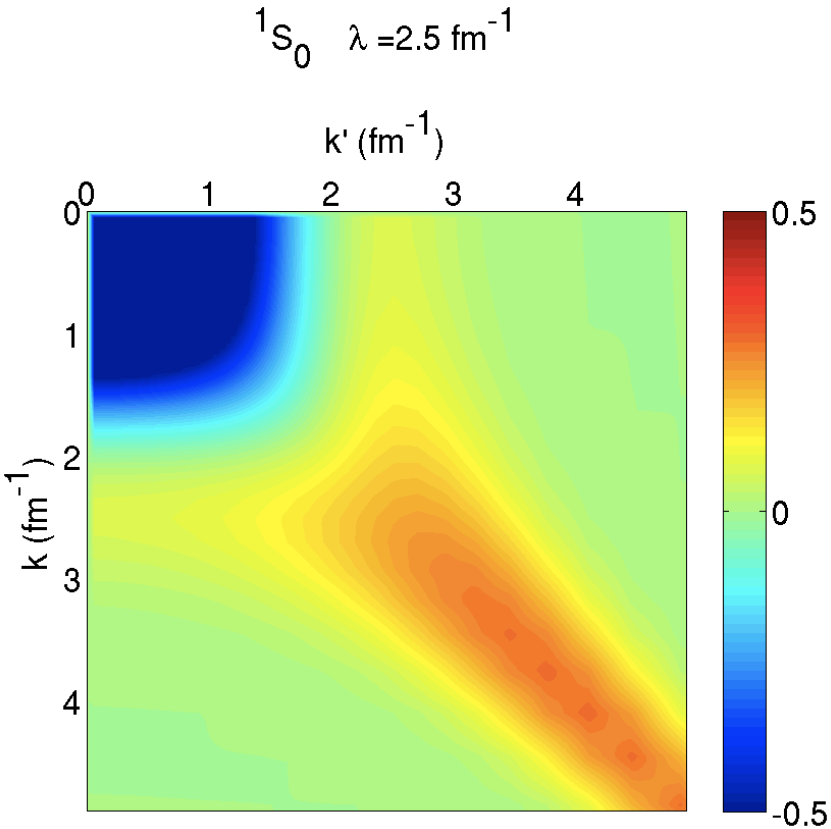


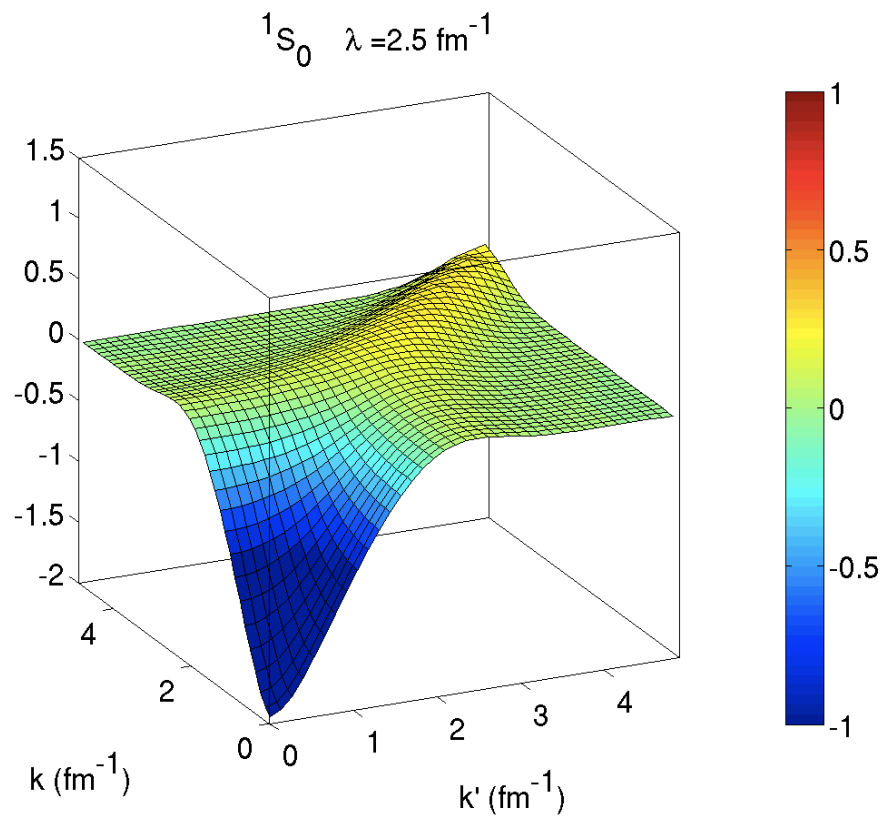

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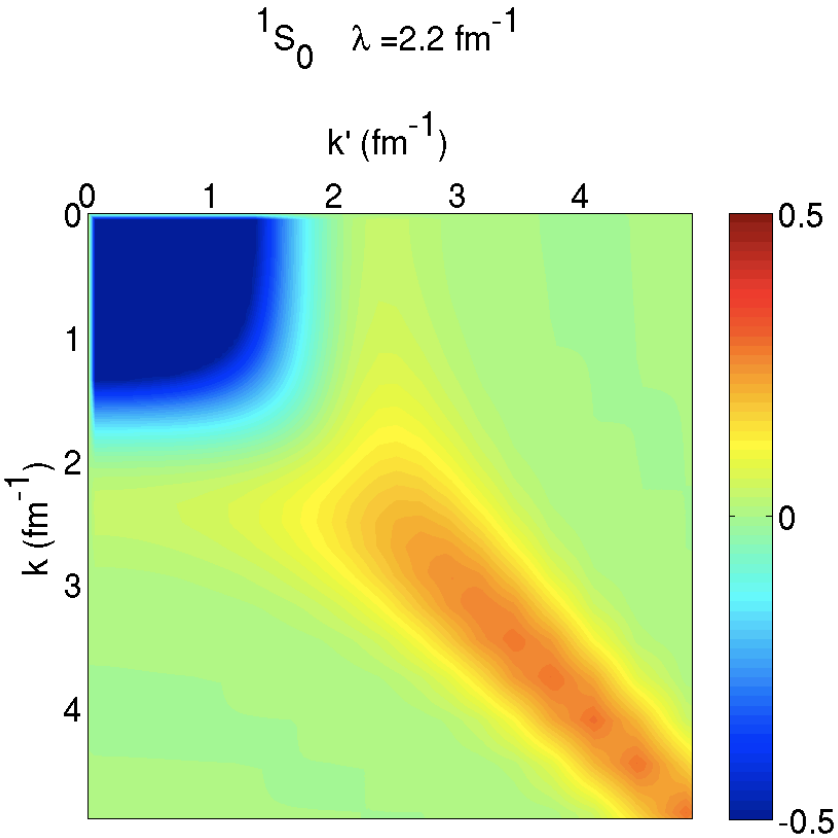


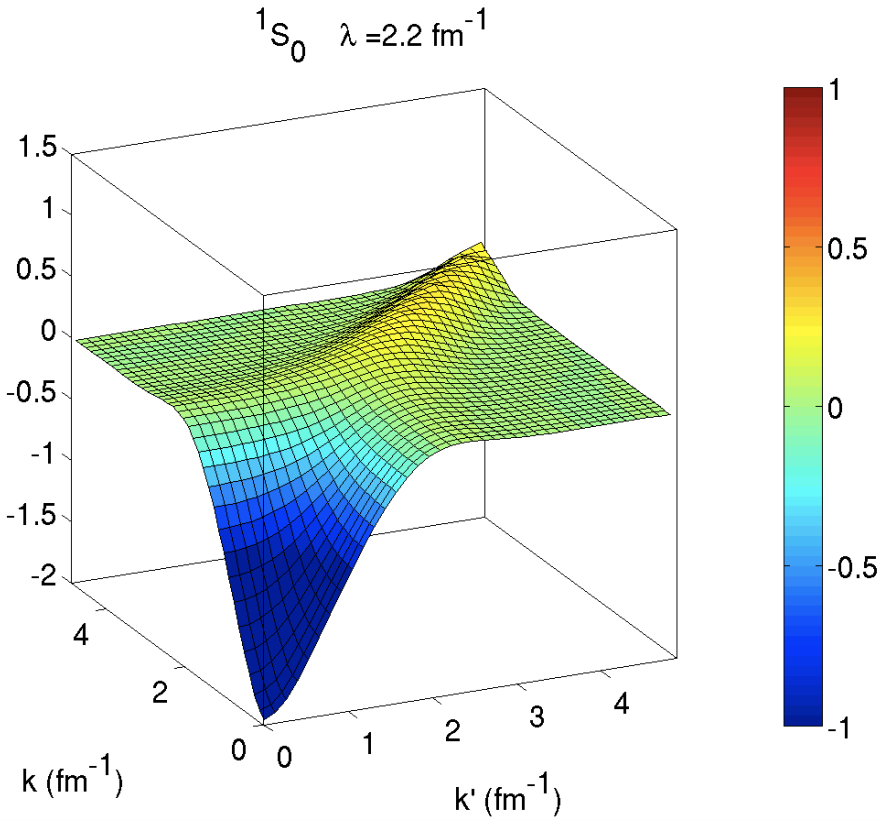

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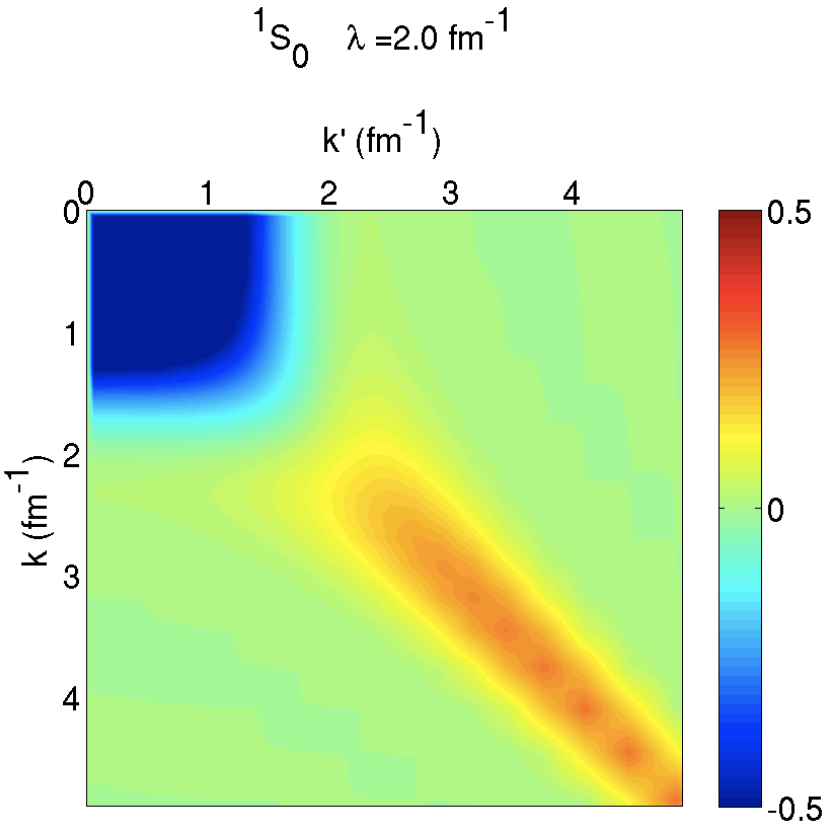
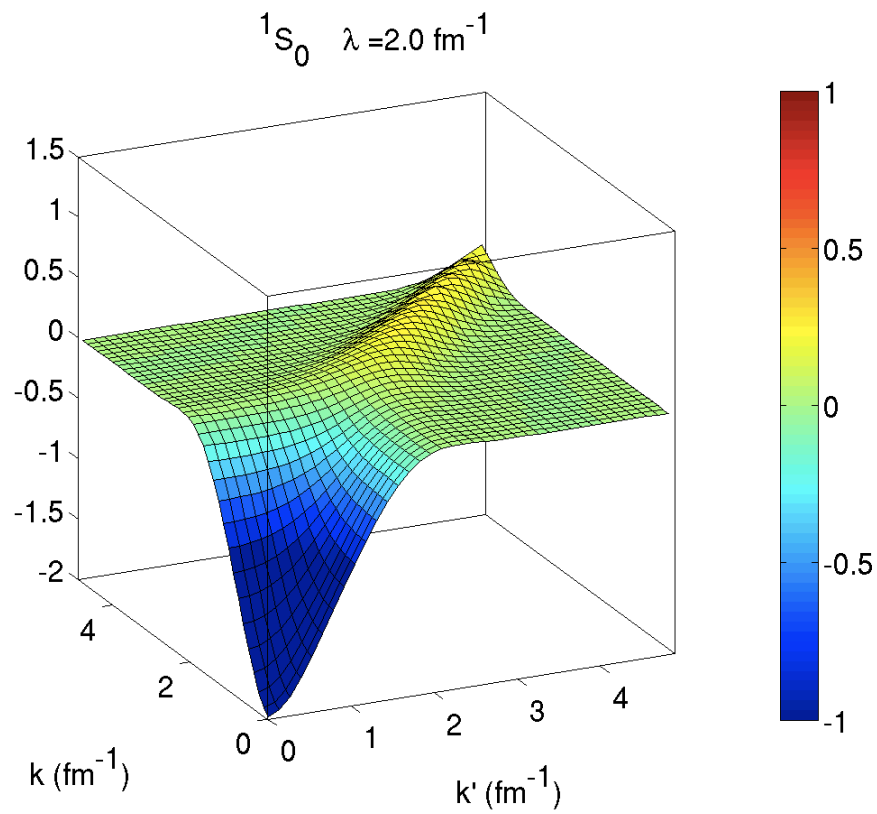
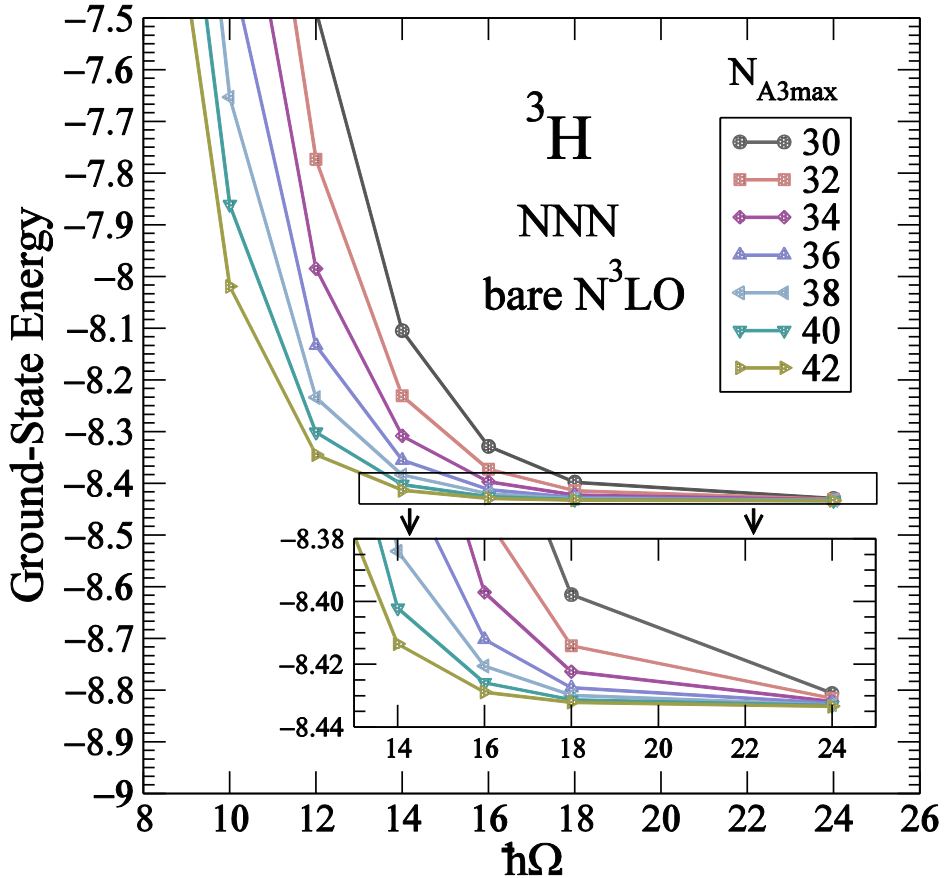


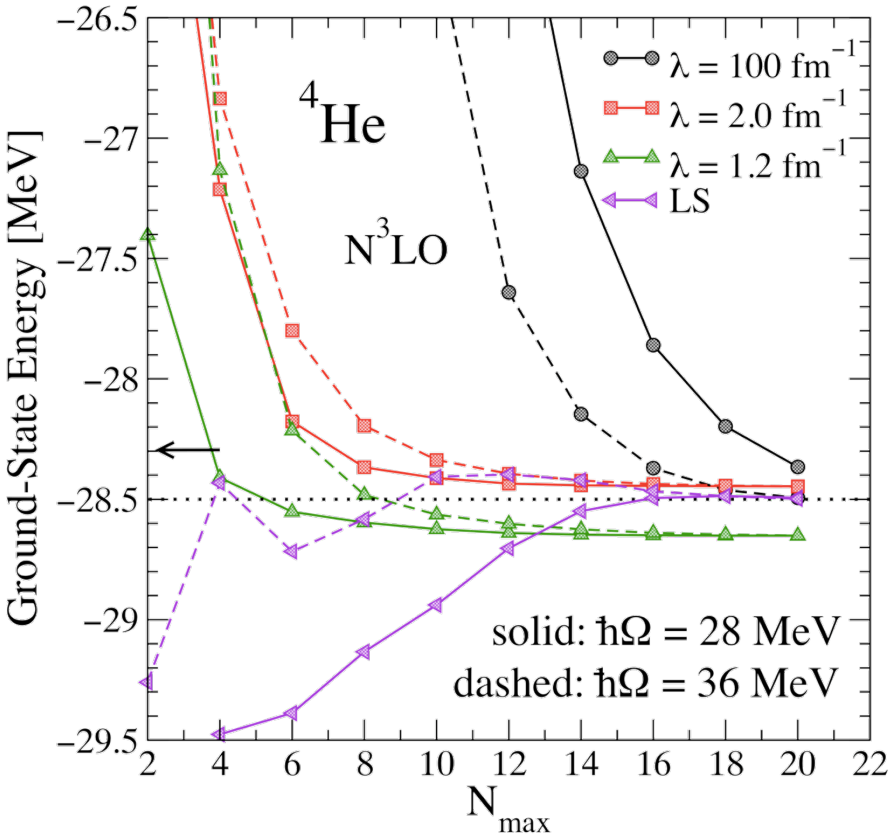
# Convergence of $^3$H

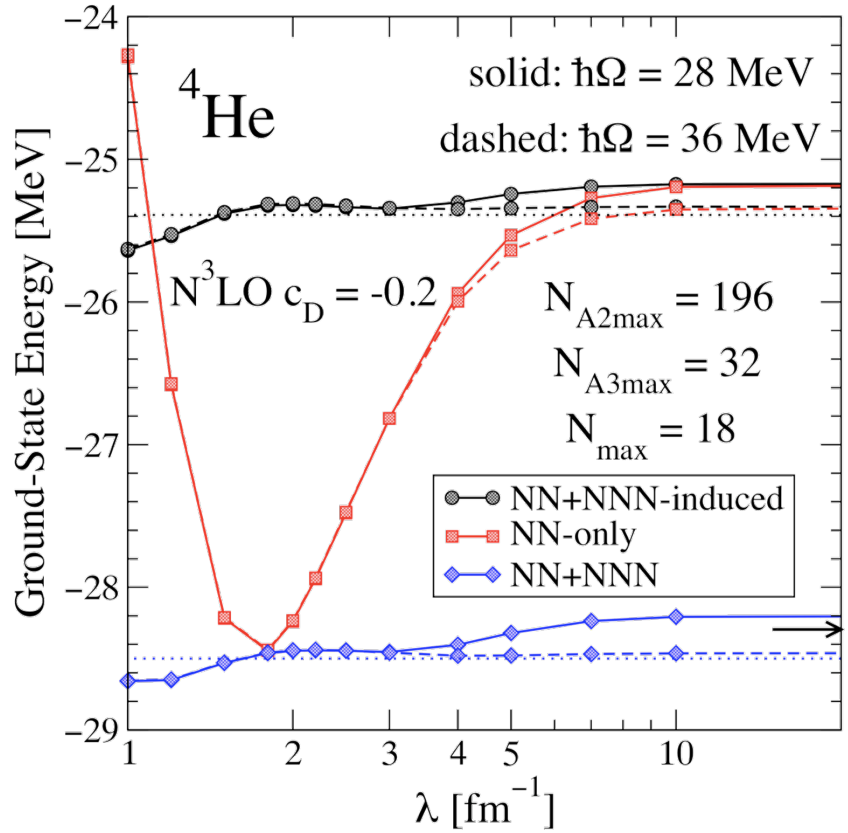
- A=3 system must converge in order to build a three-body interaction




hΩ > 16 is not well converged, and is not reliable for many-body calculations

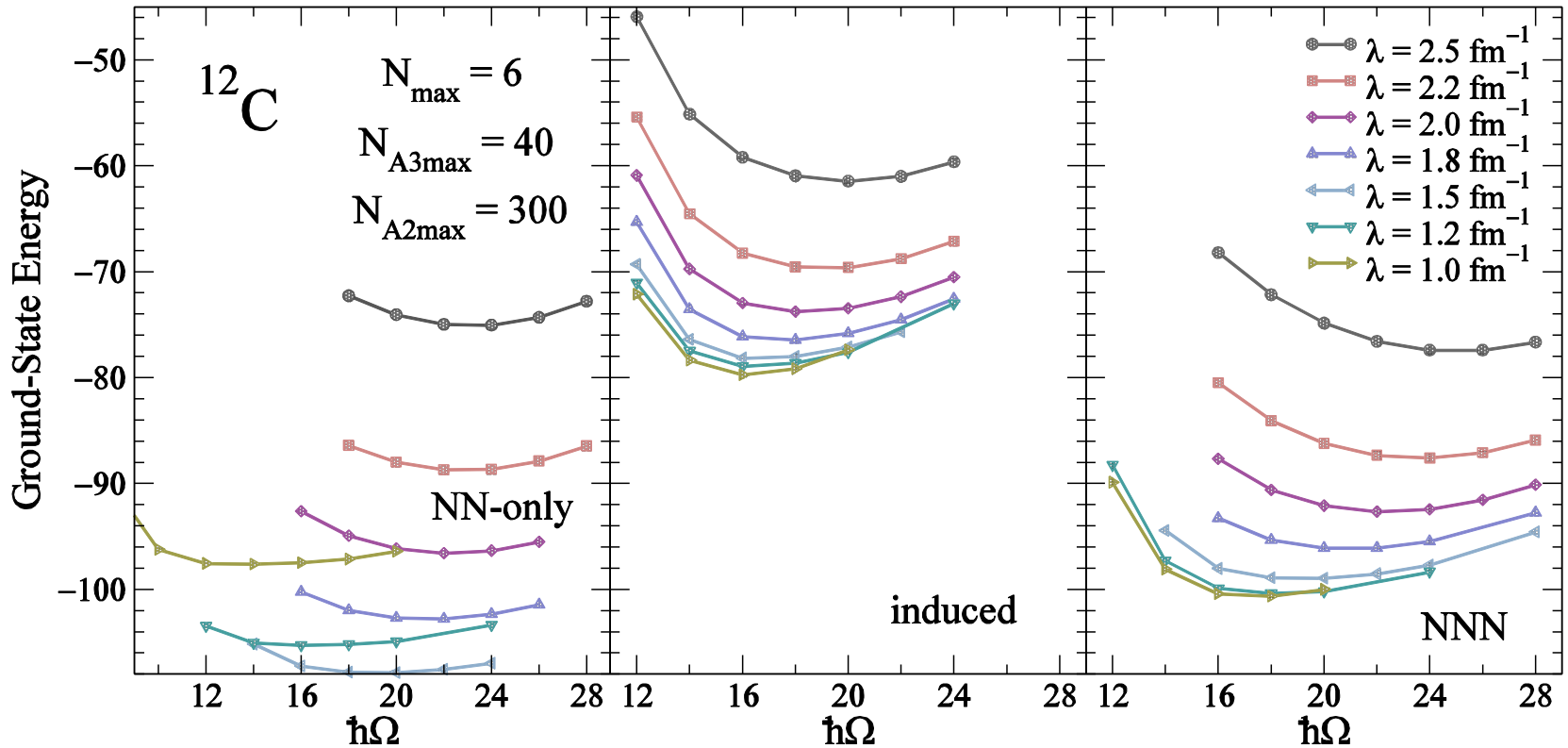
# Behavior of SRG for $^4$He



# Systematic study of p-shell nuclei

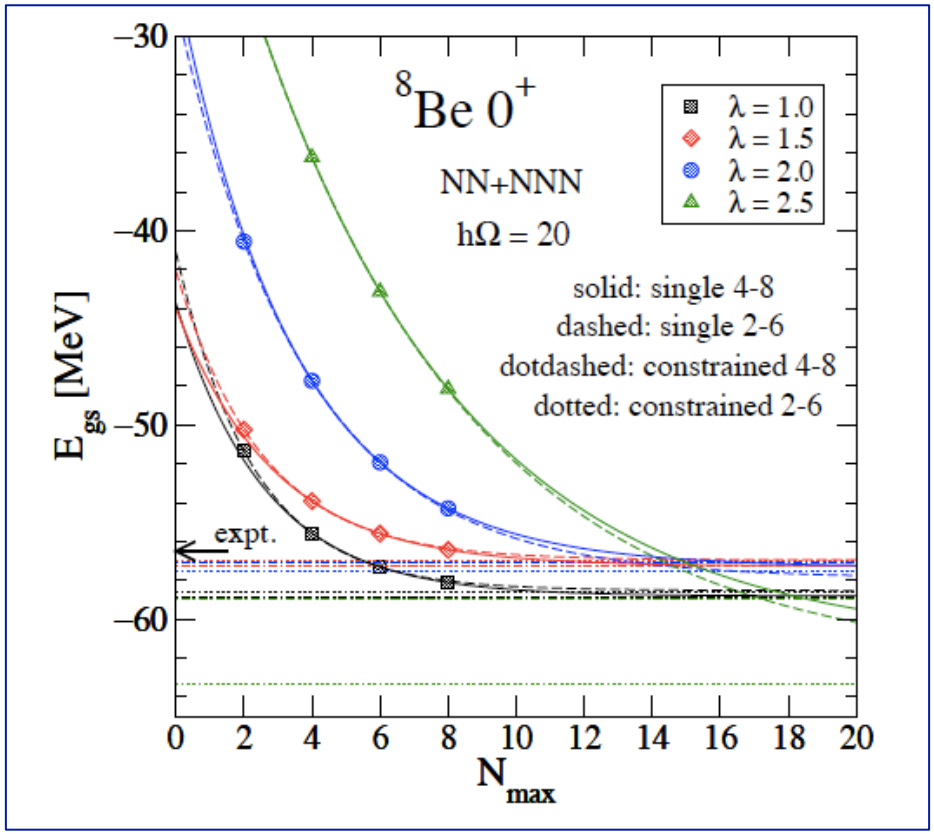
- Many-body calculations for $^{7}$Li, $^{7}$Be, $^{8}$Be, $^{10}$B, and $^{12}$C
  - $N_{max}$: 2-8, $\hbar\Omega$: 16-28, $\lambda$: 1.1-2.5
  - E. Jurgenson, P. Maris, R. Furnstahl, P. Navratil, WEO, J. Vary
  - 20M CPU hours

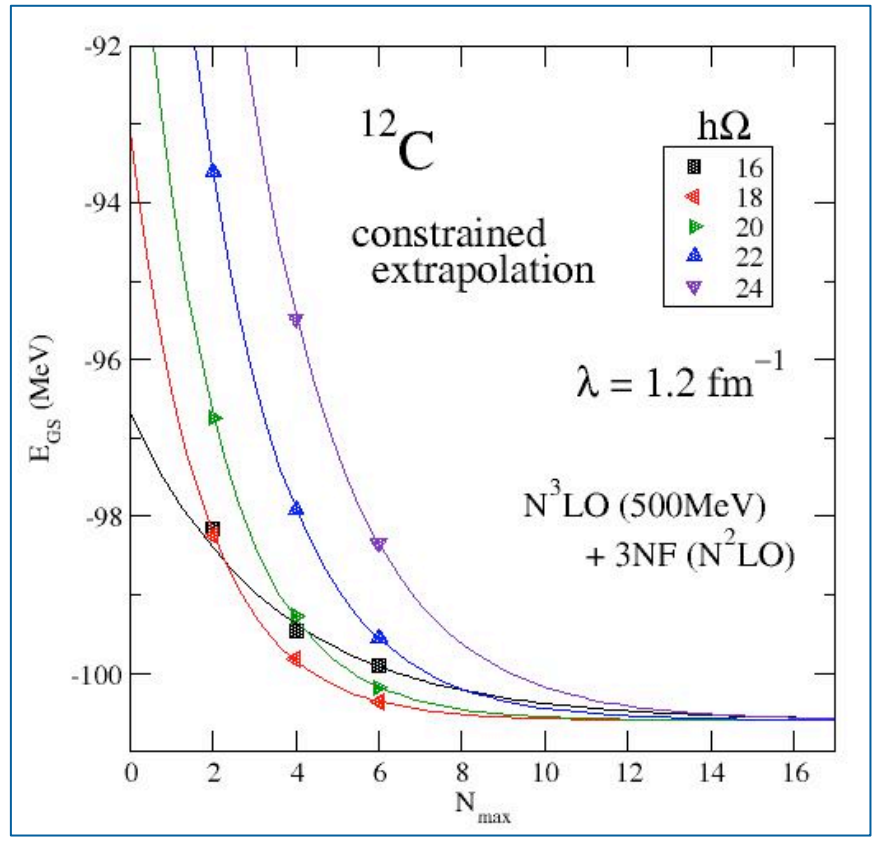


# Extrapolation to $N_{max} \rightarrow \infty$

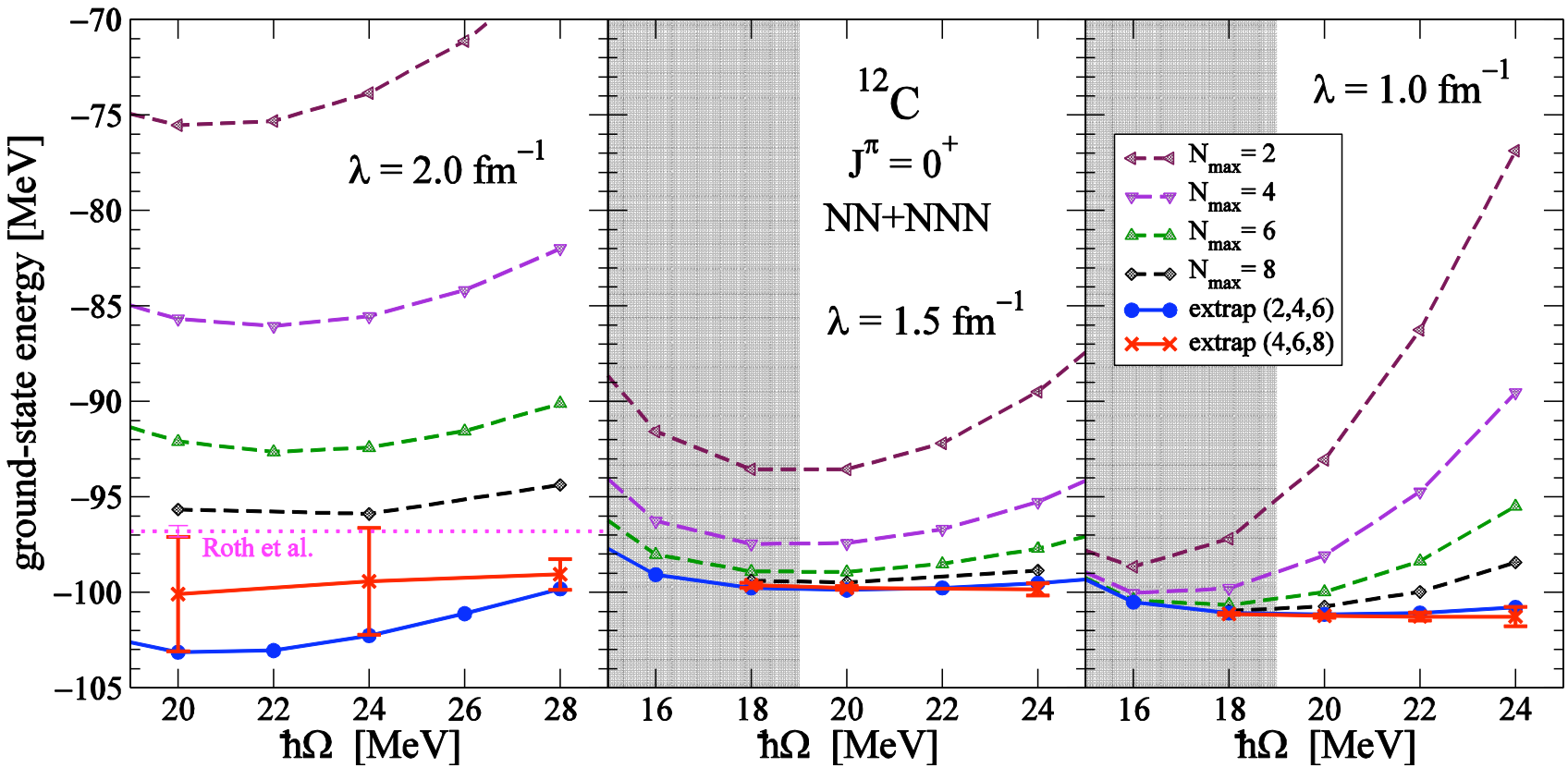
- Need to extrapolate $N_{max}$ to infinity
  - Assumed exponential behavior
  - Bracketed various sets of $N_{max}$
- Roth used importance truncation
  - Include basis states with amplitude larger than $\varepsilon$ for a given $N_{max}$

# Extrapolation to $N_{max} \rightarrow \infty$

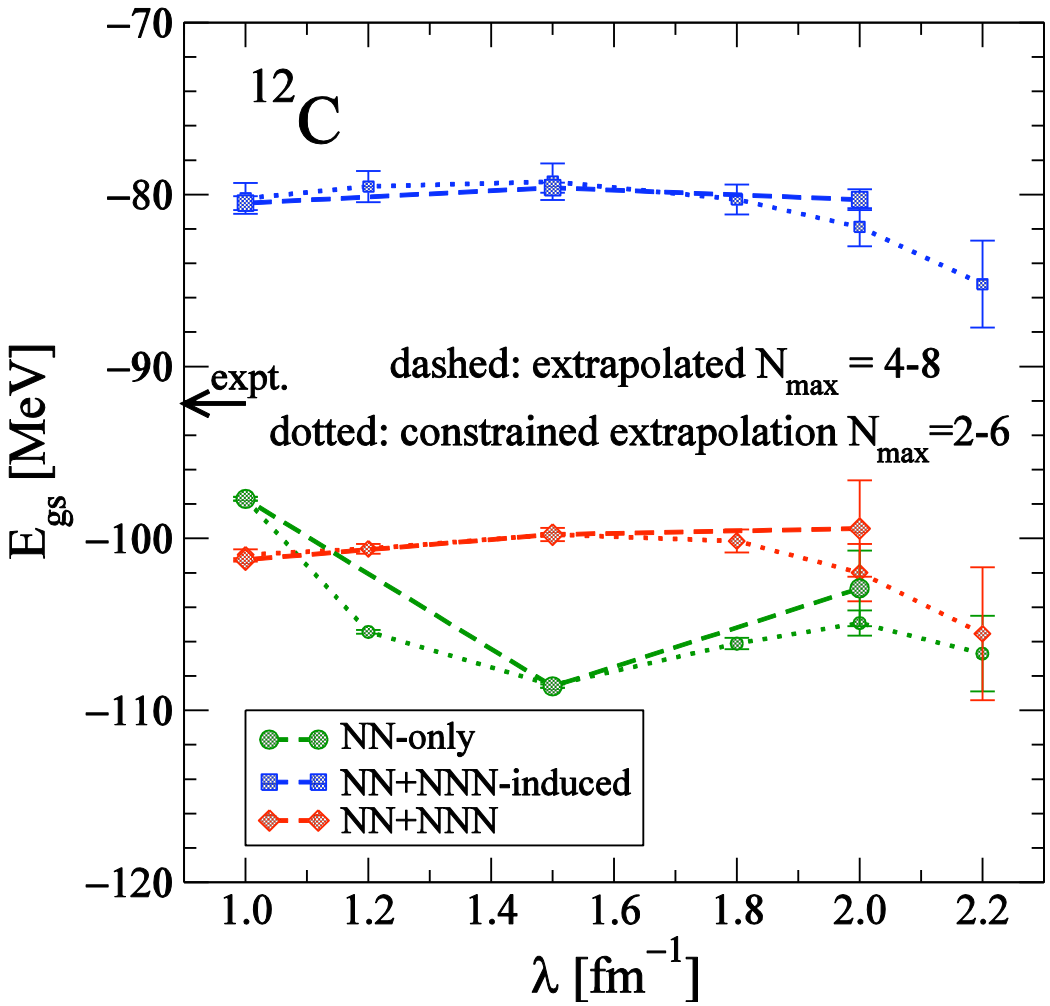


# $^{12}$C Extrapolations

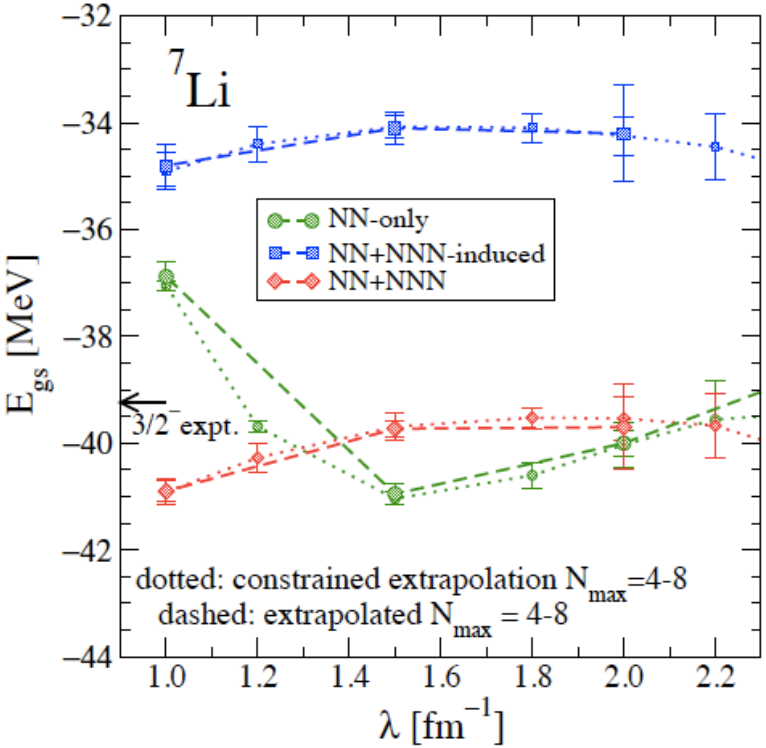


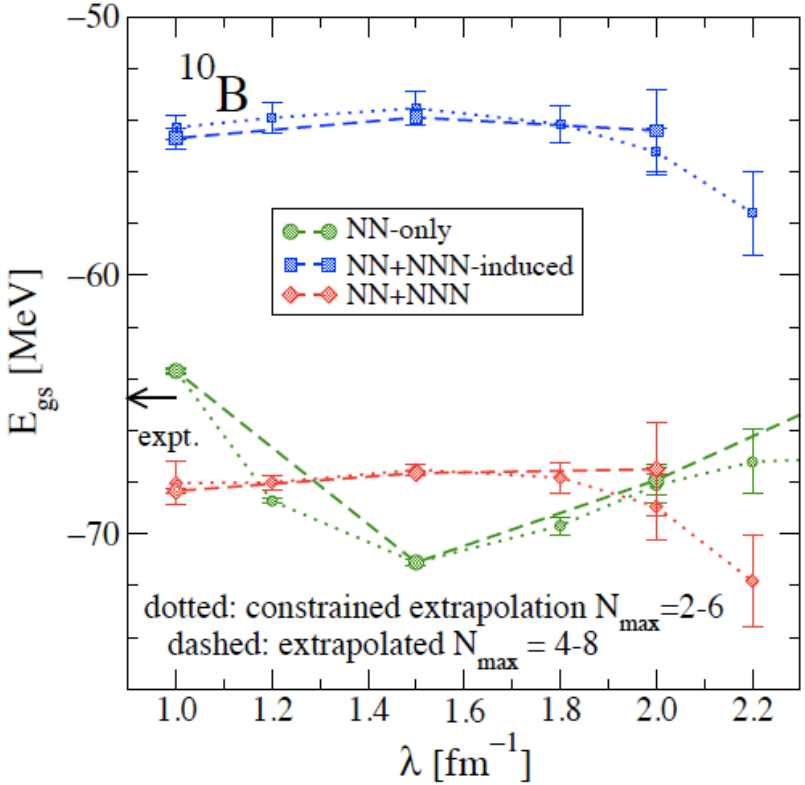
# Full-space results – $^{12}C$



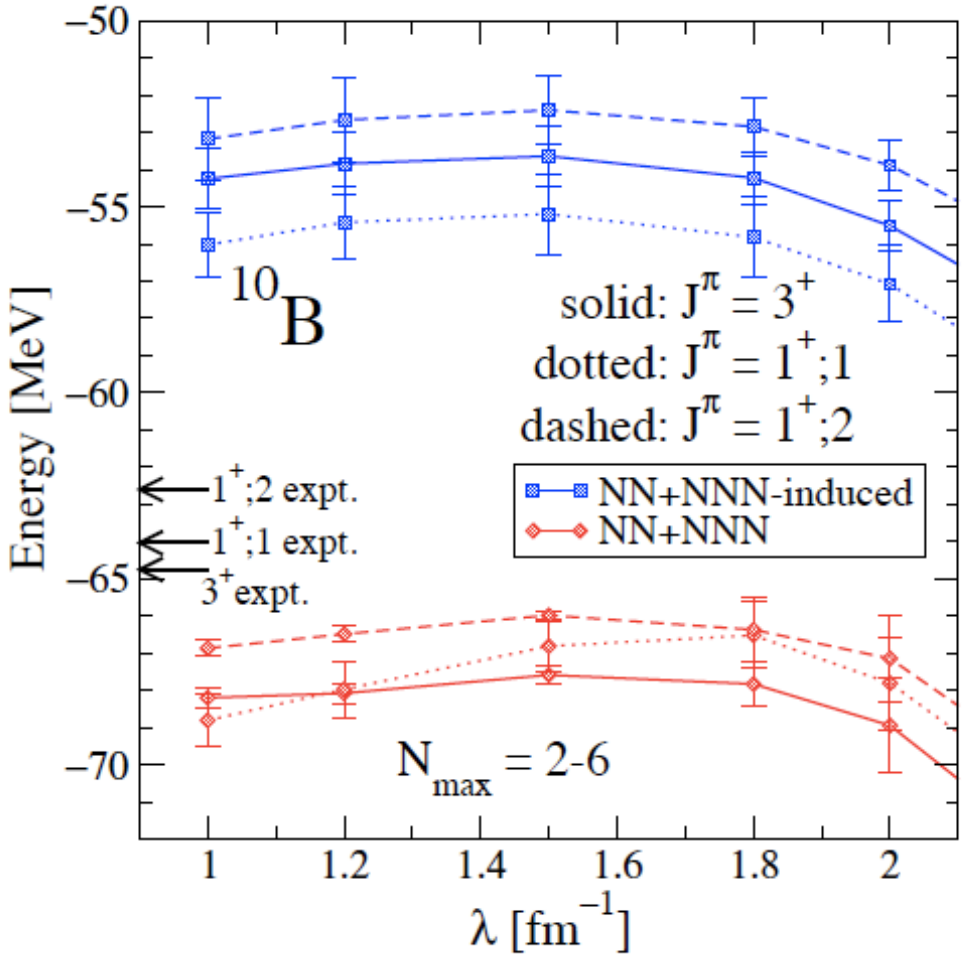

$N^2LO$ three-body interaction significantly overbinds $^{12}C$

# Full-space results



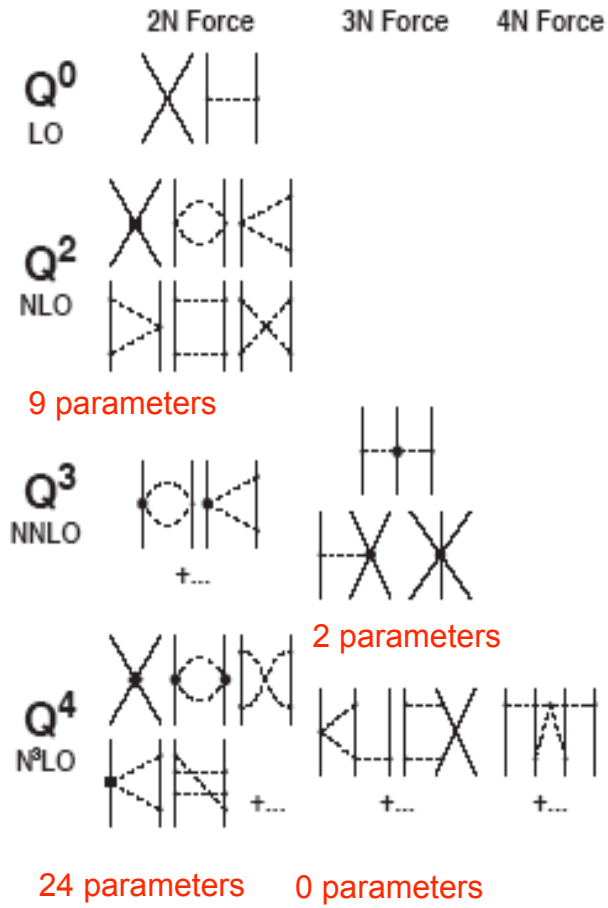



# Full-space results – $^{10}B$



# Higher-order three-body interaction

- Roth gets reasonable binding with $\Lambda \sim 400$ MeV
- Diagrams for $N^3LO$ three-body interaction have been derived
  - EFT specialists are PESSIMISTIC that that $N^3LO$ terms will have much of an effect
  - Need to include terms from higher orders or go to Delta-full EFT
    - Most likely there are TOO many terms to be consistent, must be selective




**Is this the end of formal EFT?**

# Summary

- *Ab initio* approaches to nuclear structure and reactions are maturing and providing interesting insight into nuclear processes
  - The nature of the three-body interaction in nuclei
  - The strength of electro-weak interactions – C
  - Light-ion reactions
  - Weakly-bound effects, parity inversion in $^{11}$Be
- Challenges
  - Convergence for ground-state and “intruder” states
  - Form of three-body interaction
  - Effective operators