Nonequilibrium Dynamics and Visosity

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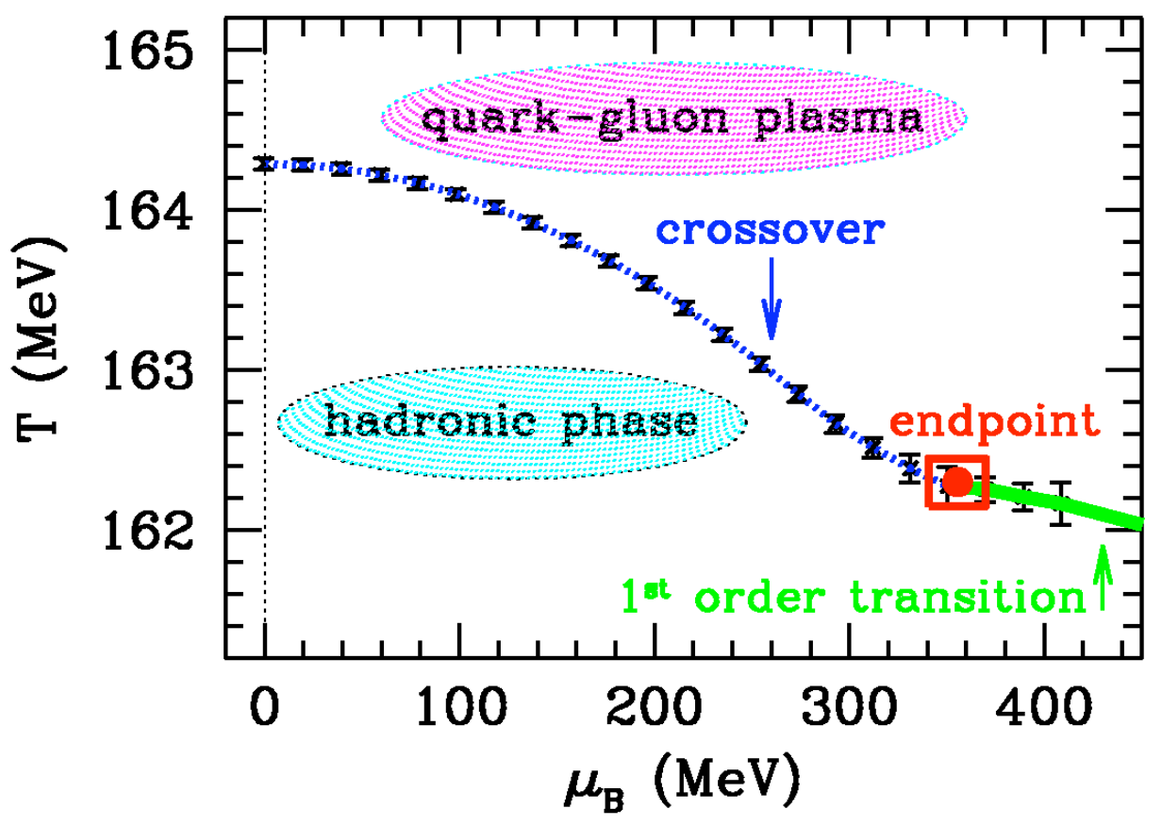
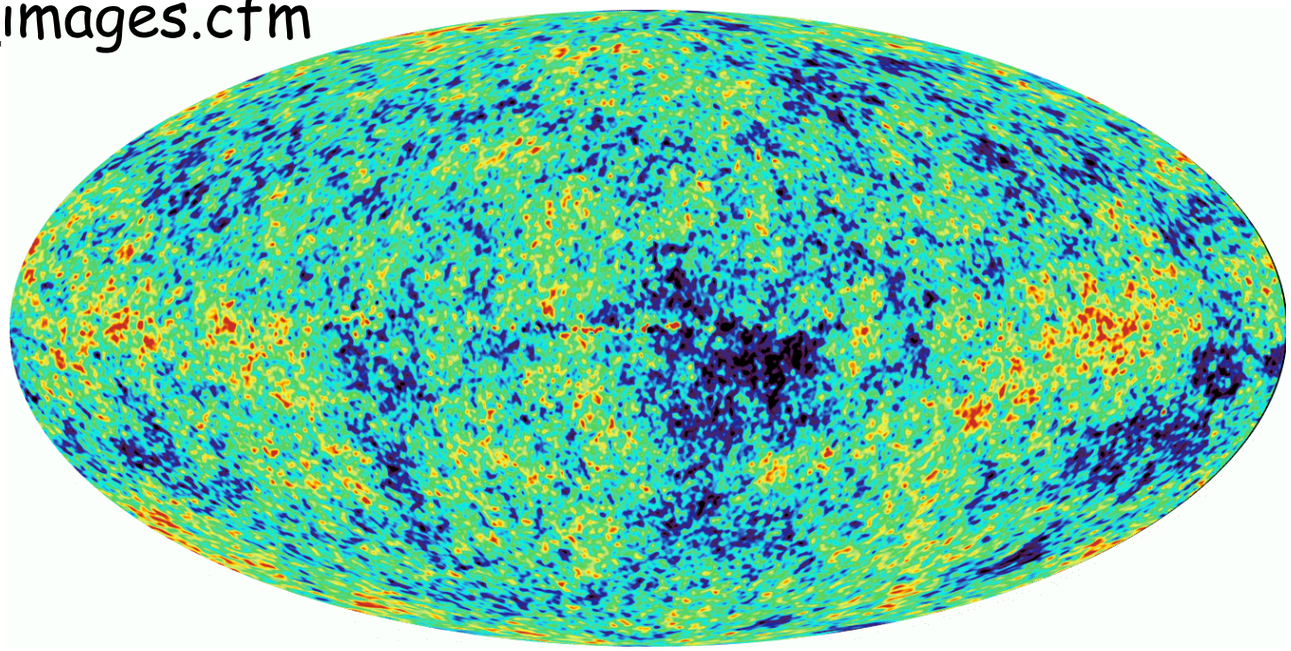
Motivation

Fodor + Katz (JHEP 0404 (2004) 050)



Motivation

WMAP Science Team
http://lambda.gsfc.nasa.gov/product/map/m_images.cfm



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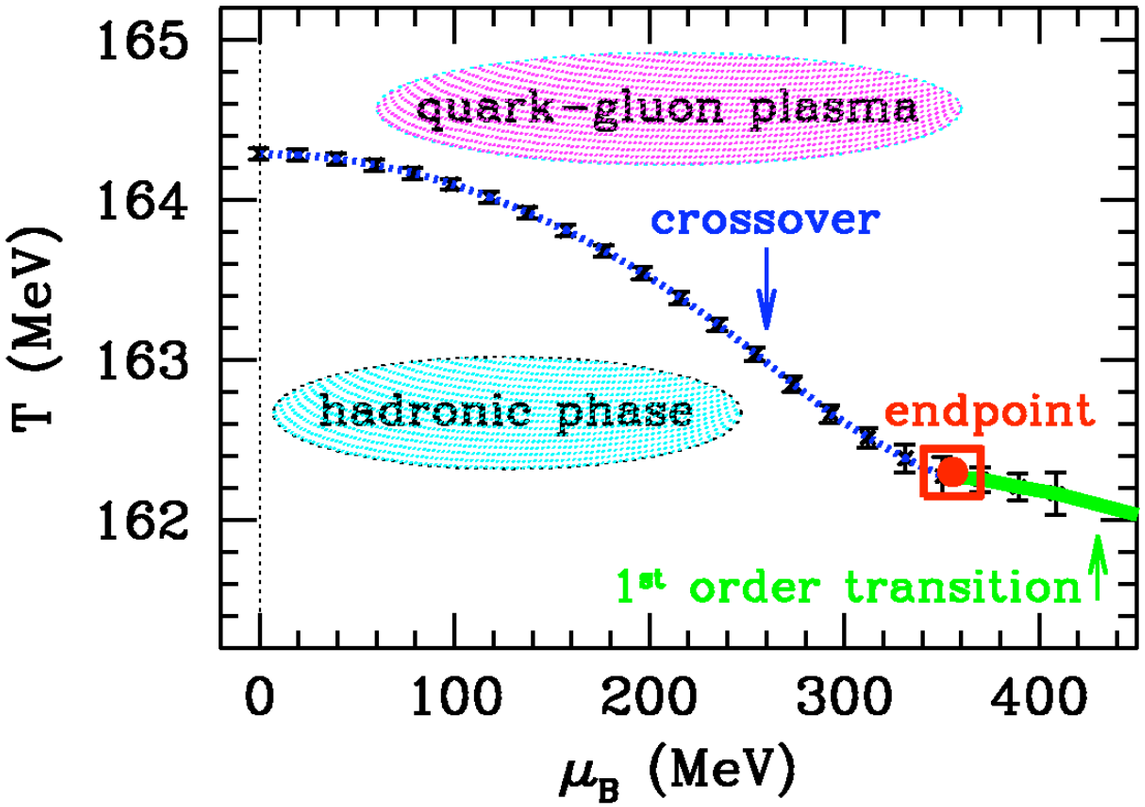

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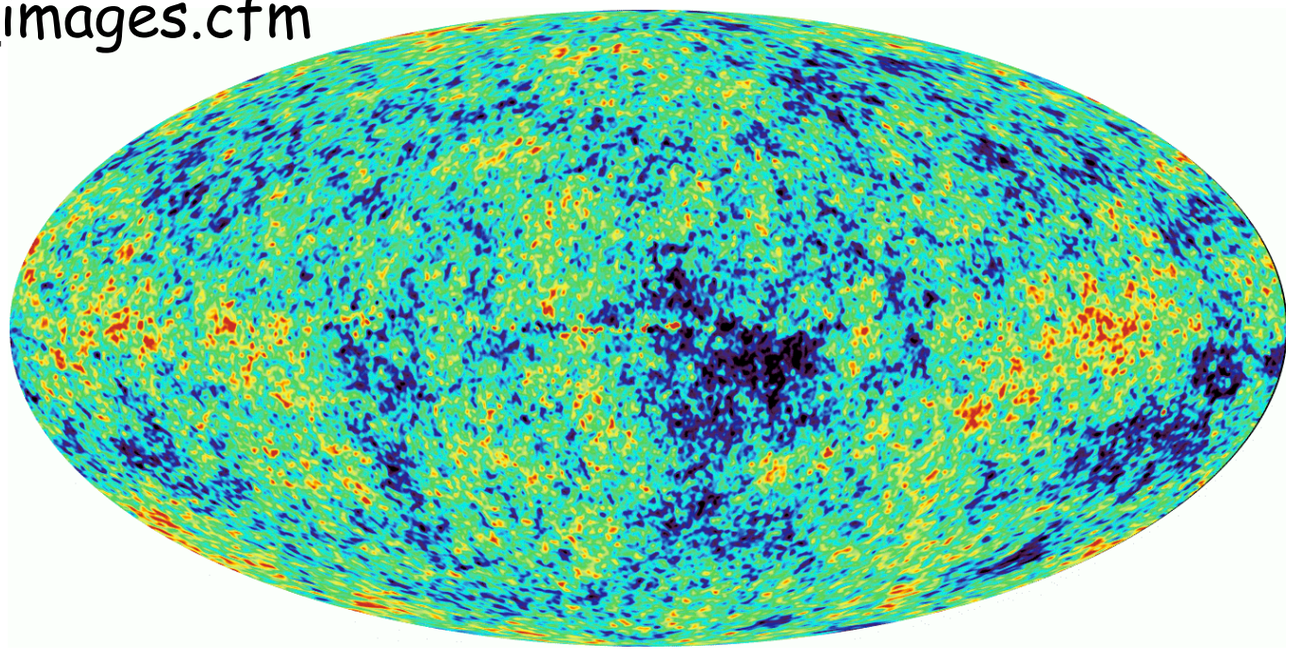

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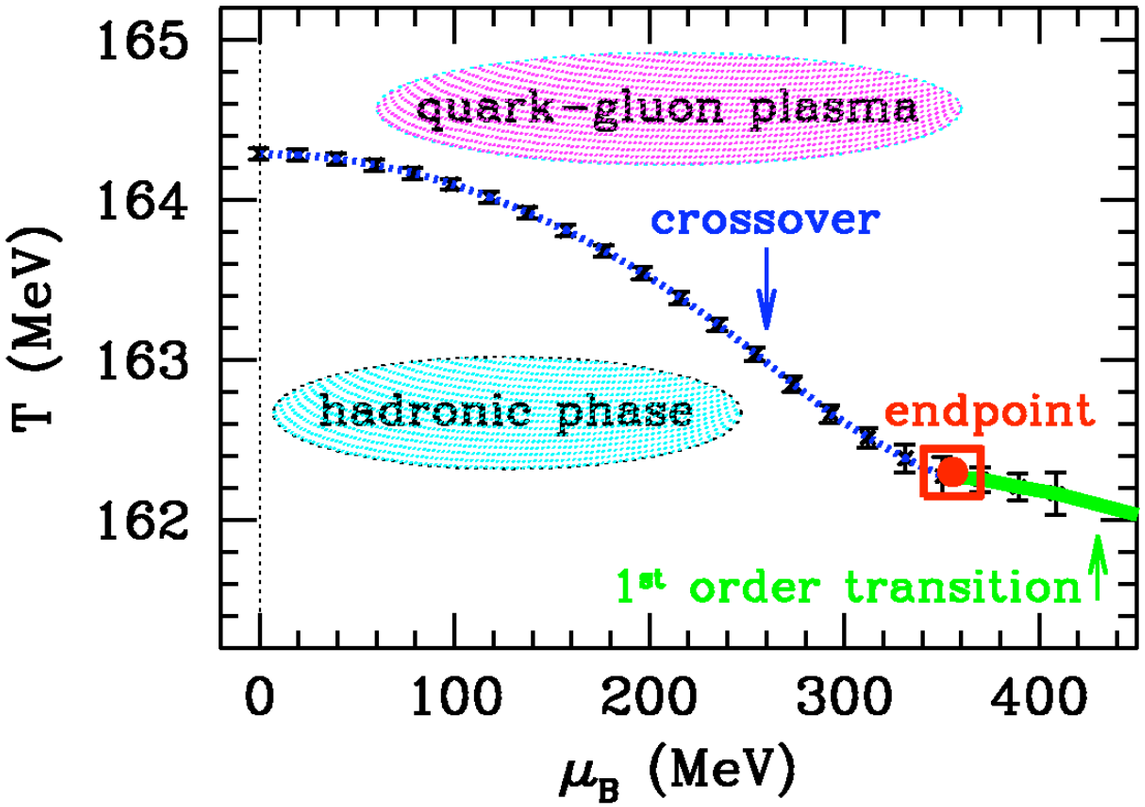

- ➡ Model: Chiral Hydro
- ➡ Application: Inhomogeneities
- ➡ A closer look: Viscosity

Gell-Mann Levy - Model

$$\mathcal{L} = \overline{q}\left[i\gamma^\mu\partial_\mu - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right] q + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U(\sigma,\vec{\pi})$$

$$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - h_q\sigma - U_0 \qquad \phi_a = (\sigma,\vec{\pi})$$

Finite Temperatures:

One-loop effective Potential

$$V_{\text{eff}}(\phi_a, T, \mu) = U(\phi_a) - d_q T \int \frac{d^3p}{(2\pi)^3}\left\{\log\left(1 + e^{-\frac{E+\mu}{T}}\right) + \log\left(1 + e^{-\frac{E-\mu}{T}}\right)\right\}$$

$$E = \sqrt{p^2 + (g\phi)^2}$$

q and $\bar{q}$ have been integrated out to generate the effective Potential for the chiral fields φ_a

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$$\mathcal{L} = \overline{q}\left[i\gamma^\mu\partial_\mu - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right] q + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U(\sigma,\vec{\pi})$$

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Chiral Field acts as Order Parameter

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Coupled Dynamics of Field and Fluid

Equations of Motion:

Field: $$\Box\phi_a + \frac{\delta V_{eff}}{\delta\phi_a} = 0$$

Chiral Hydrodynamics

Fluid: $$\partial_\mu T^{\mu\nu}_{\text{fluid}} = \partial_\mu \left[(e+p)u^\mu u^\nu - p g^{\mu\nu}\right]$$

$$\partial_\mu j^\mu = \partial_\mu(\rho u^\mu) = 0$$

$$p = p(e,\rho,\phi) = -(V_{\text{eff}}(e,\rho,\phi) - U(\phi_a))$$

Coupling of field and fluid:

$$\boxed{\partial_\mu\left(T^{\mu\nu}_{fluid} + T^{\mu\nu}_{field}\right) = 0}$$

- Scavenius, Mocsy, Mishustin, Rischke; Phys. Rev. C64 045202, 2001
- Paech, Stöcker, Dumitru Phys.Rev.C68:044907, 2003

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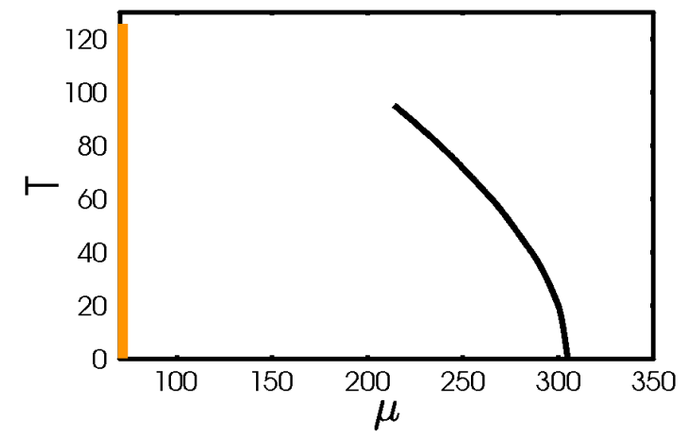
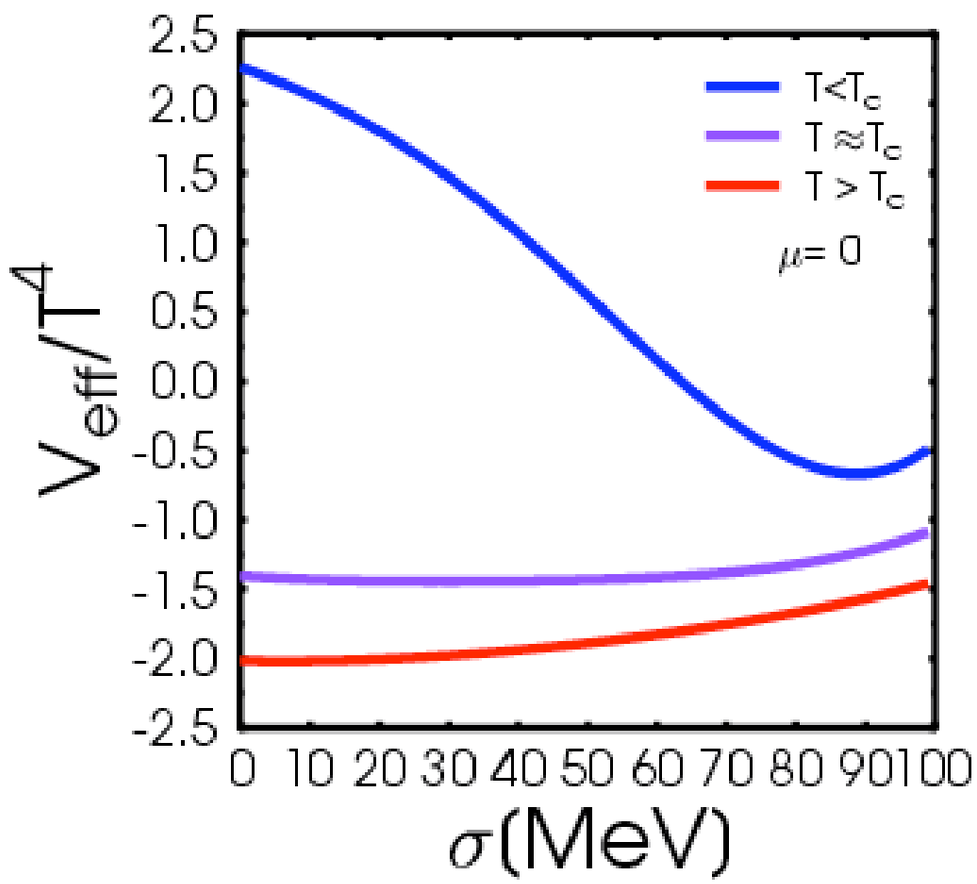
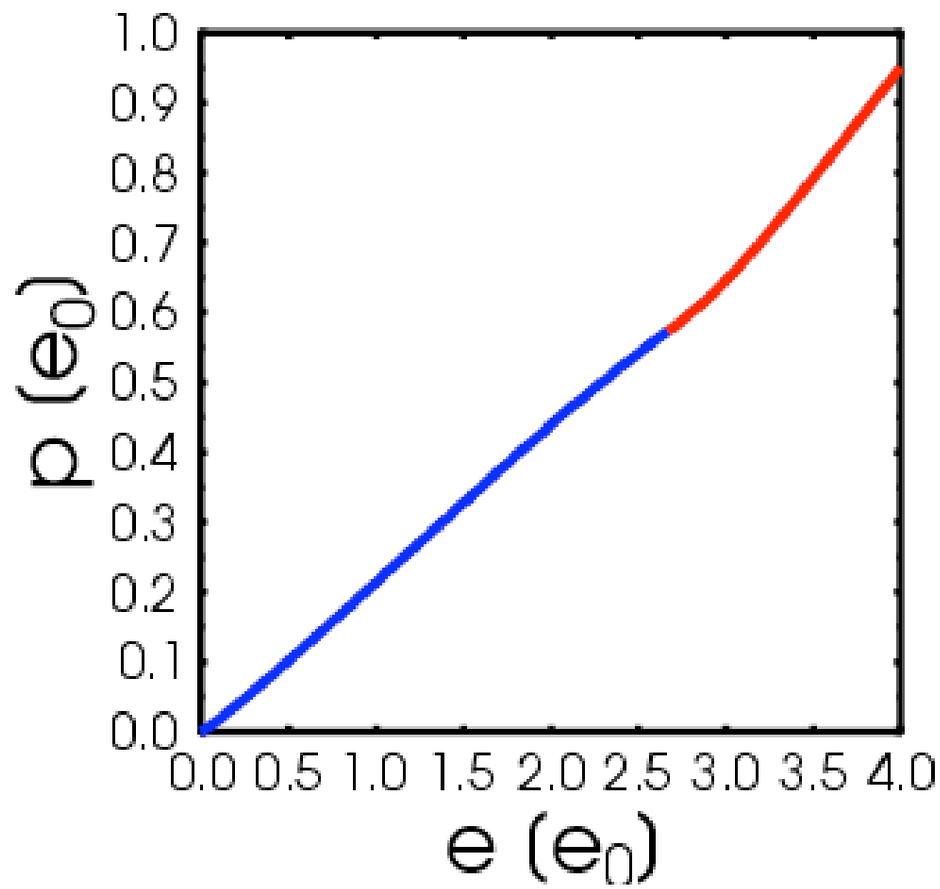
Effective Potential and Equation of State

Crossover

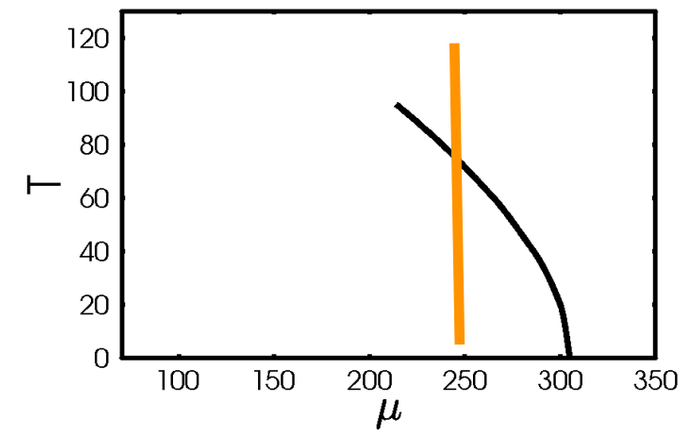


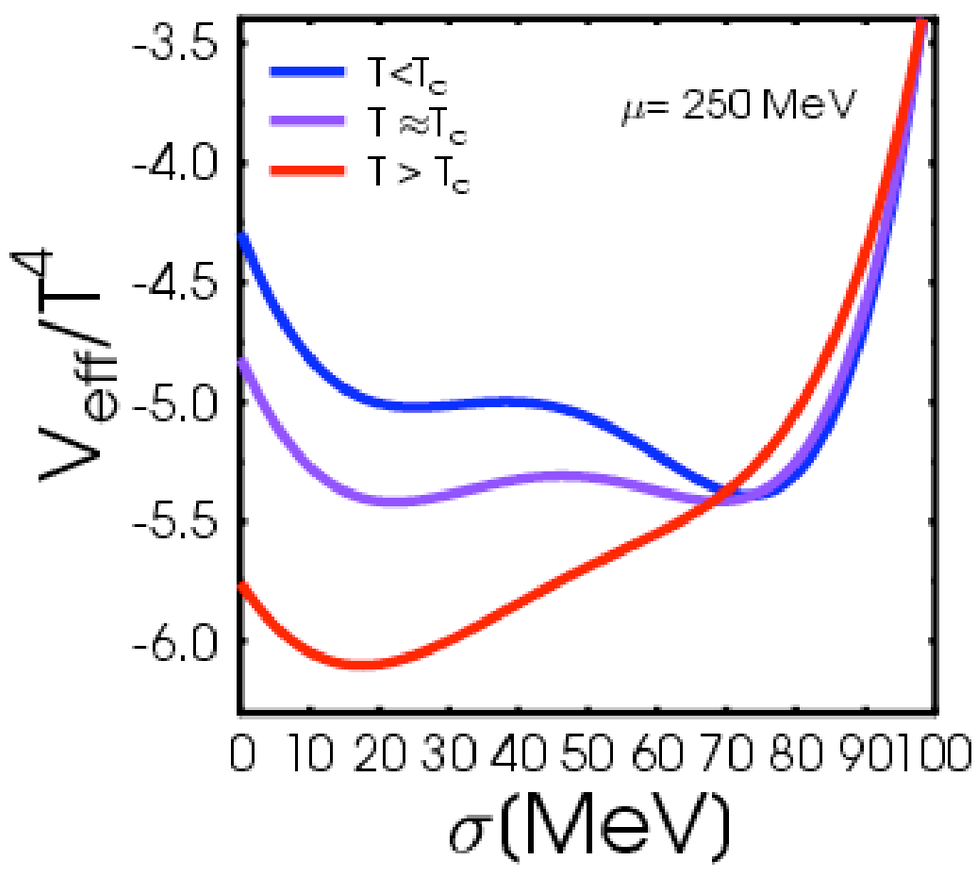


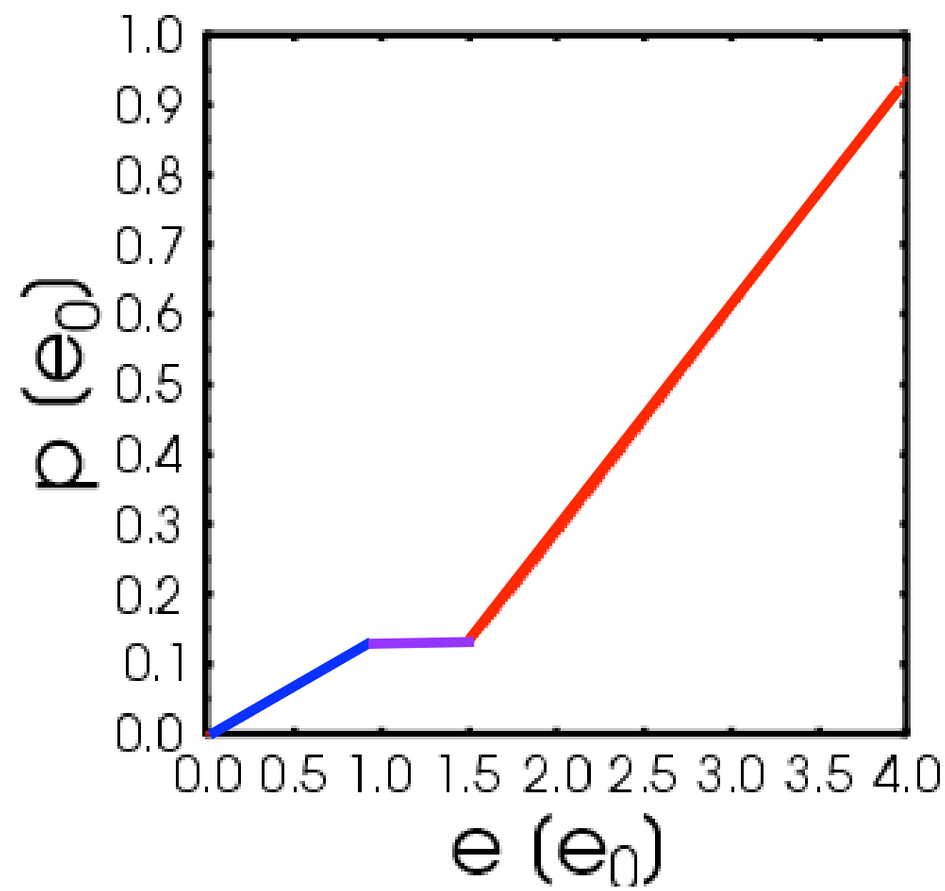


Effective Potential and Equation of State

1st Order Transition

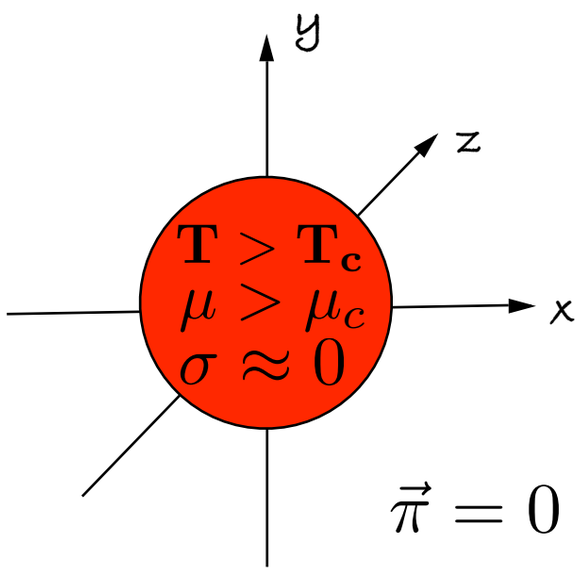




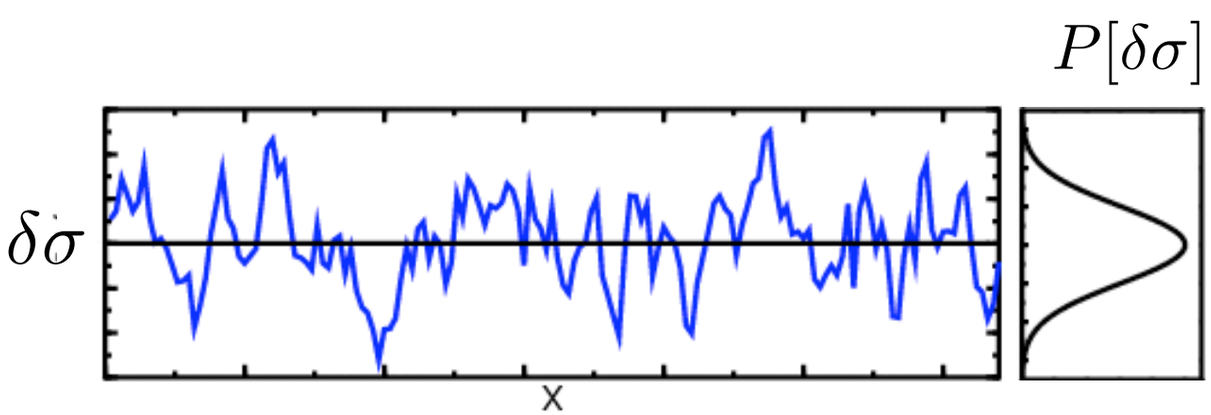


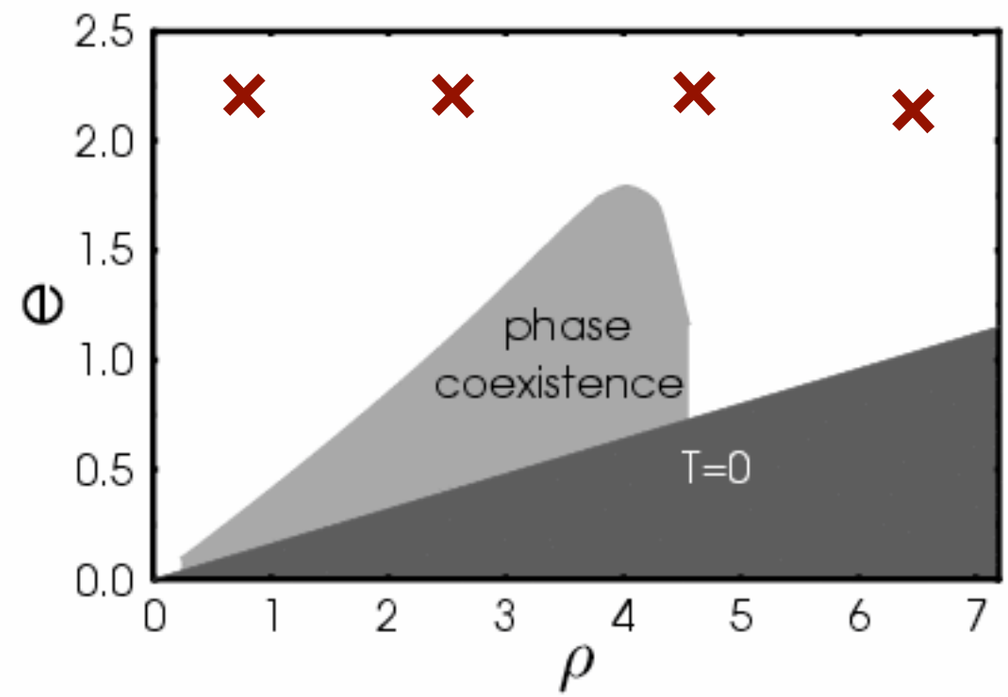
Initial Conditions

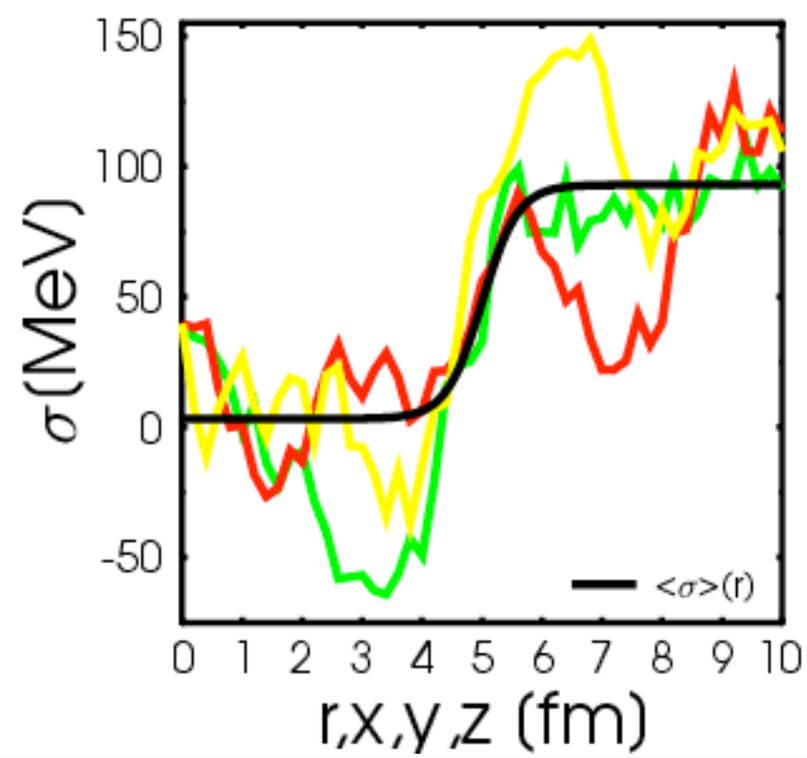
$$\mu = \mathbf{T} = \mathbf{0}$$
$$\sigma = 93\,\mathrm{MeV}$$



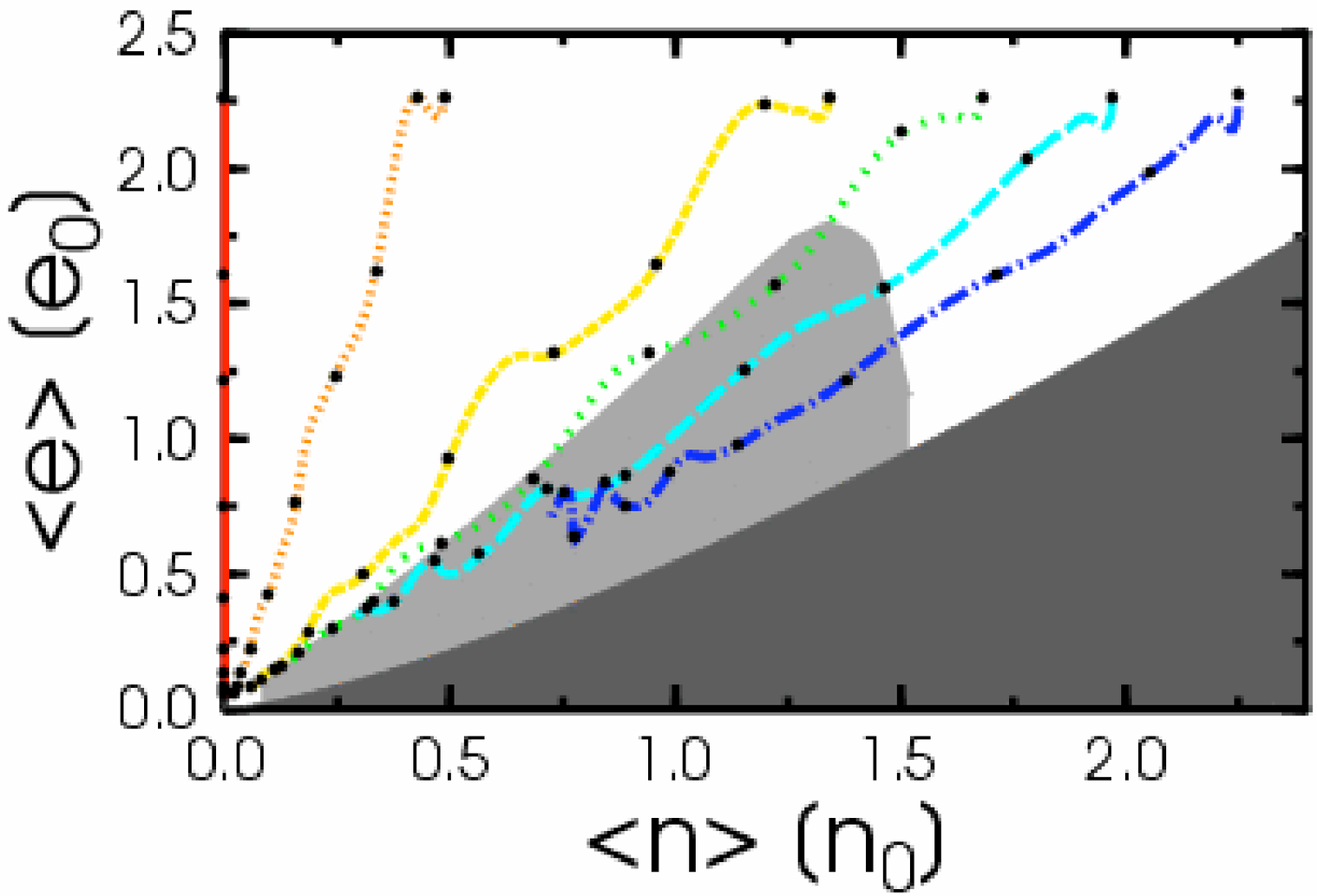
$$\vec{\pi} = 0$$



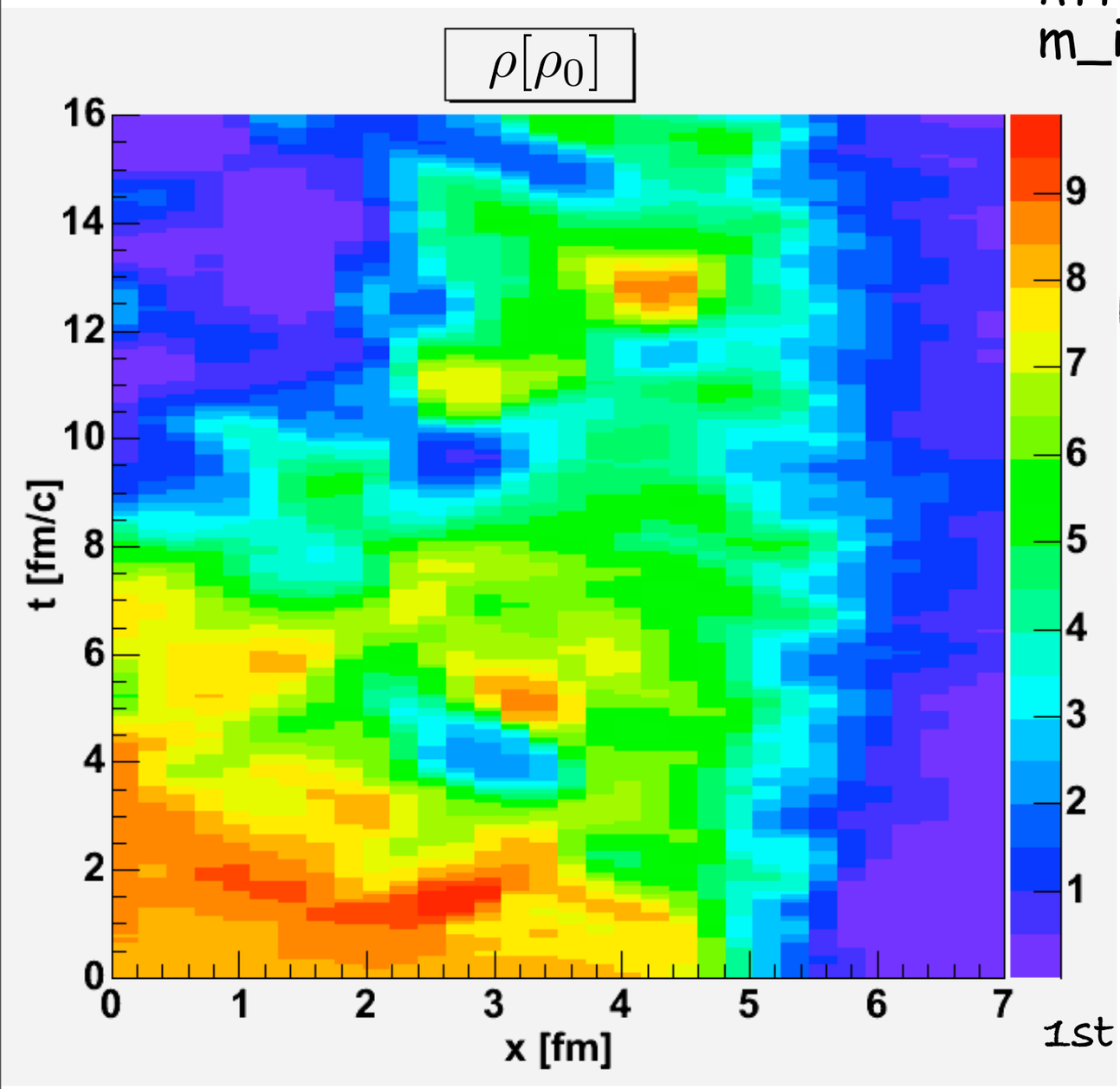




Results: Evolution of Baryon- and Energydensity



Results: Time dependence of Baryon Density



1st order

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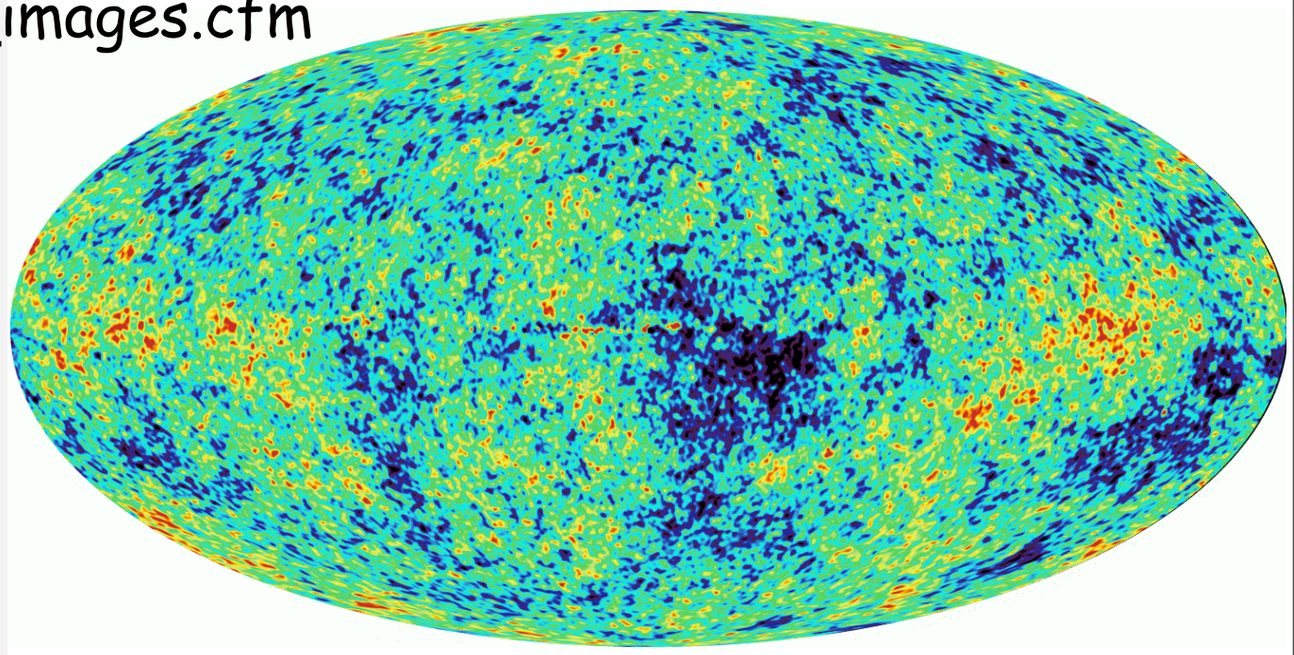
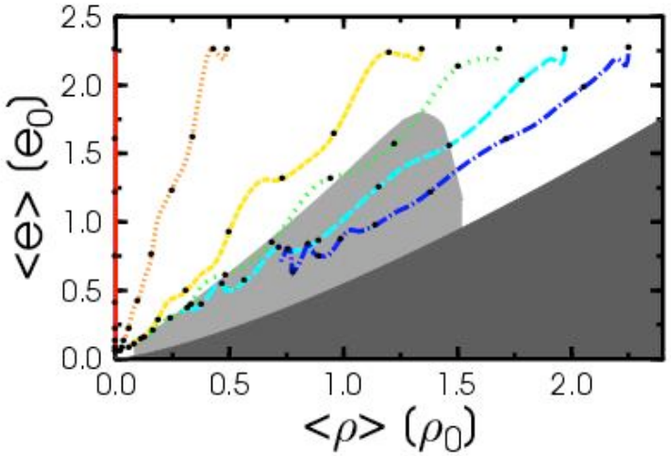
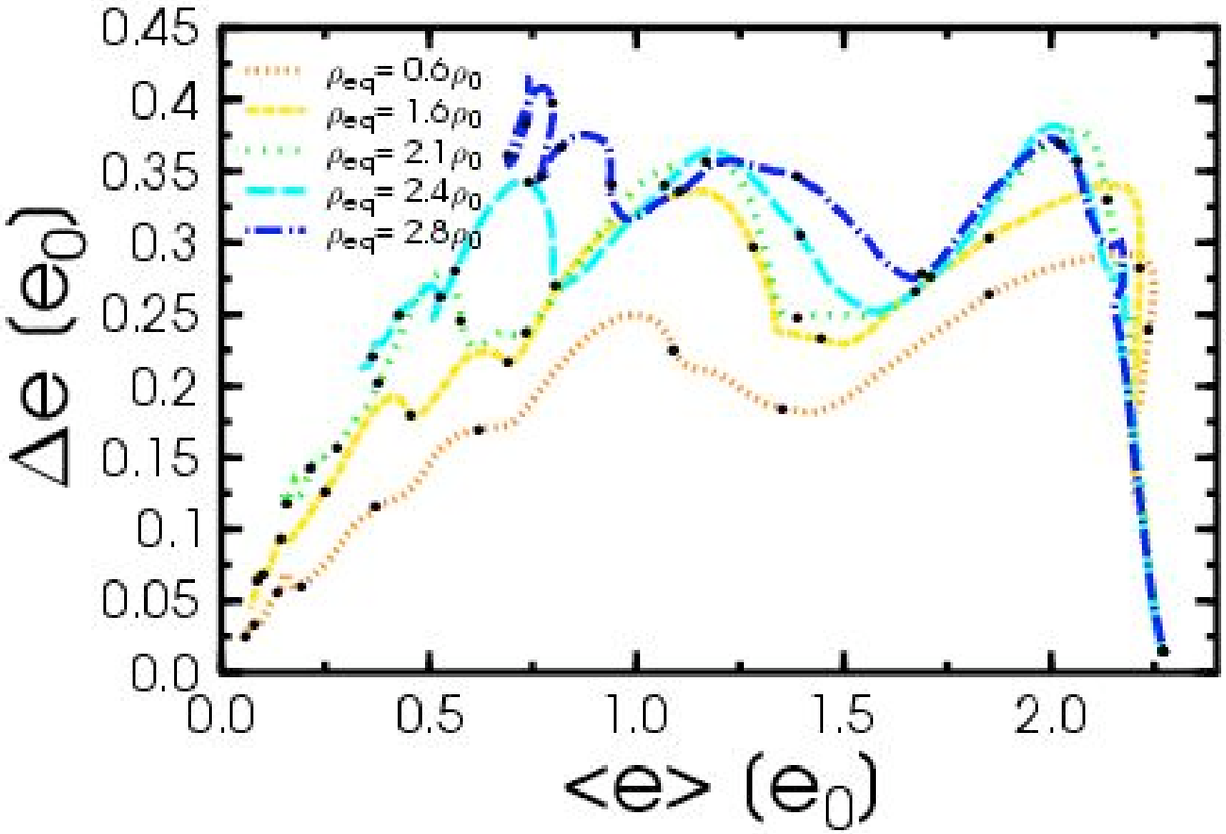
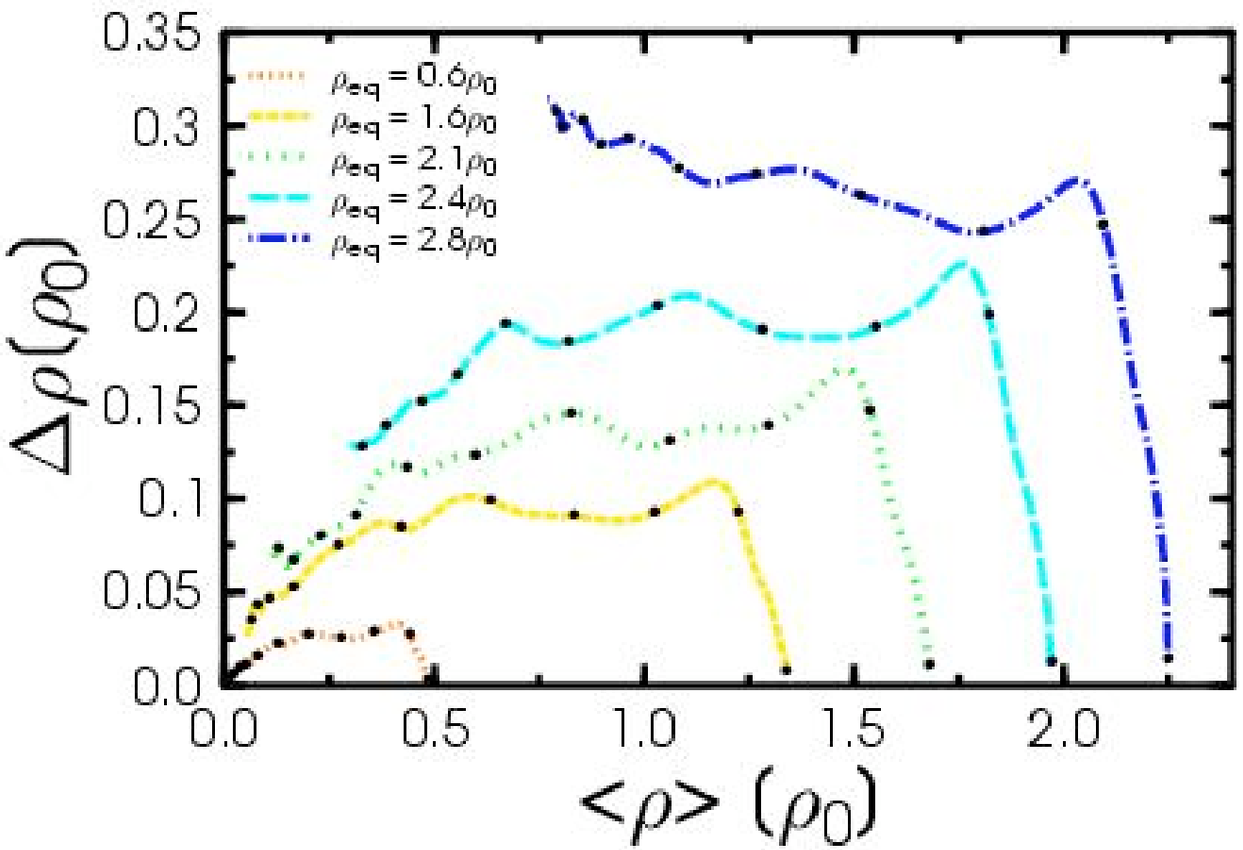


Results: Density Inhomogeneities







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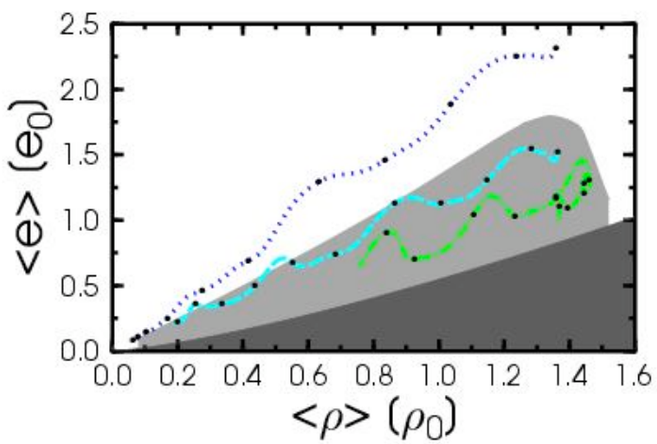
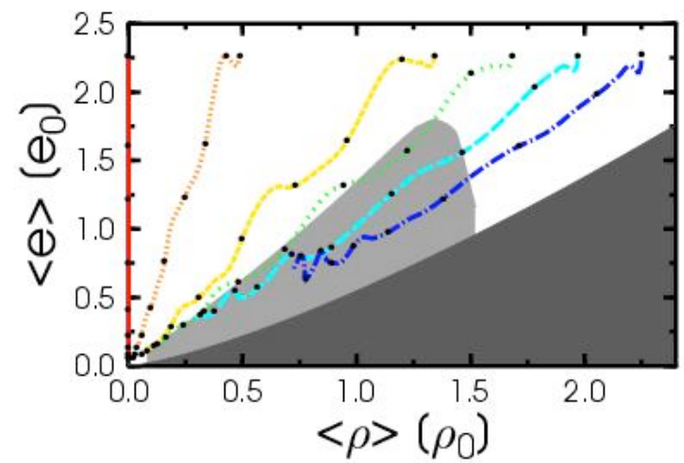
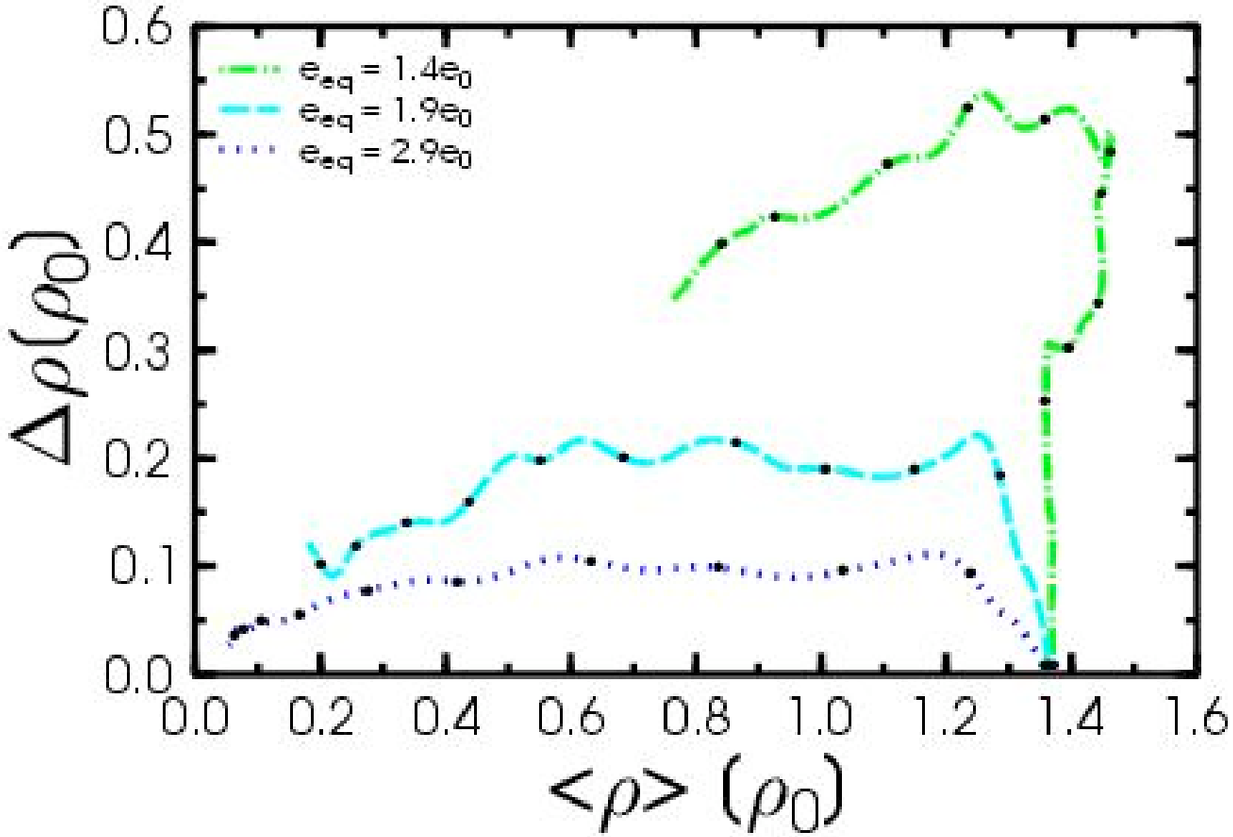
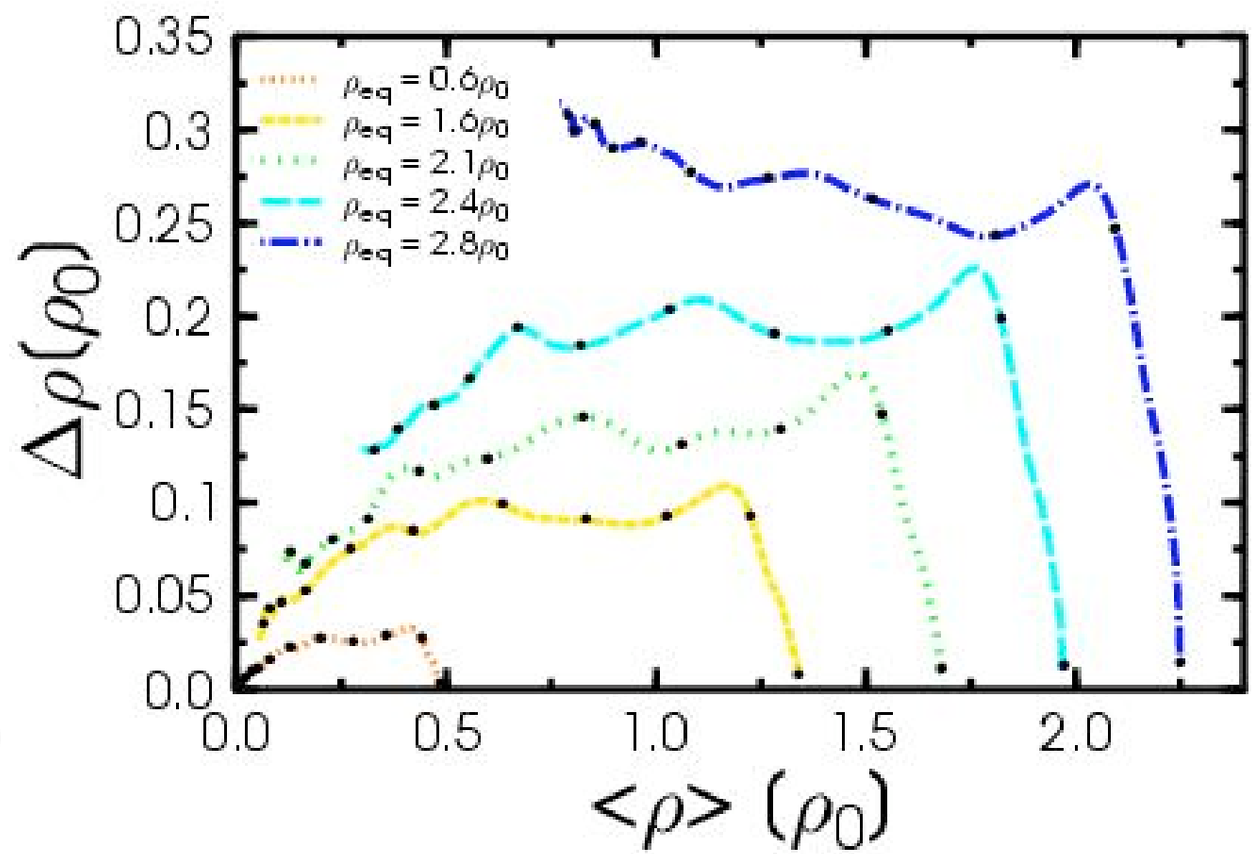








Viscosity

Ideal Hydrodynamics:

$$\begin{aligned} T^{\alpha\beta} &= (p+e)u^\alpha u^\beta - p g^{\alpha\beta} \\ &= e u^\alpha u^\beta - p H^{\alpha\beta} \end{aligned}$$

$$H_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta$$

$$N^\alpha = n u^\alpha$$

$$u_\alpha = \gamma(1, \vec{v})$$

local rest frame:

$$T^{00} = e \qquad u^0 = 1$$
$$N^0 = n \qquad u^i = 0$$

Viscosity

Non-Ideal Hydrodynamics:

$$T^{\alpha\beta} = (p+e)u^\alpha u^\beta - pg^{\alpha\beta} + \delta T^{\alpha\beta}$$
$$= eu^\alpha u^\beta - pH^{\alpha\beta} + \delta T^{\alpha\beta}$$

$$H_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta$$

$$N^\alpha = nu^\alpha + \delta N^\alpha$$

$$u_\alpha = \gamma(1, \vec{v})$$

local rest frame:

$$T^{00} = e \qquad u^0 = 1$$
$$N^0 = n \qquad u^i = 0$$

Landau frame: $\delta T^{0i} = 0$

Eckart frame: $\delta N^\alpha = 0$

Viscosity

Non-Ideal Hydrodynamics:

$$T^{\alpha\beta} = (p+e)u^\alpha u^\beta - p g^{\alpha\beta} + \delta T^{\alpha\beta}$$
$$= e u^\alpha u^\beta - p H^{\alpha\beta} + \delta T^{\alpha\beta}$$

$$H_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta$$

$$N^\alpha = n u^\alpha + \delta N^\alpha$$

$$u_\alpha = \gamma(1, \vec{v})$$

Eckart frame:

$$\delta T^{\alpha\beta} = - \eta H^{\alpha\gamma} H^{\beta\delta} W_{\gamma\delta} - \zeta H^{\alpha\beta} \partial_\gamma u^\gamma - \chi \left(H^{\gamma\alpha} u^\beta + H^{\gamma\beta} u^\alpha\right) Q_\gamma$$

shear viscosity

bulk viscosity

heat conduction

$$W_{\alpha\beta} = \partial_\beta u_\alpha + \partial_\alpha u_\beta - \frac{2}{3} g_{\alpha\beta} \partial_\gamma u^\gamma$$

$$Q_\alpha = \partial_\alpha T + T \left(u \cdot \partial\right) u_\alpha$$

viscosity

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{1}{\tau_{\mathrm{chem}}}\left(N - N_{\mathrm{eq}}\right)$$

$$\delta N \equiv N - N_{\mathrm{eq}}$$

➡ offset from equilibrium:

$$\delta N = \tau_{\mathrm{chem}}\left(\cancel{\frac{\mathrm{d}\delta N}{\mathrm{d}t}} + \frac{\mathrm{d}N_{\mathrm{eq}}}{\mathrm{d}t}\right)$$

(slow expansion)

$$\begin{aligned}\delta P &= \left(\frac{\partial p}{\partial n}\right)_e \delta n = -\left(\frac{\partial p}{\partial n}\right)_e \tau_{\mathrm{chem}} \frac{\mathrm{d}n_{\mathrm{eq}}}{\mathrm{d}t} \\ &= \boxed{\left(\frac{\partial p}{\partial n}\right)_e \tau_{\mathrm{chem}} \; n_{\mathrm{eq}}} \nabla \cdot \vec{v}\end{aligned}$$

coefficient of bulk viscosity ζ

Viscosity

$$\frac{\partial^2\sigma(t)}{\partial t^2} + \Gamma\frac{\partial\sigma(t)}{\partial t} + m_\sigma^2\left(\sigma(t) - \sigma_{\rm eq}(t)\right) = R(t)$$

scalar mean field

$$\langle R(t)R(t')\rangle = A\,\delta(t-t')$$

$$\langle\delta\sigma\rangle = -\frac{m_\sigma^2\dot\sigma_{\rm eq}}{T}\int_{-\infty}^{0} dt'\langle\sigma(0)\sigma(t')\rangle$$

linear response theory

$$\langle\delta\sigma\rangle = -\frac{\Gamma}{m_\sigma^2(T)}\frac{{\rm d}\sigma_{\rm eq}}{{\rm d}t}$$

$$\Rightarrow\ \delta P = \left(\frac{\partial p}{\partial\sigma}\right)_e \frac{\Gamma}{m_\sigma^2}\frac{{\rm d}\sigma_{\rm eq}}{{\rm d}s}\, s\,\nabla\cdot\vec{v}$$

coefficient of bulk viscosity ζ

Viscosity

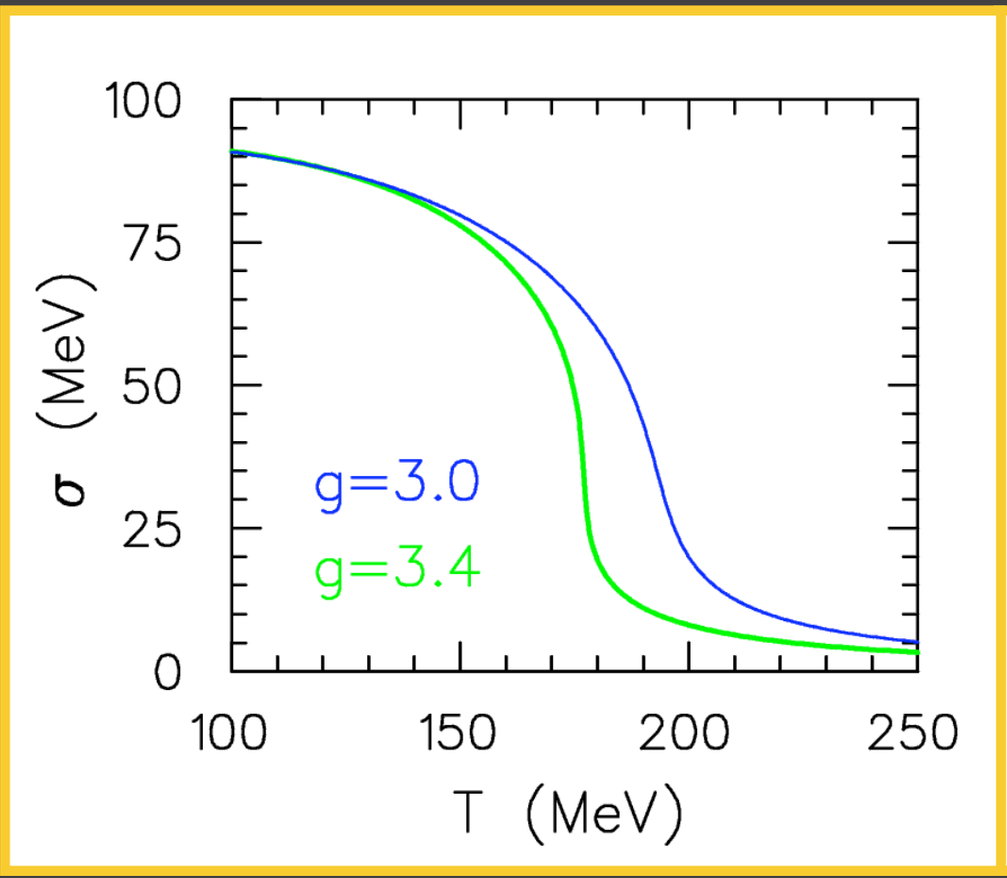
Linear Sigma Model

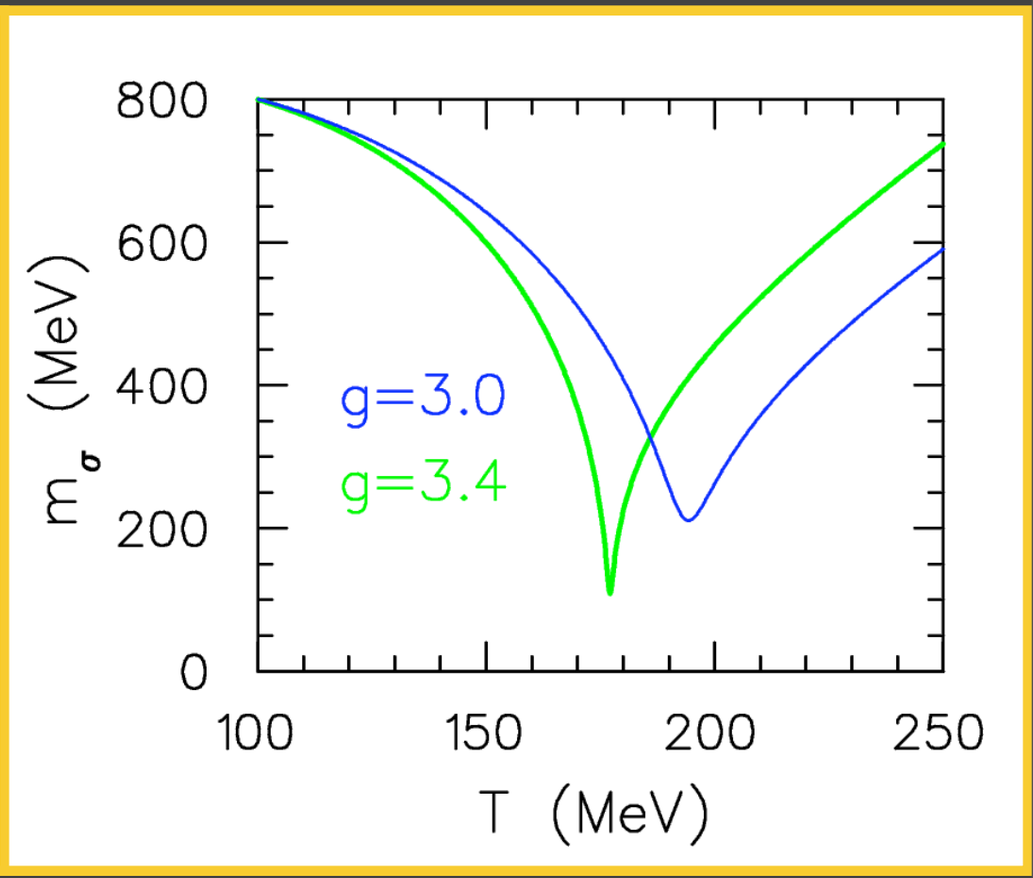
$$\zeta = -\left(\frac{\partial p}{\partial \sigma}\right)_e \frac{\Gamma}{m_\sigma^2(T)}\,\dot{\sigma}_{\rm eq}$$

For μ=0:

1st order when g > 3.5549

crossover when g < 3.5549





Viscosity

Linear Sigma Model

$$\zeta = -\left(\frac{\partial p}{\partial \sigma}\right)_e \frac{\Gamma}{m_\sigma^2(T)}\,\dot{\sigma}_{\rm eq}$$

Bjorken expansion

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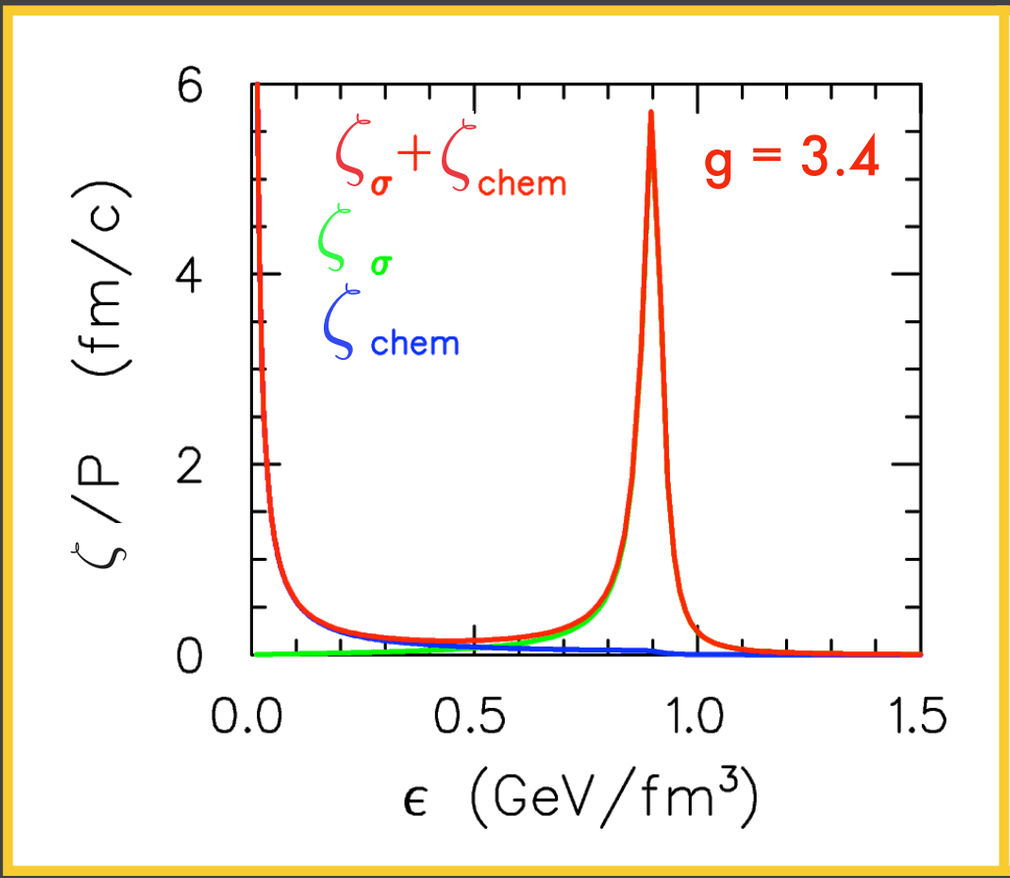

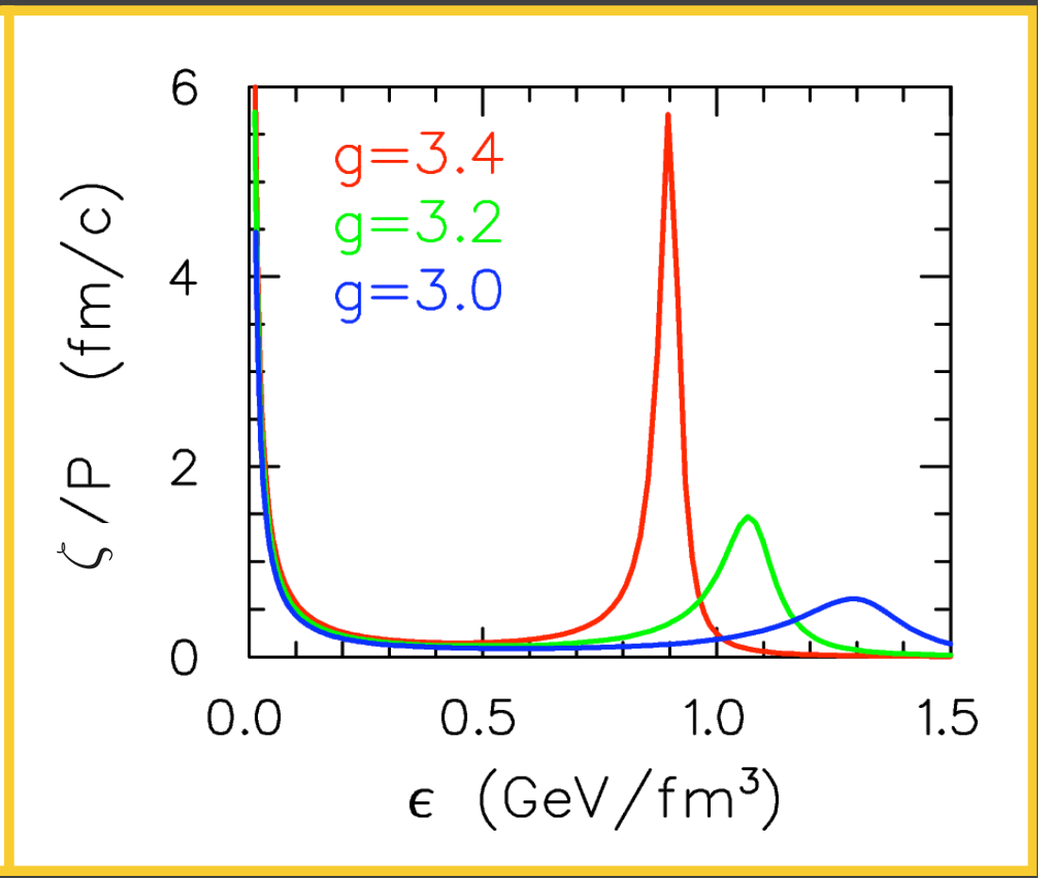

Summary and Outlook

- Non-equilibrium dynamics of the chiral phase transition
- non-eq dynamics of order parameter field can lead to large density inhomogeneities: $\Delta e/e0$, $\Delta\rho/\rho 0 \sim 1$
- Δe and $\Delta\rho$ depends on structure of V_{eff}: larger effect in the regime of 1st-O transitions than for crossover
- Bulk viscosity can alternatively can be included through explicit evolution of chemistry or fields and could become important near T_c