Introduction to the Standard Model
William and Mary PHYS 771 Spring 2014

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Class information, including syllabus and homework assignments can be found at
`http://ntc0.lbl.gov/~walkloud/wm/courses/PHYS_771/`
or
`http://cyclades.physics.wm.edu/~walkloud/wm/PHYS_771/`

Homework Assignment 2: due no sooner than Wed., 26 February

1. **[25 pts.]** For a compact Lie group, we discussed the importance of the Cartan subalgebra, H spanned by the maximal set of commuting generators,

$$H_i^\dagger = H_i, \quad [H_i, H_j] = 0, \quad \mathrm{Tr}(H_i H_j) = \lambda \delta_{ij}, \quad H_i|\mu, x, D\rangle = \mu_i|\mu, x, D\rangle, \tag{1}$$

where $?_i$ are the *weights*, x is other information needed to describe the state and D is the representation. We discussed in the adjoint representation, $[T_a]_{bc} = -if_{abc}$, the states can be described by the generators, as the same label describes both the generators (a) and the states (elements of the matrix (b, c)). This led to important properties, such as linear combinations of states correspond to linear combinations of generators

$$a|X_a\rangle + b|X_b\rangle = |aX_a + bX_b\rangle \tag{2}$$

with a convenient scalar product

$$\langle X_a|X_b\rangle = \lambda^{-1}\mathrm{Tr}(X_a^\dagger X_b) \tag{3}$$

which allowed us to determine the action of a generator on a state

$$X_a|X_b\rangle = |[X_a, X_b]\rangle \tag{4}$$

and so the states which correspond to the Cartan subalgebra satisfy

$$H_i|H_j\rangle = 0 \tag{5}$$

while other states corresponding to the rest of the generators satisfy

$$H_i|E_\alpha\rangle = \alpha_i E_\alpha \to [H_i, E_\alpha] = \alpha_i E_\alpha, \quad [H_i, E_\alpha^\dagger] = -\alpha_i E_\alpha^\dagger \to E_\alpha^\dagger = E_{-\alpha}, \tag{6}$$

and $[E_\alpha, E_{-\alpha}] = \alpha \cdot H$.

In the adjoint representation, these non zero weights α_i are called *roots* and uniquely specify the states. For each non zero pair of root vectors, $\pm\alpha$, there is an $SU(2)$ subalgebra of the group, with generators

$$E_\pm \equiv |\alpha|^{-1} E_{\pm\alpha}, \qquad E_3 \equiv |\alpha|^{-2}\alpha \cdot H. \tag{7}$$

(a) what are the commutation relations

$$[E_3, E_\pm] = ?$$
$$[E_+, E_-] = ? \tag{8}$$

[5 pts.] *Solution:* see attached notes

(b) in $SU(3)$, what are the explicit commutation relations $[E_+, E_-]$ for all $SU(2)$ subalgebras?

[6 pts.] *Solution:* see attached notes

(c) in $SU(3)$, calculate f_{147} and f_{458}.

[2 pts.] *Solution:* see attached notes

(d) $SU(2)$ subalgebra of $SU(3)$:

[6 pts.] *Solution:* see attached notes

i. show that t_1, t_2 and t_3 generate an $SU(2)$ subalgebra of $SU(3)$

ii. take t_3 as the Cartan generator of the subalgebra and the raising lowering operators as $t_\pm = (t_1 \pm it_2)/\sqrt{2}$. What are the eigenvectors in this representation, $|1\rangle, |2\rangle, |3\rangle$ and their corresponding weight vectors? Which is the lowest?

iii. Is there an invariant subspace in this representation? (do the raising/lowering operators span the space of these eigenvectors?)

(e) same as problem 1d except with t_2, t_5 and t_7

[6 pts.] *Solution:* see attached notes

2. **[15 pts.]** We discussed the need for a new quantum number, color, to explain the $|\Delta^{++}\rangle$ and $|\Omega\rangle$ decuplet states, which have totally symmetric spin and flavor wave-functions. This also leads us to require the combined spin and flavor wave-functions of the octet baryons are totally symmetric, which can be constructed for example as

$$|B\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\psi_f^S\rangle|\psi_s^S\rangle + |\psi_f^A\rangle|\psi_s^A\rangle\right) \tag{9}$$

where the mixed-symmetric and mixed-anti-symmetric spin wave-functions are

$$|\uparrow S\rangle = \frac{1}{\sqrt{6}}\left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right)$$
$$|\uparrow A\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right) \tag{10}$$

where the ordering of the labels correspond to quark 1,2 and 3 in the proton wave-function. Similarly, the proton mixed-symmetric and mixed-anti-symmetric flavor wave functions are

$$|pS\rangle = \frac{1}{\sqrt{6}}\left(|udu\rangle + |duu\rangle - 2|uud\rangle\right) ,$$
$$|pA\rangle = \frac{1}{\sqrt{2}}\left(|ud\rangle - |du\rangle\right)|u\rangle$$
$$= \frac{1}{\sqrt{2}}\left(|udu\rangle - |duu\rangle\right) , \tag{11}$$

where the total proton spin-up spin-flavor wave-function is

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\Big[\ |uud\rangle\left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right) + |udu\rangle\left(|\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\downarrow\uparrow\rangle\right) + |duu\rangle\left(|\uparrow\downarrow\uparrow\rangle + |\uparrow\uparrow\downarrow\rangle - 2|\downarrow\uparrow\uparrow\rangle\right)\Big] \tag{12}$$

and recall the ordering of the labels is correlated

$$|uud\rangle \otimes |\uparrow\downarrow\uparrow\rangle = |uud\rangle|\uparrow\downarrow\uparrow\rangle = |u\uparrow, u\downarrow, d\uparrow\rangle\,,$$
$$\langle uud|\langle\uparrow\downarrow\uparrow\,|\,|uud\rangle|\uparrow\uparrow\downarrow\rangle = 0 \tag{13}$$

This is not the only way to construct an anti-symmetric wave-function for the octet baryons. For example

$$|p\uparrow\rangle_A = \frac{1}{\sqrt{2}}\left(|pA\rangle|\uparrow S\rangle - |pS\rangle|\uparrow A\rangle\right) \tag{14}$$

is a totally anti-symmetric wave-function, without the need for color.

(a) following Eq. (14), determine the neutron spin-flavor wave-function in full detail as in Eq. (12)

[5 pts.] *Solution:* see attached notes

(b) compute the proton magnetic moment with Eq. (14) as we did in class

[8 pts.] *Solution:* see attached notes

(c) what is

$$\frac{\mu_n}{\mu_p} = ? \tag{15}$$

with these anti-symmetric, colorless wave-functions? How does it compare with experiment?

[2 pts.] *Solution:* see attached notes

3. **[20 pts.]** In class, we determined the hyper-fine quark-quark interaction Hamiltonian for the $N-\Delta$ and $\Lambda-\Sigma$ systems;

$$H_{S\cdot S'} = \frac{2}{3}\sum_{j\neq k}\kappa_j\kappa_k\vec{S}_j\cdot\vec{S}_k\,, \qquad \kappa_j = \frac{\kappa}{m_j} \tag{16}$$

where κ is a constant proportional to the quark chromo-magnetic moment, with mass dimension 3/2 and m_j is the constituent quark mass for quark flavor j (the strong interactions are flavor blind so κ is the same for all quarks). We worked out

$$H^{\Lambda\Sigma}_{S\cdot S'} = \frac{2}{3}\left[\kappa_l^2\begin{pmatrix}1/4\\1/4\\-3/4\end{pmatrix} + \kappa_l\kappa_s\begin{pmatrix}1/2\\-1\\0\end{pmatrix}\right] \text{ for } \begin{cases}\Sigma^* & S_d=1 \ \ S_T=3/2\\ \Sigma & S_d=1 \ \ S_T=1/2\\ \Lambda & S_d=0 \ \ S_T=1/2\end{cases} \tag{17}$$

where $\kappa_u = \kappa_d = \kappa_l$ for the light quarks and S_d is the spin of the light di-quark and S_T is the total spin.

(a) determine the equivalent expression, Eq. (17) for the Ξ-Ξ^* system.
[8 pts.] *Solution:* see attached notes

(b) determine $H_{S.S'}$ for the spin-0 and spin-1 mesons
[10 pts.] *Solution:* see attached notes

(c) assuming that κ in the baryons is the same as the mesons, relate $m_\Delta - m_N$ to $m_\rho - m_\pi$ and compare to the experimental values
[2 pts.] *Solution:* see attached notes

HW 2

1) a) $$[E_3, E_\pm] = \frac{1}{|\alpha|^3}[\alpha\cdot H, E_{\pm\alpha}]$$

$$= \frac{1}{|\alpha|^3}\alpha_i [H_i, E_{\pm\alpha}]$$

$$= \frac{1}{|\alpha|^3}\alpha_i(\pm\alpha_i E_{\pm\alpha})$$

$$= \frac{1}{|\alpha|^3}|\alpha|^2(\pm E_{\pm\alpha})$$

$$= \pm E_\pm \qquad \left(E_\pm = \frac{1}{|\alpha|}E_{\pm\alpha}\right)$$

$$[E_+, E_-] = \frac{1}{|\alpha|^2}[E_{+\alpha}, E_{-\alpha}]$$

$$= \frac{1}{|\alpha|^2}\alpha\cdot H$$

$$= E_3$$

b) For each pair of root vectors, $\pm\alpha$, there are SU(2) subalgebras. For SU(3), there are 3 such pairs.

The Cartan Subalgebra is

$$H = (t_3, t_8) \qquad (H_1 \equiv t_3,\ H_2 \equiv t_8)$$

The weights are

$$(\mu_1, \mu_2) = \left\{\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right), \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}\right), \left(0, -\frac{1}{\sqrt{3}}\right)\right\}$$

For SU(3), this gives the corresponding generators

$$E_{+\alpha_1} = \frac{1}{\sqrt{2}}(t_4 + it_5) \qquad E_{-\alpha_1} = \frac{1}{\sqrt{2}}(t_4 - it_5)$$

$$E_{+\alpha_2} = \frac{1}{\sqrt{2}}(t_6 + it_7) \qquad E_{-\alpha_2} = \frac{1}{\sqrt{2}}(t_6 - it_7)$$

$$E_{+\alpha_3} = \frac{1}{\sqrt{2}}(t_1 + it_2) \qquad E_{-\alpha_3} = \frac{1}{\sqrt{2}}(t_1 - it_2)$$

The roots can be determined from differences of the ~~general~~ weights

$$\alpha_1 = \mu^{(1)} - \mu^{(2)} = (1, 0) \qquad \alpha_1 \cdot \alpha_1 = 1$$

$$\alpha_2 = \mu^{(1)} - \mu^{(3)} = (1/2, \sqrt{3}/2) \qquad \alpha_2 \cdot \alpha_2 = 1$$

$$\alpha_3 = \mu^{(2)} - \mu^{(3)} = (-1/2, \sqrt{3}/2) \qquad \alpha_3 \cdot \alpha_3 = 1$$

α_1: $E_\pm = \frac{1}{|\alpha_1|} E_{\pm\alpha_1} = \frac{1}{\sqrt{2}} (t_1 \pm i t_2)$

$$[E_+, E_-] = \frac{1}{2} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} = E_3 \quad \checkmark \quad = t_3 + 0\, t_8$$

α_2: $E_\pm = \frac{1}{|\alpha_2|} E_{\pm\alpha_2} = \frac{1}{\sqrt{2}} (t_4 \pm i t_5)$

$$[E_+, E_-] = \frac{1}{2} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 1 \end{pmatrix} = E_3 \quad \checkmark \quad = \frac{1}{2} t_3 + \frac{\sqrt{3}}{2} t_8$$

α_3: $E_\pm = \frac{1}{|\alpha_3|} E_{\pm\alpha_3} = \frac{1}{\sqrt{2}} (t_6 \pm i t_7)$

$$[E_+, E_-] = \frac{1}{2} \begin{pmatrix} 0 & & \\ & 1 & \\ & & -1 \end{pmatrix} = E_3 \quad \checkmark \quad = -\frac{1}{2} t_3 + \frac{\sqrt{3}}{2} t_8$$

c) calculate f_{147} & f_{458} for SU(3)

$$[t_a, t_b] = i f_{abc} t_c$$

We can ~~tab~~ determine a specific constant using

$$f_{abc} = -2i \, tr(t_c [t_a, t_b]).$$

$$f_{147} = -2i \, tr(t_7 [t_1, t_4])$$
$$= \frac{1}{2}$$

$$f_{458} = -2i \, tr(t_8 [t_4, t_5])$$
$$= \frac{\sqrt{3}}{2}$$

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d) SU(2) subalgebras of SU(3)

i) We showed above that $t_\pm = \frac{1}{\sqrt{2}}(t_1 \pm i t_2)$

$$\Rightarrow [t_+, t_-] = t_3$$

Also, we have

$$[t_1, t_2] = i \varepsilon_{123} t_3$$

$$[t_i, t_j] = i \varepsilon_{ijk} t_k$$

ii) $t_3 = H_1$

$t_\pm = \frac{1}{\sqrt{2}}(t_1 \pm i t_2)$

What are the eigenvectors? and their weights?

Because $t_3 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$, we can pick the eigenvectors

$|1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$|2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$|3\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$t_3|1\rangle = \frac{1}{2}|1\rangle$

$t_3|2\rangle = 0|2\rangle$

$t_3|3\rangle = -\frac{1}{2}|3\rangle$

iii) $|1\rangle$ is the highest weight vector

$|3\rangle$ is the lowest weight vector

iv) There is an invariant subspace

$t_+|3\rangle = \frac{1}{\sqrt{2}}|1\rangle$

$t_+|1\rangle = 0$

$t_-|1\rangle = \frac{1}{\sqrt{2}}|3\rangle$

$t_-|3\rangle = 0$

$t_\pm|2\rangle = 0$

The ladder operators ~~don't~~ ~~change~~ project $|2\rangle$ out of space.

e) If we instead pick

$$t_2, t_5, t_7$$

i) they generate an SU(2) algebra

$$[t_2, t_5] = \frac{i}{2} t_7$$

$$[t_5, t_7] = \frac{i}{2} t_2 \quad \checkmark$$

$$[t_7, t_2] = \frac{i}{2} t_5$$

ii) lets take t_2 as the Cartan generator

$$t_\pm = t_5 \pm i t_7 \qquad [t_2, t_+] = \frac{1}{2} t_+$$

$$[t_+, t_-] = t_2 \qquad [t_2, t_-] = -\frac{1}{2} t_-$$

The eigenvectors are proportional to

$$|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -i \\ 1 \\ 0 \end{pmatrix} \qquad t_+|1\rangle = 0 \qquad t_2|1\rangle = +|1\rangle \frac{1}{2}$$

$$|2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad t_+|2\rangle = \frac{1}{\sqrt{2}}|1\rangle \qquad t_2|2\rangle = 0|2\rangle$$

$$|3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} \qquad t_+|3\rangle = \frac{1}{\sqrt{2}}|2\rangle \qquad t_2|3\rangle = -|3\rangle \frac{1}{2}$$

iii) Highest weight vec $|1\rangle$ with weight $\frac{1}{2}$

lowest $|3\rangle$ $-\frac{1}{2}$

iv) No, the representation spans the space

2) a) $|p\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|pA\rangle|\uparrow S\rangle - |pS\rangle|\uparrow A\rangle \right)$

$$= \frac{1}{\sqrt{2}}\Big[\frac{1}{\sqrt{2}}\left(|udu\rangle - |duu\rangle\right)\frac{1}{\sqrt{6}}\left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right)$$

$$- \frac{1}{\sqrt{6}}\left(|udu\rangle + |duu\rangle - 2|uud\rangle\right)\cdot\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right)\Big]$$

$$= \frac{1}{\sqrt{24}}\Big[|udu\rangle|\uparrow\downarrow\uparrow\rangle + |udu\rangle|\downarrow\uparrow\uparrow\rangle - 2|udu\rangle|\uparrow\uparrow\downarrow\rangle$$

$$- |duu\rangle|\uparrow\downarrow\uparrow\rangle - |duu\rangle|\downarrow\uparrow\uparrow\rangle + 2|duu\rangle|\uparrow\uparrow\downarrow\rangle$$

$$- |udu\rangle|\uparrow\downarrow\uparrow\rangle + |udu\rangle|\downarrow\uparrow\uparrow\rangle$$

$$- |duu\rangle|\uparrow\downarrow\uparrow\rangle + |duu\rangle|\downarrow\uparrow\uparrow\rangle$$

$$+ 2|uud\rangle|\uparrow\downarrow\uparrow\rangle - 2|uud\rangle|\downarrow\uparrow\uparrow\rangle \Big]$$

$$= \frac{1}{2\sqrt{6}}\Big[2|udu\rangle|\downarrow\uparrow\uparrow\rangle - 2|duu\rangle|\uparrow\downarrow\uparrow\rangle - 2|udu\rangle|\uparrow\uparrow\downarrow\rangle$$

$$+ 2|duu\rangle|\uparrow\uparrow\downarrow\rangle + 2|uud\rangle|\uparrow\downarrow\uparrow\rangle - 2|udd\rangle|\downarrow\uparrow\uparrow\rangle \Big]$$

To get the neutron, we simply swap $u \longleftrightarrow d$

$$|n\uparrow\rangle = \frac{1}{\sqrt{6}}\Big[|dud\rangle|\downarrow\uparrow\uparrow\rangle - |udd\rangle|\uparrow\downarrow\uparrow\rangle - \cancel{2}|dud\rangle|\uparrow\uparrow\downarrow\rangle$$

$$+ |udd\rangle|\uparrow\uparrow\downarrow\rangle + |ddu\rangle|\uparrow\downarrow\uparrow\rangle - |ddu\rangle|\downarrow\uparrow\uparrow\rangle \Big]$$

b) Compute proton μ_p

$$\mu_i = Q_i\left(\frac{e}{2m_i}\right) \quad , \quad Q_u = \frac{2}{3} \quad , \quad Q_d = -\frac{1}{3}$$

$$\mu_p = \sum_{\ell=1}^{3} \langle p\uparrow | \mu_\ell (\sigma_3)_\ell | p\uparrow \rangle$$

$$(\sigma_3)_1 = \sigma_3 \otimes 1 \otimes 1$$

$$\Rightarrow \mu_p = \frac{1}{6}\Big\{ -\mu_u + \mu_d + \mu_u + \mu_d + \mu_u - \mu_u$$
$$+ [\mu_d - \mu_u + \mu_d + \mu_u - \mu_u + \mu_u]$$
$$+ [\mu_u + \mu_u - \mu_u - \mu_u + \mu_d + \mu_d] \Big\}$$

$$= \frac{1}{6}\left[6\mu_d + 0\mu_u \right] = \mu_d$$

c) for the neutron, we just use isospin symmetry to change $u \longleftrightarrow d$

$$\mu_n = \mu_u$$ [Don't work hard when you don't have too]

$$\frac{\mu_n}{\mu_p} = \frac{\mu_u}{\mu_d} = \frac{-2\mu_d}{\mu_d} = -2$$

not very similar to -1.5 we get with the other wave functions (from class).

3] Hyperfine quark-quark interaction

$$H_{S\cdot S'} = \frac{2}{3} \sum_{j \neq k} K_j K_k \, \vec{S}_j \cdot \vec{S}_k \qquad K_j = \frac{K}{m_j}$$

(a) What is the interaction Hamiltonian for the $\Xi \cdot \Xi^*$ system?

let $q=1$ correspond to the light quark and $q=2,3$ be the strange quarks

$$H = \frac{2}{3}\left[K_\ell K_s \, \vec{S}_1 \cdot (\vec{S}_2 + \vec{S}_3) + K_s^2 \, \vec{S}_2 \cdot \vec{S}_3 \right]$$

$$S_d = S_2 + S_3$$

$$S_T^2 = S_1^2 + (S_d)^2 + 2 S_1 \cdot S_d$$

$$\vec{S}_1 \cdot \vec{S}_d = \frac{1}{2}\left[\vec{S}_T^2 - \vec{S}_1^2 - \vec{S}_d^2 \right]$$

$$\langle \vec{S}_1 \cdot (\vec{S}_2 + \vec{S}_3) \rangle = \frac{1}{2}\left[S_T(S_T+1) - \frac{3}{4} - S_d(S_d+1) \right]$$

$$\langle \vec{S}_2 \cdot \vec{S}_3 \rangle = \frac{1}{2} \langle \vec{S}_d^2 - 2\vec{S}_i^2 \rangle$$

$$= \frac{1}{2}\left[S_d(S_d+1) - 2 \cdot \frac{3}{4} \right]$$

$$H^{\Xi\Xi}_{S\cdot S} = \frac{2}{3}\left[K_\ell K_s \cdot \frac{1}{2}\left[S_T(S_T+1) - \frac{3}{4} - S_d(S_d+1) \right] + K_s^2 \cdot \frac{1}{2}\left[S_d(S_d+1) - \frac{3}{2} \right] \right]$$

If $S_T = 3/2 \Rightarrow S_d = 1$

Also, $S_T = 1/2 \Rightarrow S_d = 1$

This is because the only possible flavor wave function for the s-quarks is symmetric, which means the spin w.f. must also be symmetric

$$\boxed{\Rightarrow H^{\Xi\Xi}_{S\cdot S} = \frac{2}{3}\left[K_\ell K_s \begin{pmatrix} 1/2 \\ -1 \end{pmatrix} + K_s^2 \begin{pmatrix} 1/4 \\ 1/4 \end{pmatrix} \right] \begin{matrix} \Xi^*(3/2) \\ \Xi(1/2) \end{matrix}}$$

b) for $\bar{q}q$ mesons, knowing we are modeling QCD, we have to adjust the prefactor, $\frac{2}{3}$.

For $\bar{q}q$ mesons, the strength of the color interaction is slightly enhanced

$$H^{\phi}_{s\cdot s'} = \frac{4}{3}\sum_{j\neq k} K_j K_k \vec{S}_j \cdot \vec{S}_k$$

$$= \frac{4}{3} K_1 K_2 \vec{S}_1 \cdot \vec{S}_2$$

$$= \frac{4}{3} K_1 K_2 \left[\frac{1}{2}\vec{S}_T^2 \underbrace{- \frac{1}{2} S_1^2 - \frac{1}{2} S_2^2}\right] \qquad S_1^2 = \frac{3}{4} \qquad \frac{1}{2}\cdot\left(2\,\frac{3}{4}\right) = \frac{3}{4}$$

$$= \frac{4}{3} K_1 K_2 \left[\frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \frac{3}{2}\right] \qquad \begin{pmatrix} \rho, K^*, \ldots \\ \pi, K, \ldots \end{pmatrix}$$

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$$= \frac{4}{3} K_1 K_2 \begin{bmatrix} -1/2 \\ -3/4 \end{bmatrix} \qquad \begin{matrix} S_T = 1 & (\rho, \ldots \\ S_T = 0 & (\pi, \ldots \end{matrix} \qquad \frac{4}{3} K_1 K_2 \begin{bmatrix} 1/4 \\ -3/4 \end{bmatrix}$$

+1/4

$$m_\rho = \cancel{-\frac{2}{3} K_\ell^2 + 2m_q} = 2m_q + \frac{1}{3}K_\ell^2$$

$$m_\pi = \cancel{-2K_\ell^2 + 2m_q} = 2m_q - K_\ell^2$$

$$\underline{m_\rho - m_\pi = \frac{4}{3} K_\ell^2}$$

For the Δ-N splitting, we have from class

$$m_\Delta - m_N = K_\ell^2$$

$$\underbrace{m_\rho - m_\pi}_{\sim 630\ \text{MeV}} = \underbrace{\frac{4}{3}(m_\Delta - m_N)}_{\sim 390\ \text{MeV}}$$