Tues. Feb. 11 2014

Start w/ $\square \times \square \times \square = \square\square\square + \square + \square + \square$

mesons 0^{-+} vector mesons baryons octet decuplet

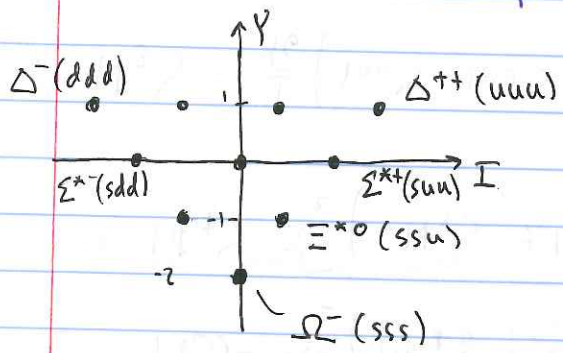
Gell-Mann – Okubo Formula

$$M_\Lambda = \frac{1}{3}\left(2M_N + 2M_\Xi - M_\Sigma\right)$$

$$1115.7\ \text{MeV} \overset{?}{=} 1106.5\ \text{MeV}$$

only ~1% off!

What about 10-d rep? $\square\square\square$



use symmetry to show this

For the mass term of a 10- there is only 1 (not two)

$10 = \square\square\square$

$10^* = \begin{smallmatrix}\square\square\square\\ \square\square\square\end{smallmatrix}$ $\quad (10^* \otimes 10 \supset 1) \quad 1 = \square$ or $\square\square\square\square$ ...

So this predicts an equal spacing rule

$$M_{\Sigma^*} - M_\Delta = M_{\Xi^*} - M_{\Sigma^*} = M_\Omega - M_{\Sigma^*}$$

When Gell-Mann was constructing this quark model – the Ω^- had not been observed

$M_\Delta \simeq 1230$ $\quad \delta M \cong 155$

$M_{\Sigma^*} \simeq 1385$ $\quad \delta M \simeq 145$

$M_{\Xi^*} \simeq 1530$

$\Rightarrow$ Gell-Mann predicted the existence of Ω and

$$M_\Omega \simeq 1680\ \text{MeV} \quad [1672.45\ \text{MeV}]!$$

This made the quark-model suddenly very promising.

And at the same time, ~~very~~ introduced a puzzle.

$|\Delta^{++}\rangle = |uuu\rangle$ completely symmetric in flavor & spin

=> ultimately, this led to the introduction of a new QN. - color

→ for mesons, GMO is for mass² operators

$$\mathcal{L}_m = \phi^\dagger [\partial^2 - m^2]\phi \quad \text{vs.} \quad \mathcal{L}_F = \bar{\Psi}(i\not{\partial} - m)\Psi$$

$$m_\eta^2 = \frac{1}{3}\left[2(m_K^2 + m_{\bar{K}}^2) - m_\pi^2\right]$$

$$0.299 \overset{?}{=} 0.321 \quad \text{GeV}^2$$

~~10% correction~~

16.7% relative diff $\frac{(\varepsilon_1 - \varepsilon_2)/2}{(\varepsilon_1 + \varepsilon_2)/2}$

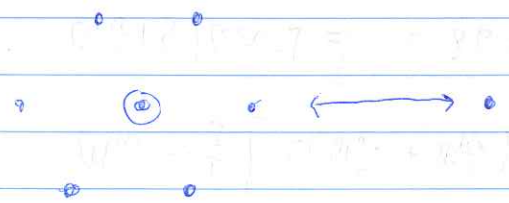
→ Vector mesons

$$m_\omega^2 = \frac{1}{3}\left[2(m_{K^*}^2 + m_{\bar{K}^*}^2) - m_\rho^2\right]$$

$$0.613\,[\text{GeV}^2] \overset{?}{=} 0.865$$

~17% relative diff.

Because there is SU(3) breaking, the eigenstates will not be pure SU(3) eigenstates



In particular, the ω and ϕ will mix (both isosinglet)
we are assuming exact isospin

$$\begin{pmatrix} \Phi_\omega \\ \Phi_\phi \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Phi_8 \\ \Phi_1 \end{pmatrix}$$

QCD eigenstates SU(3) eigenstates

$$\mathcal{L} = \bar{\Phi}_{SU(3)} \left(-\partial^2 - M^2_{SU(3)} \right) \Phi_{SU(3)} \quad \text{or} \quad \bar{\Phi}_{QCD} \left(-\partial^2 - M^2_{QCD} \right) \Phi_{QCD}.$$

$$M^2_{QCD} = \begin{pmatrix} m_\omega^2 & \\ & m_\phi^2 \end{pmatrix}$$

$$M^2_{SU(3)} = \begin{pmatrix} m_{88}^2 & m_{81}^2 \\ m_{18}^2 & m_{11}^2 \end{pmatrix}$$

The GMO formula should be a good estimate for the SU(3) basis masses

$$M^2 = \begin{pmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{pmatrix} \begin{pmatrix} m_\omega^2 & \\ & m_\phi^2 \end{pmatrix} \begin{pmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{pmatrix}$$

$$\sin\theta = \sqrt{\frac{m_{88}^2 - m_\omega^2}{m_\phi^2 - m_\omega^2}}$$

$$\sin\theta = \sqrt{\frac{0.859 - (0.782)^2}{(1.020)^2 - (0.783)^2}} = 0.76$$

$$\theta \simeq 49.5^\circ \quad \cancel{(0.86} \quad (0.27\pi)$$

This means there is near maximal mixing between ω, ϕ mesons

Suppose $\theta = \pi/4$ (maximal mixing) Then the QCD eigenstates would be

$$|\phi_\omega\rangle = \frac{\sqrt{2}}{2}\left(|\phi_8\rangle + |\phi_1\rangle\right)$$

$$|\phi_\phi\rangle = \frac{\sqrt{2}}{2}\left(-|\phi_8\rangle + |\phi_1\rangle\right)$$

$$|\phi_8\rangle = \frac{1}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle\right)$$

$$|\phi_1\rangle = \frac{\sqrt{2}}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\right)$$

$$|\phi_\omega\rangle = \frac{1}{\sqrt{12}}\left((1+\sqrt{2})(|u\bar{u}\rangle + |d\bar{d}\rangle - (2-\sqrt{2})|s\bar{s}\rangle\right)$$

$$|\phi_\phi\rangle = \frac{1}{\sqrt{12}}\left((\sqrt{2}-1)(|u\bar{u}\rangle + |d\bar{d}\rangle) + (\sqrt{2}+2)|s\bar{s}\rangle\right)$$

$|\phi_\omega\rangle$: $\frac{(1+\sqrt{2})^2}{12}$ probability to be $|u\bar{u}\rangle$ & $|d\bar{d}\rangle$ 48.6% each

$\frac{(2-\sqrt{2})^2}{12}$ probability to be $|s\bar{s}\rangle$ ~ 2.9%

$|\phi_\phi\rangle$: $\frac{(\sqrt{2}-1)^2}{12} \simeq 1.4\%$ to be $|u\bar{u}\rangle$ or $|d\bar{d}\rangle$

$\frac{(\sqrt{2}+2)^2}{12} \simeq 97\%$ to be $|s\bar{s}\rangle$

It is found

$$\frac{\Gamma(\phi \to K K^+)}{\Gamma(\phi \to X)} \simeq 83\% \quad , \quad \frac{\Gamma(\phi \to \pi\pi\pi)}{\Gamma} \sim 15\%$$

This is consistent w/ model that $|\phi\rangle$ is predominantly $|s\bar{s}\rangle$

as the phase space $(m_\phi - 3m_\pi)$ is much larger for $(m_\phi - 2m_K)$ 3 π's than KK^+.

What about mixing for pseudo-scalars?

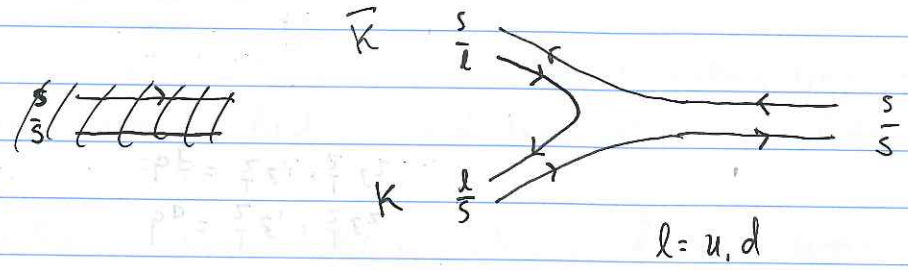
$$\cancel{|\eta\rangle = |\phi_8\rangle}$$

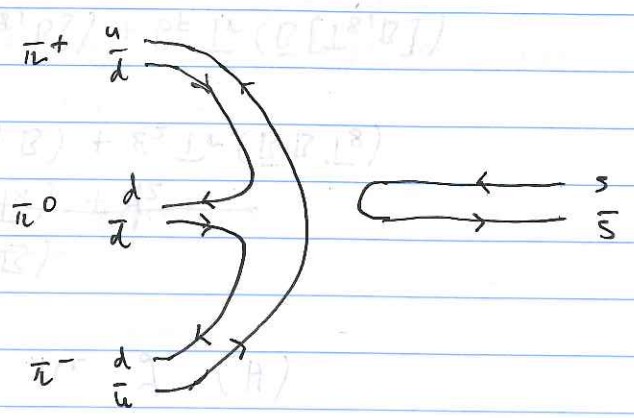
$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |8\rangle \\ |1\rangle \end{pmatrix}$$

$$\Rightarrow \sin\theta \simeq \frac{\sqrt{0.321 - (0.548)^2}}{\sqrt{(0.958)^2 - (0.548)^2}} \simeq 0.19 \Rightarrow \theta \simeq 11^\circ \simeq \frac{\pi}{16.5}$$

This is another example of how pseudo-scalars are special

Suppose $|\phi\rangle$ is mostly $|s\bar{s}\rangle$, then decay modes can be understood with "quark-flow" diagrams





Historically, the suppression of diagrams like the 2nd type were known as OZI rule (Okubo, Zweig, Iizuka).

We now understand there are intermediate gluons (at least 3) in this case.

OZI suppression occurs whenever you can cut a quark-flow diagram and only cut virtual gluons (not drawn)

Lets look at masses ~~more~~ in more detail.

In QED, positronium receives a hyperfine splitting

$$H_{ss} = -\frac{2}{3}\vec{\mu}_1\cdot\vec{\mu}_2\,|\psi(0)|^2$$

$$= +\frac{2}{3}\,4\pi\alpha\,\frac{\vec{S}_1\cdot\vec{S}_2}{m_1 m_2}\,|\psi(0)|^2$$

$$\vec{\mu}_i = \frac{e_i}{m_i}\,\frac{\vec{\sigma}_i}{2}$$

For QCD (SU(3) vector gauge theory $\gamma \to g_a$)

$\alpha \to$	$\frac{2}{3}\alpha_s$	(qqq)
	$\frac{4}{3}\alpha_s$	$(\bar{q}q)$

We expect a similar interaction.

Consider first the N-Δ system.

$$H_{s.s} = \frac{2}{3}\frac{k^2}{m_q^2}\underbrace{\left(\vec{S}_1\cdot\vec{S}_2 + \vec{S}_1\cdot\vec{S}_3 + \vec{S}_2\cdot\vec{S}_3\right)}$$

$$\frac{1}{2}\left\{(\vec{S}_T)^2 - \sum_i \vec{S}_i^2\right\} = \begin{cases} 3/4 & S_T = 3/2 \\ -3/4 & S_T = 1/2 \end{cases}$$

$$\frac{1}{2}\left\{S_T(S_T+1) - \frac{9}{4}\right\}$$

$$\Rightarrow \boxed{\begin{aligned} m_N &= 3m_q - \frac{1}{2}\frac{k^2}{m_q^2} = 3m_q - \frac{1}{2}\mu_q^2 \quad ; \quad \mu_q \equiv \frac{k}{m_q} \\ m_\Delta &= 3m_q + \frac{1}{2}\frac{k^2}{m_q^2} = 3m_q + \frac{1}{2}\mu_q^2 \end{aligned}}$$

$m_q = \frac{1}{6}(m_\Delta + m_N) \simeq 360$ MeV Constituent quark mass

$\mu_q^2 = (m_\Delta - m_N) \simeq 290$ MeV $\mu_q \propto$ magnetic moment of constituent quark

What if we change one of light quarks to strange?

$(N, \Delta) \rightarrow (\Lambda, \Sigma, \Sigma^*)$

$N \doteq$ $I = 1$

$I = 0$

$\Sigma \quad I = 1$

$\Lambda \quad I = 0$

$\Rightarrow$ remaining 2 light quarks $I = 1 \Rightarrow \Sigma$

$I = 0 \Rightarrow \Lambda$

Now H_{ss}

$$H_{ss} = \frac{2}{3} \sum_{j \neq k} \cancel{\mu_j} \cancel{\mu_k} \, \vec{S}_j \cdot \vec{S}_k$$

if $j = k = 1$ corresponds to s-quark

$$H_{ss} = \frac{2}{3} \left[\cancel{\mu_q^2} \, \vec{S}_2 \cdot \vec{S}_3 + \cancel{\mu_q} \cancel{\mu_s} \, \vec{S}_1 \cdot (\vec{S}_2 + \vec{S}_3) \right]$$

$$\vec{S}_T^{\,2} = \vec{S}_1^{\,2} + \underbrace{(\vec{S}_2 + \vec{S}_3)^2}_{\vec{S}_d^{\,2}} + 2 \vec{S}_1 \cdot (\vec{S}_2 + \vec{S}_3)$$

$$\vec{S}_1 \cdot (\vec{S}_2 + \vec{S}_3) = \vec{S}_1 \cdot \vec{S}_d = \frac{1}{2} \left[\vec{S}_T^{\,2} - \vec{S}_1^{\,2} - \vec{S}_d^{\,2} \right]$$

↑ diquark spin

$$\langle \vec{S}_1 \cdot (\vec{S}_2 + \vec{S}_3) \rangle = \frac{1}{2} \left[S_T (S_T + 1) - \frac{3}{4} - S_d (S_d + 1) \right]$$

S_d

$$\langle \vec{S}_2 \cdot \vec{S}_3 \rangle = \frac{1}{2} \langle \vec{S}_d^{\,2} - 2 \vec{S}_i^{\,2} \rangle = \frac{1}{2} \left[S_d (S_d + 1) - 2 \cdot \frac{3}{4} \right]$$

$$H_{ss}^{\Lambda\Sigma} = \frac{2}{3}\left[\cancel{\mu_q^2}\cdot \vec{S}_2\cdot\vec{S}_3 + \cancel{\mu_q}\,\cancel{\mu_s}\,\vec{S}_1\cdot(\vec{S}_2+\vec{S}_3)\right]$$

$$= \frac{2}{3}\left[\cancel{\mu_q^2}\begin{pmatrix}\frac{1}{4}\\ \frac{1}{4}\\ -3/4\end{pmatrix} + \cancel{\mu_q}\,\mu_s\begin{pmatrix}1/2\\ -1\\ 0\end{pmatrix}\right] \quad \begin{cases} \Sigma^* & S_d = 1 & S_T = 3/2 \\ \Sigma & S_d = 1 & 1/2 \\ \Lambda & S_d = 0 & 1/2 \end{cases}$$

$$m_\Lambda = 2m_q + m_s - \frac{1}{2}\cancel{\mu_q^2}$$

$$m_\Sigma = 2m_q + m_s + \frac{1}{6}\cancel{\mu_q^2} - \frac{2}{3}\cancel{\mu_q}\,\cancel{\mu_s}$$

$$m_{\Sigma^*} = 2m_q + m_s + \frac{1}{6}\cancel{\mu_q^2} + \frac{1}{3}\cancel{\mu_q}\,\cancel{\mu_s}$$

$$m_\Lambda - m_N = m_s - m_q \Rightarrow m_s = 536 \text{ MeV}$$

$$m_{\Sigma^*} - m_\Sigma = \cancel{\mu_q}\,\cancel{\mu_s}$$

$$1385 - 1193$$

$$= \cancel{\mu_q^2}\cdot\frac{m_q}{m_s} = 290 * \frac{360}{536}$$

$$192 \overset{?}{\simeq} 195$$

$$H_{ss}^{\Xi\Xi^*} = \frac{2}{3}\left[\cancel{\mu_s^2}\begin{pmatrix}\frac{1}{4}\\ 1/4\end{pmatrix} + \cancel{\mu_q}\,\cancel{\mu_s}\begin{pmatrix}1/2\\ -1\end{pmatrix}\right] \quad \begin{cases} \Xi^* & S_+ \; 3/2 \\ \Xi & 1/2 \end{cases}$$

$$m_\Xi = m_q + 2m_s + \frac{1}{6}\cancel{\mu_s^2} - \frac{2}{3}\cancel{\mu_q}\,\mu_s$$

$$m_{\Xi^*} = m_q + 2m_s + \frac{1}{6}\cancel{\mu_s^2} + \frac{1}{3}\cancel{\mu_q}\,\cancel{\mu_s}$$

M_B			M_T		
N	$3m_q$	$-\frac{1}{2}\cancel{\mu_q}^2$	Δ	$3m_q$	$+\frac{1}{2}\cancel{\mu_q}^2$
Λ	$3m_q+\Delta_s$	$-\frac{1}{2}\cancel{\mu_q}^2$			
Σ	$3m_q+\Delta_s$	$-\frac{1}{6}(4\cancel{\mu_q}\cancel{\mu_s}-\cancel{\mu_q}^2)$	Σ^*	$3m_q+\Delta_s$	$+\frac{1}{6}(2\cancel{\mu_q}\cancel{\mu_s}+\cancel{\mu_q}^2)$
Ξ	$3m_q+2\Delta_s$	$-\frac{1}{6}(4\cancel{\mu_q}\cancel{\mu_s}-\cancel{\mu_s}^2)$	Ξ^*	$3m_q+2\Delta_s$	$+\frac{1}{6}(2\cancel{\mu_q}\cancel{\mu_s}+\cancel{\mu_s}^2)$
			Ω^-	$3m_q+3\Delta_s$	$+\frac{1}{2}\cancel{\mu_s}^2$

$$m_\Lambda - \frac{1}{3}(2m_N+2m_\Xi - m_\Sigma)$$

$$= -\frac{1}{9}(\cancel{\mu_s}-\cancel{\mu_q})^2$$

$O(t_8^2)$ consistent w/ GMO

$$\frac{290}{9}\cdot\left(\frac{m_q}{m_s}-1\right)^2 \simeq 3.5\ \text{MeV}$$

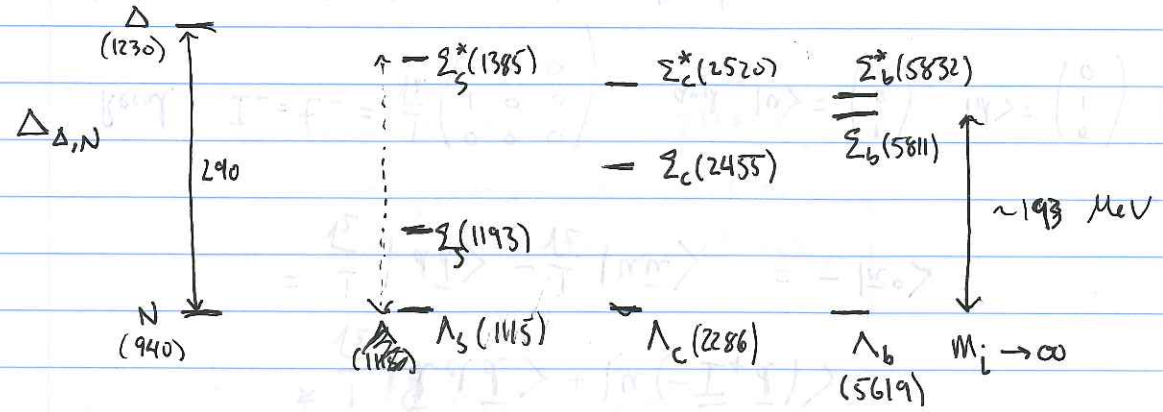
$$m_{\Omega^-} - m_{\Xi^*} = \Delta_s + \frac{2}{6}(\cancel{\mu_s}^2-\cancel{\mu_s}\cancel{\mu_q}) = \Delta_s + \frac{2}{6}\cancel{\mu_s}(\cancel{\mu_s}-\cancel{\mu_q})$$

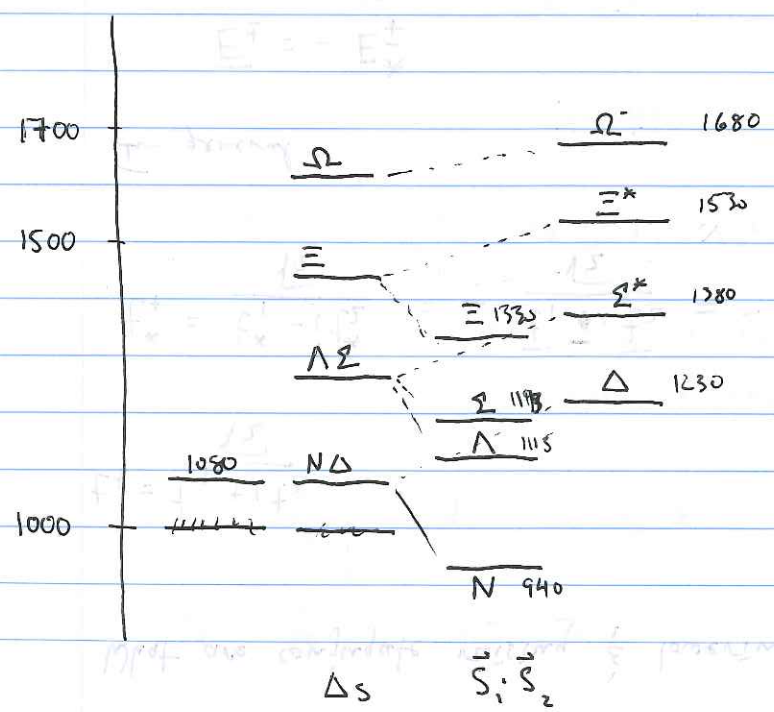
$$m_{\Xi^*} - m_{\Sigma^*} = \Delta_s + \frac{1}{6}(\cancel{\mu_s}^2-\cancel{\mu_q}^2) = \Delta_s + \frac{1}{6}(\cancel{\mu_s}+\cancel{\mu_q})(\cancel{\mu_s}-\cancel{\mu_q}).$$

$$m_{\Sigma^*} - m_\Delta = \Delta_s + \frac{1}{6}(2\cancel{\mu_q}\cancel{\mu_s} - 2\cancel{\mu_q}^2) = \Delta_s + \frac{2}{6}\cancel{\mu_q}(\cancel{\mu_s}-\cancel{\mu_q})$$

<u>Notice</u> we could replace $s \to c$ in our previous discussion - what would we expect?

for fixed (s,c) the splittings are suppressed w/ $1/m_{s,c}$



So this simple model very** qualitatively describes observed hadron spectrum.

For g.s. Hadrons (B,T)