Tues, 8 April

Using this full W propagator, the νe scattering unitarity problem is solved.

But it turns out to leave other unitarity problems, related to the longitudinal modes of the W propagator

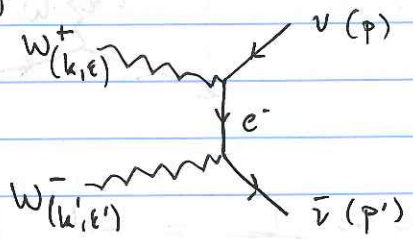
$$\mathcal{L} = -\frac{1}{4}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^\nu - \partial^\nu W^\mu) + M_W^2 W_\mu^+ W^\mu$$

$$\Rightarrow G_{\mu\nu} = -i\,\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2 + i\varepsilon}$$

for large k, $G_{\mu\nu} \sim \frac{1}{M_W^2}\frac{k_\mu k_\nu}{k^2}$

The unitarity problems can be observed in e.g.

$\nu\bar{\nu} \to W^+W^-$

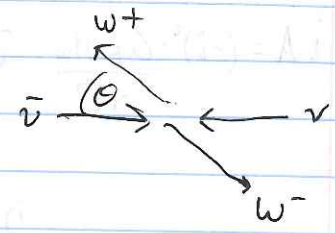


The longitudinal amplitude

$$iA = \bar{V}(p')\left(\frac{-ig\not{\varepsilon}'}{\sqrt{2}}\right)\frac{1-\gamma_5}{2}\,\frac{i}{\not{p}-\not{k}-m_e+i\varepsilon}\left(\frac{-ig\not{\varepsilon}}{\sqrt{2}}\right)\frac{1-\gamma_5}{2}\,u(p)$$

$k_\mu = (E, 0, 0, k)$, $k = \sqrt{E^2 - M_W^2}$

kinematics (relativistic)



$$p_{\bar{\nu}}^\mu = (Q, 0, 0, Q)$$

$$p_{\bar{\nu}}^{*\mu} = (Q, 0, 0, -Q)$$

$$k_+^{*\mu} = (Q, k\sin\theta, 0, k\cos\theta) = k_+^\mu$$

$$k_-^{*\mu} = (Q, -k\sin\theta, 0, -k\cos\theta) = k_-^\mu$$

$$k = \sqrt{Q^2 - M_W^2}$$

$$p_e^\mu = p_{\bar{\nu}}^\mu - k_+^{*\mu}$$

$$= (0, -k\sin\theta, 0, Q - k\cos\theta) \quad , \quad s = 4Q^2$$

The longitudinal polarization vectors

$$\varepsilon_\pm^\mu = \left(\frac{k}{M_W}, \frac{Q}{M_W}\hat{k}_\pm\right) \longrightarrow \frac{k_\pm^\mu}{M_W} \quad \text{for } Q, k \gg M_W$$

~~$\varepsilon^\mu \cdot k = 0$ ($k^2 = 0$ for $|k| \gg M_W$)~~

$\varepsilon_\mu \varepsilon^\mu =$

$$\varepsilon_+^\mu k_{+\mu} = \frac{k}{M_W}\cdot Q - \frac{Q}{M_W}\cdot k(\sin^2\theta + \cos^2\theta)$$

$$= 0 \quad \checkmark$$

$$\varepsilon^\mu \varepsilon_\mu = \frac{k^2}{M_W^2} - \frac{Q^2}{M_W^2}\cdot\frac{k^2(\sin^2\theta + \cos^2\theta)}{k^2}$$

$$= \frac{(Q^2 - M_W^2) - Q^2}{M_W^2}$$

$$= -1 \quad \checkmark$$

$$p_e^2 = -k^2\sin^2\theta - (Q - k\cos\theta)^2$$

$$= -k^2 - Q^2 + 2Qk\cos\theta$$

$$\to -2Q^2(1 - \cos\theta)$$

$= t$

$$p_e = p_{\bar{\nu}} - k_+ = (0, \vec{p}_{\bar{\nu}} - \vec{k}_+)$$

~~$p_e^2 = -2\,p_{\bar{\nu}}\cdot k_+ + p_{\bar{\nu}}^2 + k_+^2$~~

~~$\simeq -2Q^2\cos\theta$~~

$$p_e^2 = -[\vec{p}_{\bar{\nu}}^{\,2} + \vec{k}_+^{\,2} - 2\,\vec{p}_{\bar{\nu}}\cdot\vec{k}_+]$$

$$= -2Q^2(1 - \cos\theta) \quad \checkmark$$

$$iA = (-i)^2 \cdot \frac{G_F M_W^2}{\sqrt{2}} \bar{v}_{p'} \not{\epsilon}'^* (1-\gamma_5) \frac{i(\not{p}_e + m_e)}{p_e^2 - m_e^2} \not{\epsilon}_+^* (1-\gamma_5) u_p$$

ignore masses

$$= -\frac{iG_F M_W^2}{\sqrt{2}} \cdot \bar{v}_{p'} \frac{\not{k}_-}{M_W^2} (1-\gamma_5) \frac{\not{p}_e}{p_e^2} \frac{\not{k}_+}{M_W} (1-\gamma_5) u_p$$

$\not{k}_+ \not{\epsilon}_+^* = 0$

$p_e = p_\nu - k^+$

$k^+ = p_\nu - p_e$

$p_e - k^- = p'_{\bar{\nu}}$

$k^- = -p'_{\bar{\nu}} + p_e$

Ignoring neutrino masses

$\not{k}_+ u_p = -\not{p}_e u_p$

$\bar{v}_{p'} \not{k}_- = \bar{v}_{p'} \not{p}_e$

$$iA = \frac{-iG_F}{\sqrt{2}} \bar{v}_{p'} \not{p}_e (1-\gamma_5) \frac{\not{p}_e \not{p}_e (-)}{p_e^2} (1-\gamma_5) u_p$$

$$= +i\sqrt{2} G_F \bar{v}_{p'} \not{p}_e (1-\gamma_5) u_p \qquad \left(\frac{1-\gamma_5}{2} \cdot \frac{1-\gamma_5}{2} = \frac{1-\gamma_5}{2} \right)$$

$$A = \sqrt{2} G_F \bar{v}_{p'} \not{p}_e (1-\gamma_5) u_p$$

$$= \sqrt{2} G_F \bar{v}_{p'} \not{k}' (1-\gamma_5) u_p$$

What is the helicity structure of this amplitude?

Work in χ-basis: $\bar{v}_{p'} k' (1-\gamma_5) u_p$

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma^\mu = (1, \vec{\sigma})$$
$$\bar{\sigma}^\mu = (1, -\vec{\sigma})$$

~~$u(p) = \sqrt{p \cdot \sigma}$~~

$$u(p) = \begin{pmatrix} \sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar{\sigma}} \end{pmatrix} \phi, \quad \phi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$v(p) = \begin{pmatrix} \sqrt{p\cdot\sigma} \\ -\sqrt{p\cdot\bar{\sigma}} \end{pmatrix} \chi \qquad \chi_+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \chi_- = -\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$u_\pm = \sqrt{\frac{E+m}{2}} \begin{pmatrix} \lambda_\pm \phi_\pm \\ \lambda_\mp \phi_\pm \end{pmatrix} \qquad v_\pm = \sqrt{\frac{E+m}{2}} \begin{pmatrix} \lambda_\mp \chi_\pm \\ -\lambda_\pm \chi_\pm \end{pmatrix}$$

$$\lambda_\pm = 1 \mp \frac{p}{E+m}$$

In relativistic limit $\lambda_+ = 0$, $\lambda_- = 2$

$$u_+ = \sqrt{2E} \begin{pmatrix} 0 \\ \phi_+ \end{pmatrix} \qquad v_+ = \sqrt{2E} \begin{pmatrix} \chi_+ \\ 0 \end{pmatrix}$$
$$u_- = \sqrt{2E} \begin{pmatrix} \phi_- \\ 0 \end{pmatrix} \qquad v_- = \sqrt{2E} \begin{pmatrix} 0 \\ -\chi_- \end{pmatrix}$$

$$\bar{v}_+ = \sqrt{2E} (\chi_+ \; 0) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sqrt{2E} (0 \; \chi_+)$$

$$\bar{v}_- = \sqrt{2E} (0 \; -\chi_-) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -\sqrt{2E} (\chi_- \; 0)$$

$\bar{v}_{p'} \not{k}' (1-\gamma_5) u_p$

$1-\gamma_5 = \begin{pmatrix} ? & 0 \\ 0 & 0 \end{pmatrix}$

only $u_{p(-)}$ contributes

$u_{p-} = \sqrt{2E} \begin{pmatrix} \phi_- \\ 0 \end{pmatrix}$

$= \bar{v}_{p'} \begin{pmatrix} 0 & k^\circ - \vec{k}\cdot\vec{\sigma} \\ k^\circ - \vec{k}'\cdot(-\vec{\sigma}) & 0 \end{pmatrix} 2\sqrt{2E} \begin{pmatrix} \phi_- \\ 0 \end{pmatrix}$ $\qquad \vec{k}' = -\vec{k}$

$= 2\sqrt{2E}\, \bar{v}_{p'} \begin{pmatrix} 0 \\ (E - k\vec{\sigma}\cdot\vec{e})\phi_- \end{pmatrix}$ $\qquad \vec{e} = (\sin\theta, 0, \cos\theta]$

The only state contributing is $\bar{v}_{p+}$

$= 2\cdot 2E \begin{pmatrix} 0 & \chi_+ \end{pmatrix} \begin{pmatrix} 0 \\ (E - k\vec{\sigma}\cdot\vec{e})\phi_- \end{pmatrix}$ $\qquad \phi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

So the E-term & σ_z-term will contribute

$= 4E \cdot E (1 + \cos\theta)$

So, the differential cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{|\bar{A}|^2}{32\pi s}$$ (extreme relativistic limit so ignoring all masses)

$$|\bar{A}|^2 = 2G_F^2\, Tr\left[\bar{v}_{p'}\not{p}_e(1-\gamma_5)u(p)\,\bar{u}(p)\gamma^0(1-\gamma_5)(\not{p}_e)^\dagger\gamma^0 v(p')\right]$$

$$= 2G_F^2\, Tr\left[\not{p}_e(1-\gamma_5)\not{p}_\nu(1+\gamma_5)\not{p}_e\not{p}_{\bar{\nu}}\right]$$

$$= 4G_F^2\, Tr\left[(1-\gamma_5)\not{p}_\nu\not{p}_e\not{p}_{\bar{\nu}}\not{p}_e\right]$$

$$= 16G_F^2\left[2\,p_\nu\cdot p_e\; p_{\bar{\nu}}\cdot p_e - p_\nu\cdot p_{\bar{\nu}}\; p_e\cdot p_e\right]$$

$p_e = p_\nu - k_+$

$p_e = k_- - p_{\bar{\nu}}$

$p_e^2 = -2Q^2(1-\cos\theta)$

$$= 16G_F^2\left[2\cdot(p_\nu^2 - p_\nu\cdot k_+)(p_{\bar{\nu}}\cdot k_- - p_{\bar{\nu}}^2) - \frac{1}{2}s(-2Q^2(1-\cos\theta)\right]$$

$$= 16G_F^2\left[-2\,p_\nu\cdot k_+\; p_{\bar{\nu}}\cdot k_- + \frac{1}{4}s^2(1-\cos\theta)\right]$$

$$p_\nu\cdot k_+ = \frac{1}{2}\left[(p_\nu - k_+)^2 - p_\nu^2 - k_+^2\right]$$

$$= \frac{1}{2}\left[-2\vec{p}_\nu\cdot\vec{k}_+ - 0\right]$$

$$= -Q^2\cos\theta$$

$$p_\nu\cdot k_+ = Q^2 - \vec{p}_\nu\cdot\vec{k}_+$$

$$= Q^2(1-\cos\theta)$$

$$p_{\bar{\nu}}\cdot k_- = Q^2 - \vec{p}_{\bar{\nu}}\cdot\vec{k}_-$$

$$= Q^2(1-\cos\theta)$$

$$= 16G_F^2\left[-2Q^4(1-\cos\theta)^2 + 4Q^4(1-\cos\theta)\right]$$

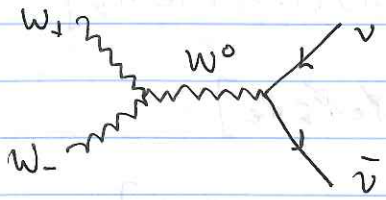
$$= 32G_F^2\left[\underbrace{2(1-\cos\theta) - (1-2\cos\theta+\cos^2\theta}_{1-\cos^2\theta}\right]Q^4$$

$$= 32G_F^2\frac{s^2}{16}\sin^2\theta$$

$$\frac{d\sigma}{d\cos\theta} = \frac{2G_F^2 s^2 \sin^2\theta}{32\pi s}$$

so again, we have a unitarity problem

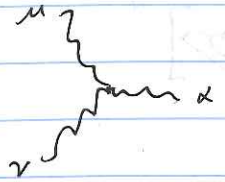
There is another process which can contribute to this scattering.



If a neutral vector boson is added, with a 3-boson vertex, this also contributes.

Requiring unitarity in this and other amplitudes leads to algebraic relations

~~$[J^-, J^+] \sim J^0$~~ $\quad [J^+, J^-] = 2J^0$



$$L_{\alpha\mu\nu} = -ig\left[(k'-k)_\alpha g_{\mu\nu} - (2k'+k)_\nu g_{\alpha\mu} + (2k+k')_\mu g_{\alpha\nu}\right]$$

This is the structure of non-abelian gauge fields.

This is what led Glashow to propose a theory of massive $SU(2)\times U(1)$ gauge bosons.

This theory still had trouble w/ preserving gauge invariance & renormalizability.

Weinberg & Salam independently proposed using the Higgs mechanism to generate the W, Z masses, with only couplings to left handed fermions.

$$e_L = \frac{1-\gamma_5}{2}\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad e_R = \frac{1+\gamma_5}{2} e$$

so the left-handed fields form an SU(2) doublet of weak flavor, while the right handed neutrino doesn't couple to the weak interactions and since it's mass is so small, the right handed neutrino was initially left of of the SM.

In contrast, the right-handed electron is kept, and it is a singlet under this weak charge.

Recal from SU(3) quark model, the electric charge was given by

$$Q = I_3 + \frac{Y}{2}$$

I_3 = 3rd component of isospin

Y = hypercharge

We are going to do the same thing ~~for~~ with the Weak interactions, but the Weak Isospin & Weak hypercharge will be gauged symmetries

For the leptons, we have

$$Y_L = -1 \qquad Y_R = -2$$
$$I_L = \frac{1}{2} \qquad I_R = 0 \quad \Rightarrow$$
$$I_{L,3} = \pm\frac{1}{2}$$

$$Q_L = \begin{cases} 0 \\ -1 \end{cases}$$
$$Q_R = -1$$